Maximum-stability dispatch policy for shared autonomous vehicles

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Driven to Discover®

Shared autonomous vehicles (SAVs)

- SAV service currently in testing on public roads
- SAVs have safety driver


Company
Taxation
Accident
& Insurance
Fuel
Finance /
Depreciation
+
Maintenance

SAV : personal vehicle replacement rates

- 1 SAV : 10 personal vehicles[a]
- 1 SAV : 9 personal vehicles[b]
- 1 SAV : 3 personal vehicles[c]

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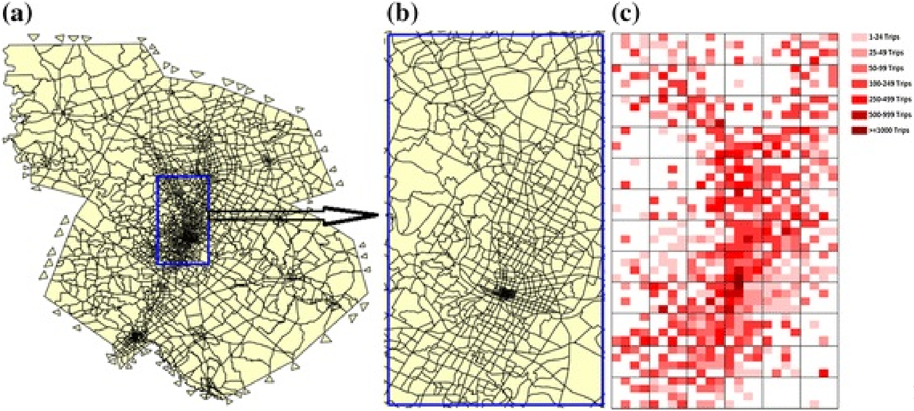
[a] Daniel J Fagnant and Kara M Kockelman. "The travel and environmental implications of shared autonomous vehicles, using agent-based model scenarios". In: *Transportