Joint Design and Control of Electric Vehicle Propulsion Systems

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***Abstract*— This paper presents models and optimization methods for the design of electric vehicle propulsion systems. Specifically, we first derive a bi-convex model of a battery electric powertrain including the transmission and explicitly accounting for the impact of its components' size on the energy consumption of the vehicle. Second, we formulate the energy-optimal sizing and control problem for a given driving cycle and solve it as a sequence of second-order conic programs. Finally, we present a real-world case study for heavy-duty electric trucks, comparing a single-gear transmission with a continuously variable transmission (CVT), and validate our approach with respect to state-of-the-art particle swarm optimization algorithms. Our results show that, depending on the electric motor technology, CVTs can reduce the energy consumption and the battery size of electric trucks between up to 10%, and shrink the electric motor up to 50%.**

I. Introduction

THE ROAD transportation sector is undergoing an extensive electrification process. Battery electric vehicles (BEVs) are finding their way into the passenger car and bus market, and sales are rapidly increasing [1]. Nevertheless, this process does not yet affect the heavy-duty transportation sector to the same degree, as the development of battery electric trucks is still focused on feasibility studies in terms of economical viability [2]–[4] and technological research [5], [6]. In this context, design studies investigating the deployment of battery electric trucks play a crucial role in defining a technological road-map for the electrification of heavy-duty road transport. Specifically, the optimal powertrain design problem is particularly critical due to the high costs entailed by the operation of freight transportation vehicles in terms of energy consumption and load capacity (due maximum weight regulations, reducing the empty-vehicle's mass results in a higher freight capacity). Given a powertrain topology, this problem consists of finding the optimal components' size minimizing a cost (for instance, energy consumption) conditional on the intended usage. This calls for numerical methods that optimize the sizing of the battery, the electric machines and the transmission together with the overall powertrain operation strategies in an integrated fashion. In this paper, we present a general bi-convex optimization framework to jointly compute the optimal powertrain design and control strategies for generic BEVs and showcase it on heavy-duty trucks.

Literature review: The state-of-the-art for powertrain design mainly consists of nonlinear optimization methods and convex optimization approaches. Critically, the first class of methods sacrifices computational tractability and global optimality guarantees for the sake of model accuracy, whilst the second one approximates the models for the sake of computational time and theoretical optimality guarantees. In the following, we revise them in turn.

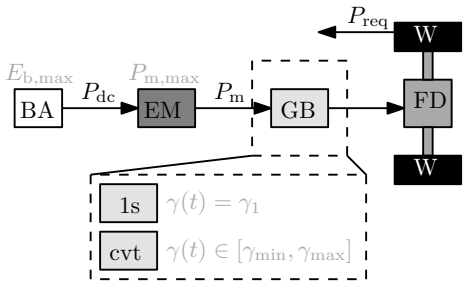

Fig. 1. Overview of the battery electric powertrain studied including a battery (BA), an electric machine (EM), a gearbox/transmission (GB), and a final drive (FD) connected to the rear wheels (W) of the vehicle. The parameters in gray indicate the design parameters of each component, namely, battery capacity, electric motor power, and gear-ratio (for the single-gear transmission) and minimum and maximum ratio (for the CVT).

The first class of methods combines high-fidelity (often map-based) simulation models with derivative-free optimization methods, such as particle swarm optimization (PSO) algorithms. Such methods have been extensively applied to battery and motor sizing problems for hybrid electric vehicles [7], [8] and BEVs [9], sometimes also including a transmission consisting of a multi-speed gearbox or of a continuously variable transmission (CVT) [10], [11]. However, these methodologies do not provide global optimality guarantees and must rely on a large number of simulations usually entailing high computation times.

A second class of methods leverages convex optimization algorithms. Overall, these methodologies rely on model approximations and relaxations to accommodate the problem in convex optimization frameworks. They have the advantage that the solution can be computed in polynomial time and is guaranteed to be globally optimal. These approaches have been extensively applied to compute the fuel-optimal control strategies for hybrid electric vehicles [12]–[14], sometimes also optimizing the size of the battery, the engine and the motor [15]–[17]. Nevertheless, they consider the transmission design to be fixed and treat its operation as a pre-computed exogenous signal (which sometimes is separately optimized in an iterative, multi-level fashion). Therefore, to the best of the authors' knowledge, there are no convex optimization frameworks to jointly design electric powertrains, including the transmission, and optimize their operation in an integrated fashion.

Statement of contributions: To bridge this gap, this paper

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presents a model to jointly optimize the design and the operation of the battery electric powertrain shown in Fig. 1, and our contribution is threefold: First, we formulate the energy-optimal design and operation problem for a given driving cycle in a bi-convex form, whereby we identify an electric motor model that is convex with respect to power and speed, while still capturing its map characteristics in detail, and two types of transmission technologies: a single-gear transmission and a CVT. Second, solving the problem as a sequence of second-order conic programs (SOCPs), we compute the globally optimal solution comprising the components' sizing and the operation of the electric motor and the CVT, when present. Finally, we apply our approach to design the electric powertrain of the heavy-duty truck, considering both a single-gear transmission and a CVT, and compare it with state-of-the-art PSO methods [11]. Our results show that, compared to a single-gear configuration, a CVT can improve the energy consumption between 1% and 10% and allow to reduce the electric motor size by 20% to 50%, depending on the electric motor technology, paving the way to extensive design studies for different powertrain architectures.

Organization: The remainder of this paper is structured as follows: We identify a bi-convex model of a BEV and formulate the energy-optimal design and operation problem in Section II. Section III presents numerical results for both types of transmission on different driving cycles. We conclude the paper in Section IV with a discussion and an outlook on future research directions.

II. Methodology

This section introduces a bi-convex optimization approach to jointly optimize the components' sizing and the powertrain controls of the BEV shown in Fig. 1. We first present a model of the BEV and its transmission in Section II-A. Second, we model the electric motor and the battery in Sections II-B and II-C, respectively, and present a model for the components' mass in Section II-D. Finally, we formulate the energy-optimal sizing and operation problem in Section II-E and discuss our modeling assumptions and their limitations in Section II-F.

A. Vehicle and Transmission

In line with common practices we use the quasi-static modeling approach of [18]. For the sake of simplicity, we drop dependence on time whenever it is clear from the context. Consider a given driving cycle consisting of an exogenous speed trajectory $v(t)$, acceleration trajectory $a(t)$ and road grade trajectory $\alpha(t)$. Accounting only for the longitudinal dynamics of the vehicle, whereby lateral effects such as crosswinds and turning are neglected, the power required to drive P_{req} consists of the drag power resulting from the aerodynamic resistance, rolling friction and gravitational force, and the inertial power as

$$\begin{aligned} P_{\mathrm{req}} =& m_{\mathrm{v}} \cdot (c_{\mathrm{r}} \cdot g \cdot \cos(\alpha) + g \cdot \sin(\alpha) + a) \cdot v \\ &+ \frac{1}{2} \cdot \rho \cdot c_{\mathrm{d}} \cdot A_{\mathrm{f}} \cdot v^3, \end{aligned} \tag{1}$$

where m_{v} is the total mass of the vehicle subject to optimization, c_{r} the rolling friction coefficient, g the gravitational acceleration, ρ the air density, c_{d} the aerodynamic drag coefficient and A_{f} the frontal area of the vehicle. Given the transmission ratio γ subject to optimization, the speed of the electric motor is

$$\omega = \gamma \cdot \frac{v \cdot \gamma_{\mathrm{f}}}{r_{\mathrm{w}}}, \tag{2}$$

where γ_{f} is the fixed transmission ratio of the final drive and r_{w} is the wheels' radius. For the transmission ratio it holds

$$\gamma(t) \begin{cases} = \gamma_1 & \text{if single-gear} \\ \in [\gamma_{\min}, \gamma_{\max}] & \text{if CVT} \end{cases} \quad \forall t, \tag{3}$$

where $\gamma_1 > 0$ is the fixed ratio of the single-gear, and $\gamma_{\min} > 0$ and $\gamma_{\max} > 0$ are the minimum and maximum transmission ratios achievable by the CVT. Considering a fixed final-drive and transmission efficiency η_{f} and η_{g}, respectively, the mechanical power provided by the motor P_{m} is related to the requested power as

$$P_{\mathrm{req}} = \begin{cases} \eta_{\mathrm{f}} \cdot \eta_{\mathrm{g}} \cdot P_{\mathrm{m}} & \text{if } P_{\mathrm{m}} \geq 0 \\ \frac{1}{\eta_{\mathrm{f}} \cdot \eta_{\mathrm{g}} \cdot r_{\mathrm{brk}}} \cdot P_{\mathrm{m}} - P_{\mathrm{brk}} & \text{if } P_{\mathrm{m}} < 0, \end{cases}$$

where r_{brk} is the fraction of braking power that the electric motor can exert via the rear axle of the vehicle without destabilizing the vehicle and the power exerted by the mechanical brakes is $P_{\mathrm{brk}} \geq 0$. Therefore, the requested power can be relaxed to

$$\begin{aligned} P_{\mathrm{req}} &\leq \eta_{\mathrm{f}} \cdot \eta_{\mathrm{g}} \cdot P_{\mathrm{m}} \\ P_{\mathrm{req}} &\leq \frac{1}{\eta_{\mathrm{f}} \cdot \eta_{\mathrm{g}} \cdot r_{\mathrm{brk}}} \cdot P_{\mathrm{m}}. \end{aligned} \tag{4}$$

Finally, we enforce the vehicle to be able to start driving with a road grade of α_0 as

$$m_{\mathrm{v}} \cdot g \cdot \sin(\alpha_0) \cdot r_{\mathrm{w}} \leq \eta_{\mathrm{f}} \cdot \eta_{\mathrm{g}} \cdot T_{\mathrm{m,max}} \cdot \begin{cases} \gamma_1 & \text{if single-gear} \\ \gamma_{\max} & \text{if CVT} \end{cases} \tag{5}$$

where $T_{\mathrm{m,max}}$ is the maximum electric motor torque.

B. Electric Motor

Starting from a standard DC circuit model jointly capturing the motor and the inverter, we relate the electric power provided P_{dc} to a corrected mechanical power $P_{\mathrm{m,corr}}$ as

$$P_{\mathrm{dc}} = \frac{P_{\mathrm{m}}^2}{P_{\mathrm{m,eff}}} + P_{\mathrm{m,corr}},$$

where $P_{\mathrm{m,eff}}$ is a speed-dependent quadratic efficiency term accounting for electric losses, and the corrected mechanical power captures speed-dependent friction and linear efficiency terms as

$$P_{\mathrm{m,corr}} = c_0(t) + c_1(t) \cdot \omega + c_2(t) \cdot \omega^2 + \eta_{\mathrm{m}}(t) \cdot P_{\mathrm{m}},$$

whereby it holds that

$$\big(\{c_i(t)\}_i, \eta_{\mathrm{m}}(t)\big) = \begin{cases} \big(\{c_i^+\}_i, \eta_{\mathrm{m}}^+\big) & \text{if } P_{\mathrm{m}}(t) > 0 \\ \big(\{c_i^-\}_i, \eta_{\mathrm{m}}^-\big) & \text{if } P_{\mathrm{m}}(t) < 0 \\ \big(0, 0\big) & \text{if } P_{\mathrm{m}}(t) = 0. \end{cases}$$

Since the electric motor is the only mover of the powertrain, the sign of the motor power can be directly assessed in the pre-processing phase. Specifically, fixing the vehicle mass to the base value $\bar{m}_v$ (whereby we use the notation $\bar{(\cdot)}$ to indicate the original size of the components), we can compute an exogenous requested power trajectory $\bar{P}_{\text{req}}$ from (1). This way, we can pre-compute the value of the coefficients of (8) as exogenous functions of time as

$$\big(\{c_i(t)\}_i, \eta_{\text{m}}(t)\big) = \begin{cases} \big(\{c_i^+\}_i, \eta_{\text{m}}^+\big) & \text{if } \bar{P}_{\text{req}}(t) > 0 \\ \big(\{c_i^-\}_i, \eta_{\text{m}}^-\big) & \text{if } \bar{P}_{\text{req}}(t) < 0 \\ (0, 0) & \text{if } \bar{P}_{\text{req}}(t) = 0. \end{cases} \quad (6)$$

Similar as in [19], we use lossless relaxations to transform the model to a convex form using second-order conic constraints. We relax the electric motor power equation to

$$(P_{\text{dc}} - P_{\text{m,corr}}) \cdot P_{\text{m,eff}} \geq P_{\text{m}}^2$$

and formulate it as a second-order conic constraint as

$$\left\| \begin{matrix} P_{\text{dc}} - P_{\text{m,corr}} - P_{\text{m,eff}} \\ 2 \cdot P_{\text{m}} \end{matrix} \right\|_2 \leq P_{\text{dc}} - P_{\text{m,corr}} + P_{\text{m,eff}}. \quad (7)$$

Since our objective is to minimize energy consumption, constraint (7) will always hold with equality, as it is inefficient to pick a higher value of P_{dc} (this statement holds also for other types of cost, such as money or lap time in racing applications). Following a similar reasoning, we relax the corrected mechanical power as

$$P_{\text{m,corr}} \geq c_0(t) + c_1(t) \cdot \omega + c_2(t) \cdot \omega^2 + \eta_{\text{m}}(t) \cdot P_{\text{m}}. \quad (8)$$

For the sake of brevity, in the remainder of the paper we abstain from showing why the relaxations performed are lossless, as the explanation follows from the same rationale. We assume the efficiency-loss power to be piecewise affine in the motor speed as

$$P_{\text{m,eff}} = a_{\text{m}}^k \cdot \omega + b_{\text{m}}^k \quad \text{if } \omega \in \left[\omega^{k-1}, \omega^k\right] \ \forall k \in [1, ..., K],$$

where $a_{\text{m}}^k \geq a_{\text{m}}^{k+1} \, \forall k = [1, ..., K-1]$ and K is the number of affine lines. Our model is fitted to the measured motor map [20] with a root mean squared error (RMSE) of 1.45%. The original efficiency map of a permanent magnet electric machine is compared with our convex approximation in Fig. 2. To preserve convexity, we relax the efficiency power equation to

$$P_{\text{m,eff}} \leq a_{\text{m}}^k \cdot \omega + b_{\text{m}}^k \quad \forall k \in [1, ..., K]. \quad (9)$$

The motor power is limited by the maximum power $P_{\text{m,max}}$ as well as by the maximum torque $T_{\text{m,max}}$ as

$$\begin{aligned} P_{\text{m}} &\in [-P_{\text{m,max}}, P_{\text{m,max}}] \\ P_{\text{m}} &\in [-T_{\text{m,max}}, T_{\text{m,max}}] \cdot \omega. \end{aligned} \quad (10)$$

Furthermore, the motor speed is constrained as

$$\omega \in [0, \omega_{\text{max}}]. \quad (11)$$

Finally, to keep the motor map consistent, we scale the motor size as a function of the maximum power $P_{\text{m,max}}$ while keeping the intersection between the maximum torque and maximum power lines at a constant speed:

$$\begin{aligned} T_{\text{m,max}} &= \bar{T}_{\text{m,max}} \cdot \frac{P_{\text{m,max}}}{\bar{P}_{\text{m,max}}} \\ c_i^{\pm} &= \bar{c}_i^{\pm} \cdot \frac{P_{\text{m,max}}}{\bar{P}_{\text{m,max}}} && \forall i \in \{1, 2, 3\} \\ a_{\text{m}}^k &= \bar{a}_{\text{m}}^k \cdot \frac{P_{\text{m,max}}}{\bar{P}_{\text{m,max}}} && \forall k \in \{1, \dots, K\} \\ b_{\text{m}}^k &= \bar{b}_{\text{m}}^k \cdot \frac{P_{\text{m,max}}}{\bar{P}_{\text{m,max}}} && \forall k \in \{1, \dots, K\}. \end{aligned} \quad (12)$$

This way, the electric motor model is convex for a given $P_{\text{m,max}}$.

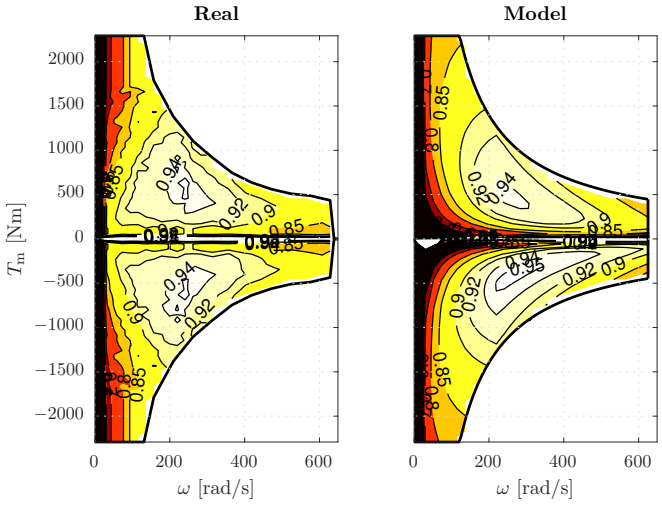

Fig. 2. Electric motor efficiency map (including the inverter) from real data [20] (left) and fitted convex model (right).

C. Battery

We model the battery as a DC circuit model with state of energy E_{b}, resistance R and open-circuit voltage U_{oc}. The power extracted at the terminal P_{b} is

$$P_{\text{b}} = P_{\text{dc}} + P_{\text{a}}, \quad (13)$$

where P_{a} is a constant auxiliary power. The terminal power is related to the internal battery power P_{i} through the open-circuit voltage U_{oc} and the internal resistance R as

$$P_{\text{i}} = P_{\text{b}} + \frac{R}{U_{\text{oc}}^2} \cdot P_{\text{i}}^2$$

that can be written using the open-circuit power $P_{\text{oc}} = \frac{U_{\text{oc}}^2}{R}$ as

$$(P_{\text{i}} - P_{\text{b}}) \cdot P_{\text{oc}} = P_{\text{i}}^2.$$

Assuming that the battery capacity $E_{\text{b,max}}$ is changed by modifying the number of cells' strings *in parallel*, while keeping the number of cells per string constant, the open-circuit voltage is not affected by the battery size and can be expressed as $U_{\text{oc}} = u_1 \cdot \frac{E_{\text{b}}}{E_{\text{b,max}}} + u_0$, with $u_1 > 0$ and $u_0 > 0$, and the internal resistance is $R = R_0 \cdot \frac{\bar{E}_{\text{b,max}}}{E_{\text{b,max}}}$, with $R_0 > 0$

and $\bar{E}_{\mathrm{b,max}}$ denoting a base battery capacity. Therefore, we get that

$$\begin{aligned} P_{\mathrm{oc}} =& \left(u_1 \cdot \frac{E_{\mathrm{b}}}{E_{\mathrm{b,max}}} + u_0 \right)^2 \cdot \frac{E_{\mathrm{b,max}}}{R_0 \cdot \bar{E}_{\mathrm{b,max}}} \\ =& \frac{u_1^2 \cdot \frac{E_{\mathrm{b}}^2}{E_{\mathrm{b,max}}} + 2 \cdot u_1 \cdot u_0 \cdot E_{\mathrm{b}} + u_0^2 \cdot E_{\mathrm{b,max}}}{R_0 \cdot \bar{E}_{\mathrm{b,max}}} \end{aligned}$$

Since it usually holds that the open-circuit voltage slope u_1 is significantly smaller than its base value u_0, we approximate $u_1^2 \approx 0$. This way, the open-circuit power becomes

$$P_{\mathrm{oc}} = a_{\mathrm{b}} \cdot E_{\mathrm{b}} + b_{\mathrm{b}} \cdot E_{\mathrm{b,max}}, \tag{14}$$

where $a_{\mathrm{b}} > 0$ and $b_{\mathrm{b}} > 0$ are two given parameters subject to identification. Similarly as for the electric motor, we relax the battery power equation to

$$(P_{\mathrm{i}} - P_{\mathrm{b}}) \cdot P_{\mathrm{oc}}(E_{\mathrm{b}}) \geq P_{\mathrm{i}}^2,$$

which can be expressed as a second-order conic constraint as

$$\left\| \begin{matrix} P_{\mathrm{i}} - P_{\mathrm{b}} - P_{\mathrm{oc}} \\ 2 \cdot P_{\mathrm{i}} \end{matrix} \right\|_2 \leq P_{\mathrm{i}} - P_{\mathrm{b}} + P_{\mathrm{oc}}. \tag{15}$$

The internal battery power is limited as

$$P_{\mathrm{i}} \in [-P_{\mathrm{i,max}}, P_{\mathrm{i,max}}], \tag{16}$$

whereby

$$P_{\mathrm{i,max}} = a_{\mathrm{b,max}} \cdot E_{\mathrm{b}} + b_{\mathrm{b,max}} \cdot E_{\mathrm{b,max}}, \tag{17}$$

where $a_{\mathrm{b,max}} > 0$ and $b_{\mathrm{b,max}} > 0$ are two given parameters subject to identification. The battery state of energy is limited by the battery size $E_{\mathrm{b,max}}$ as

$$E_{\mathrm{b}} \in [r_{\mathrm{b,min}}, r_{\mathrm{b,max}}] \cdot E_{\mathrm{b,max}}, \tag{18}$$

where $r_{\mathrm{b,min}}$ and $r_{\mathrm{b,max}}$ represent the relative minimum and maximum state of energy levels allowed. Finally, the battery dynamics are given by

$$\frac{\mathrm{d}}{\mathrm{d}t} E_{\mathrm{b}} = -P_{\mathrm{i}}. \tag{19}$$

The proposed model is fitted to the original Lithium-ion battery data used in [11] with a RMSE of 0.84%.

D. Mass

The vehicle is assumed to have a total mass m_{v} comprising a base mass m_0 accounting for structure, fixed components and load, the motor mass m_{m}, the battery mass m_{b} and the gearbox mass m_{g}, i.e.,

$$m_{\mathrm{v}} = m_0 + m_{\mathrm{m}} + m_{\mathrm{b}} + m_{\mathrm{g}}. \tag{20}$$

In line with current practices [21], we model the mass of the motor to be linear in its maximum power as

$$m_{\mathrm{m}} = \rho_{\mathrm{m}} \cdot P_{\mathrm{m,max}}, \tag{21}$$

where ρ_{m} represents the power-specific mass of the motor also including the mass of the inverter. As we modify the battery by changing the number of cells *in parallel*, which linearly affects the battery capacity $E_{\mathrm{b,max}}$, we can assume the mass of the battery to be

$$m_{\mathrm{b}} = \rho_{\mathrm{b}} \cdot E_{\mathrm{b,max}}, \tag{22}$$

where ρ_{b} represents its energy-specific mass. Finally, we assume the transmission mass m_{g} to be quadratic in the transmission ratio as

$$m_{\mathrm{g}} = \begin{cases} \rho_{\mathrm{g}} \cdot \gamma_1^2 & \text{if single-gear} \\ m_{\mathrm{cvt},0} + \rho_{\mathrm{cvt}} \cdot \gamma_{\mathrm{max}}^2 & \text{if CVT,} \end{cases}$$

where ρ_{g}, $m_{\mathrm{cvt},0}$ and ρ_{cvt} are used to model the mass of the single-gear transmission and the CVT, respectively. Finally, we relax the mass of the gearbox as

$$m_{\mathrm{g}} \geq \begin{cases} \rho_{\mathrm{g}} \cdot \gamma_1^2 & \text{if single-gear,} \\ m_{\mathrm{cvt},0} + \rho_{\mathrm{cvt}} \cdot \gamma_{\mathrm{max}}^2 & \text{if CVT.} \end{cases} \tag{23}$$

E. Energy-optimal Sizing and Operation Problem

As the objective of the optimal sizing and control problem we choose the energy consumption over the driving cycle, i.e.,

$$J(E_{\mathrm{b}}) = E_{\mathrm{b}}(0) - E_{\mathrm{b}}(t_{\mathrm{f}}), \tag{24}$$

where t_{f} is the length of the driving cycle. We state the energy-optimal sizing and operation problem as follows:

Problem 1 (Energy-optimal Sizing and Operation Problem)**.** *Given the battery electric powertrain architecture shown in Fig. 1, the optimal components' size and control strategies are the solution of*

$$\begin{aligned} &\min_{x_{\mathrm{p}}, x_{\mathrm{c}}} \; J(E_{\mathrm{b}}) \\ &\textit{s.t.} \; (1)\text{–}(24), \end{aligned}$$

with the sizing parameters $x_{\mathrm{p}} = \{P_{\mathrm{m,max}}, E_{\mathrm{b,max}}, \gamma_{\mathrm{x}}\}$*, where* $\mathrm{x} = 1$ *for the single-gear and* $\mathrm{x} \in \{\min, \max\}$ *for the CVT. The control variables are* $x_{\mathrm{c}} = \{P_{\mathrm{m}}, \gamma\}$ *(where* γ *is present for the CVT only).*

Problem 1 is bi-convex in $P_{\mathrm{m,max}}$ and $\{x_{\mathrm{p}}, x_{\mathrm{c}}\} \backslash \{P_{\mathrm{m,max}}\}$. In particular, for a given value of $P_{\mathrm{m,max}}$, Problem 1 is a SOCP in $\{x_{\mathrm{p}}, x_{\mathrm{c}}\} \backslash \{P_{\mathrm{m,max}}\}$. Since $P_{\mathrm{m,max}}$ is a scalar variable, Problem 1 can be solved computing the optimal solution for a range of given values of $P_{\mathrm{m,max}}$ as a sequence of SOCPs. Critically, if the chosen range is fine enough, the solution found is globally optimal. Alternative methods could directly leverage the problem structure [22]. Yet, this approach also delivers a sensitivity analysis on the motor size which can be used for validation purposes, as is done in Section III-C.

F. Discussion

A few comments are in order. First, we consider a given driving cycle whereby the vehicle speed trajectory is exogenous. This approach is in line with most of the literature related to the control of hybrid electric vehicles [18] and widely used in sizing problems [7]. Second, we use a pre-computed requested power trajectory based on a guess of the overall vehicle's mass to assess when the electric motor will be operated in motor or generator mode. This assumption

TABLE I

VEHICLE PARAMETERS.

Parameter	Symbol	Truck	Unit
Base vehicle weight	m_0	37.900	kg
Wheel radius	r_{w}	0.492	m
Rear axle brake fraction	r_{brk}	0.25	-
Final drive ratio	γ_{f}	1	-
Air drag coefficient	c_{d}	0.73	-
Frontal area	A_{f}	9.75	m^2
Rolling resistance coefficient	c_{r}	0.006	-
Air density	ρ	1.225	$\mathrm{kg/m}^3$
Gravitational constant	g	9.81	$\mathrm{m/s}^2$
Constant auxiliary power	P_{a}	4.86	kW
Transmission efficiency	η_g	0.97	-
Final drive efficiency	η_f	0.97	-
Battery energy-specific mass	ρ_{b}	6.7	kg/kWh
EM power-specific mass	ρ_{m}	0.9	kg/kW
Single gear density	ρ_{g}	1.68	kg
CVT density	ρ_{cvt}	1	kg
CVT base mass	$m_{\mathrm{cvt},0}$	50	kg

is in order for standard driving cycles, whereby the sign of the requested power is mostly well-defined, and especially well-suited for heavy-duty applications, as the base mass is significantly larger than the mass of the powertrain. Third, according to (12), we scale the electric motor by scaling both the maximum torque $T_{\mathrm{m,max}}$ line and the efficiency map linearly in the maximum power $P_{\mathrm{m,max}}$. Since the maximum torque is proportional to the motor length, this enables us to scale the motor mass linearly in $P_{\mathrm{m,max}}$. We leave to future research the problem of modifying the motor characteristics such as maximum speed, maximum torque and efficiency map in a separate fashion. Fourth, we modify the battery size only by changing the number of cells *in parallel*. Changing the arrangement of the battery cells also *in series* to future research. Finally, the convex relaxations proposed make the optimization Problem 1 *non-physical*. However, as the objective is to minimize the energy consumption (24), the optimal solution will have the constraints holding with equality, as it would be suboptimal otherwise, guaranteeing that the relaxations are lossless and the optimal solution *physical*.

III. RESULTS

This section presents the results obtained by solving Problem 1 for the different powertrain configurations shown in Fig. 1, namely, a battery, a motor and a single-gear or a CVT. We first outline the experiments in Section III-A, present our results and compare them with a state-of-the-art PSO method [11] in Section III-B, and discuss their validity in Section III-C. Note that the PSO method is applied to the nonlinear version of the model described in [11] and not to the convexified model described in this paper, which is based on the nonlinear model of [11].

A. Experimental Design

We consider the same heavy-duty battery electric truck as in [11]. The parameters of the vehicle are given in Table I. We consider two different 100 km long driving cycles: a shortened version of a *long-haul* driving cycle, and a *delivery* driving cycle comprising extra urban routes. Both cycles are obtained from the Vehicle Energy consumption Calculation Tool (VECTO) [23] and their velocity and slope profiles are shown in Fig. 3.

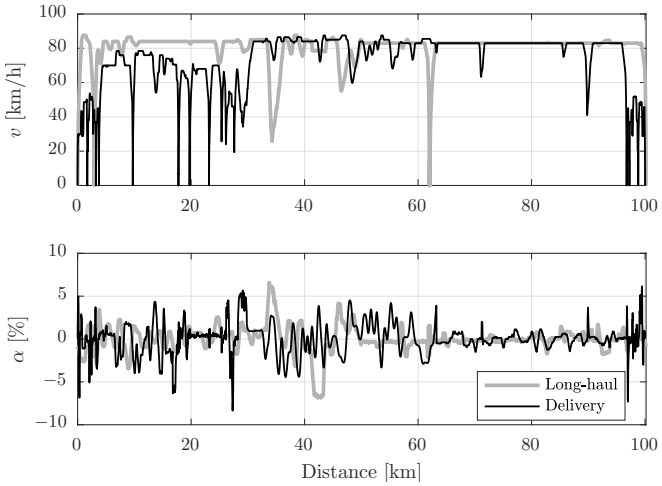

Fig. 3. Speed and grade profile of the VECTO regional long-haul cycle and the VECTO regional delivery cycle.

TABLE II

RESULTS FOR THE LONG-HAUL AND THE DELIVERY CYCLE.

Method	Bi-convex			PSO		
Transmission	1s	CVT		1s	CVT	
Long-haul cycle						
J [kWh]	175.9	174.4	-0.8%	177.3	174.7	-1.4%
$E_{\mathrm{b,max}}$ [kWh]	294	291	-0.8%	295	291	-1.4%
$P_{\mathrm{m,max}}$ [kW]	460	350	-23.9%	481	433	-10.1%
$\gamma_1/\gamma_{\mathrm{min}}$ [-]	7.04	4.51		6.03	3.14	
γ_{max} [-]	-	11.17		-	11.77	
m_{v} [kg]	40366	40342	-0.1%	40371	40427	+0.1%
Delivery cycle						
J [kWh]	192.8	190.8	-1.1%	195.6	192.2	-1.8%
$E_{\mathrm{b,max}}$ [kWh]	322	318	-1.1%	325	320	-1.7%
$P_{\mathrm{m,max}}$ [kW]	440	380	-13.6%	547	483	-11.8%
$\gamma_1/\gamma_{\mathrm{min}}$ [-]	7.36	4.51		7.51	2.44	
γ_{max} [-]	-	12.37		-	12.85	
m_{v} [kg]	40541	40577	+0.1%	40665	40691	+0.1%

B. Numerical Results

In line with current practices, for each configuration and driving cycle we discretize Problem 1 using the Euler forward method with a 1 s sampling time. Thereafter, we parse it with YALMIP [24] and solve it using ECOS [25]. Specifically, we solved Problem 1 for fixed values of $P_{\mathrm{m,max}}$ ranging between 300 kW and 600 kW. The solver took about 3 s per solution. Given the chosen discretization of 10 kW, the resulting computational time is about one and a half minutes.

Table II shows the results obtained with the proposed bi-convex approach and the PSO method from [11] (that took about 20–30 min per case to converge). The results of the PSO method differ from the results presented in [11] because different battery boundaries are used here.

Both approaches obtain similar results on both driving cycles in terms of energy consumption suggesting that a CVT can improve the minimum energy consumption by about 1%, in keeping with the results of the study in [11]. This result

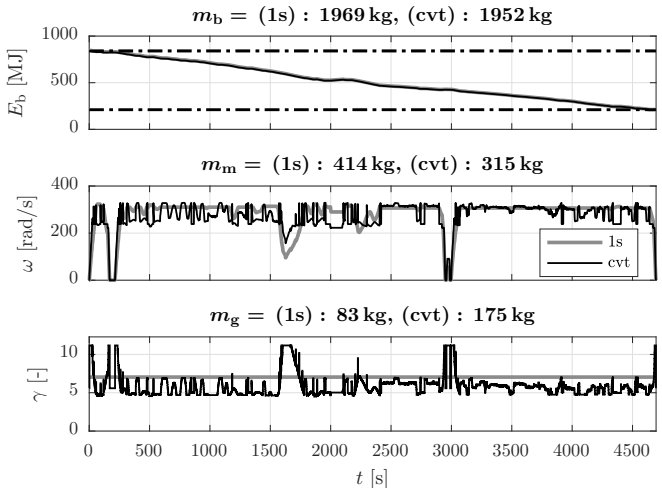

Fig. 4. Optimal solution of the convex method for the single-gear (1s) and CVT (cvt) on the VECTO regional long-haul cycle. The first plot shows the battery trajectory over the drive cycle, the second plot shows the rotational speed of the motor and the last plot shows the transmission ratio.

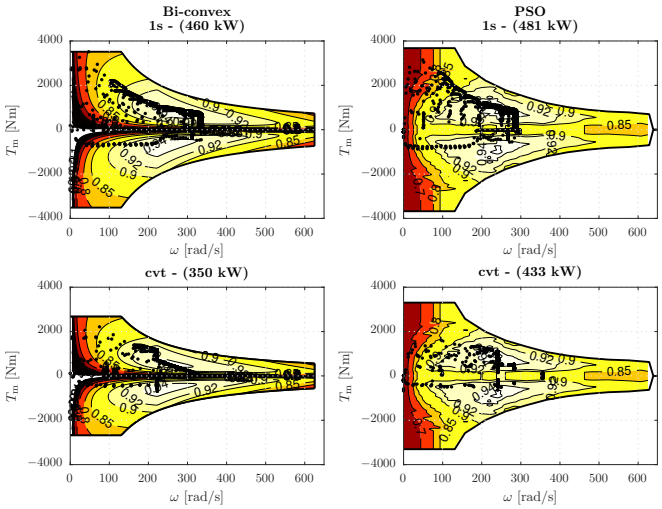

Fig. 5. Workpoints, indicated with black dots, of the electric machine on the VECTO regional long-haul cycle in case of a single-gear (1s) and CVT (cvt) transmission. On the left for the optimal solution of the bi-convex method, and on the right for the PSO method. The plot includes the maximum and minimum torque lines of the electric machines in black

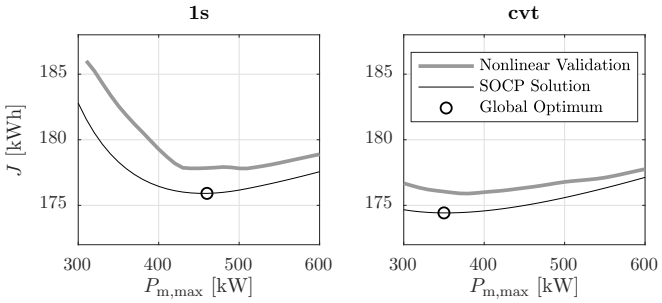

Fig. 6. Solution of Problem 1 for different values of $P_{\mathrm{m,max}}$ (SOCP Solution) and validation through simulation in the nonlinear model (Nonlinear Validation) for the long-haul driving cycle. For both the single-gear (1s) and the CVT the nonlinear model simulation results in slightly higher values. Yet, the trends are in line with each other.

can be ascribed to the fact that a CVT can operate the motor in a more efficient way by changing its operational speed as shown in Fig. 4 for the long-haul cycle. The optimal motor operation enabled by the CVT and the single-gear is shown in detail in Fig. 5 for the long-haul cycle, for both optimization methods, indicating that when a CVT is present the workpoints of the EM are located closer to the region of high efficiency of the electric machine.

In terms of optimal component sizing, we find similar battery sizes whose difference corresponds to the difference in energy consumption. Regarding the maximum motor size, the bi-convex model results in a smaller motor w.r.t. the PSO approach. Yet, for both driving cycles both models show that the CVT enables to shrink the electric machine. Partly related to the difference in electric machine sizes, the PSO finds a larger optimal ratio spread for the CVT, i.e., a larger difference between $\gamma_{\min}$ and $\gamma_{\max}$. Finally, the total vehicle's mass m_{v} is not significantly influenced, as the increase in transmission mass resulting from the CVT is close to the reduction in powertrain mass resulting from a smaller battery and electric machine.

C. Validation

To further validate our approach, we simulate the optimal solution found by the bi-convex method in the nonlinear model. The optimal single-gear solution achieved an energy consumption that was higher by 1.1% (long-haul cycle) and 1.9% (delivery cycle), whilst the consumption with the optimal solution for the CVT configuration increased the consumption by 0.4% (long-haul cycle) and 1.4% (delivery cycle). Overall, the results shown in Fig. 6 for different values of $P_{\mathrm{m,max}}$ on the long-haul cycle are not completely in line due to inevitable model inconsistencies. Yet, our model is able to capture the overall *relative* changes. Moreover, the energy consumption benefits obtained when comparing the optimal CVT configuration with the optimal single-gear transmission in the nonlinear simulator are 1.5% (long-haul cycle) and 1.6% (delivery cycle), and correspond to the findings of the PSO approach.

D. Electric Motor Technology

Since the impact of the electric motor technology can be significant [26], we leverage our method to investigate a different motor. Specifically, we perform the same study as in Section III-B for another permanent magnet electric motor [27] on the long-haul cycle. Overall, this motor results in a higher consumption when compared to the one from [20] due to its lower efficiency. Interestingly, in this case the CVT can significantly improve the overall results compared to the single-gear configuration. Specifically, the achievable energy consumption drops from 198.6 kWh to 179.9 kWh by more than 10% (with the PSO approach resulting in an 8% reduction), whilst the motor shrinks from 500 kW to 260 kW by 48% (the PSO obtained 54%). What is more, in this case the mass of the truck is reduced by 300 kg, i.e., almost 1% (the PSO achieved the same result). These results highlight (i) the importance of the motor technology, and (ii)

that it is worthwhile investigating the application of CVT technologies to BEVs.

IV. Conclusions

In this paper we explored the possibility of jointly optimizing the components' sizing and the control strategies of electric vehicle propulsion systems using convex optimization methods. Thereby, we presented a bi-convex model to capture the impact of the size of the powertrain components and their operation on the achievable energy consumption in an integrated fashion. First, we combined convex approximation and relaxation techniques to formulate the energy-optimal design and operation problem for a vehicle equipped with a battery, a single motor and a single-gear transmission or a continuously variable transmission (CVT), and computed the globally optimal solution by solving a sequence of second-order conic programs. We applied our method to design a battery electric heavy-duty truck and compared our results to a state-of-the-art particle swarm optimization approach relying on high-fidelity models. Whilst achieving similar results, our approach is significantly faster and is guaranteed to deliver solutions that are globally optimal. Specifically, we showed that, compared to standard single-speed transmissions, a CVT can reduce the energy consumption and the battery size of heavy-duty battery electric trucks in the order of 1% to 10%, and significantly shrink the electric motor size by 20% to 50%. These improvements are significant over the life-cycle of the truck due to reduced investment costs and operational costs, and increased revenues.

This work opens the field for several extensions: The framework proposed is not constrained to the powertrain topology studied in this paper nor to heavy-duty trucks, but is readily applicable to e-scooters, cars and motorbikes, and could be extended to more complex powertrain architectures.

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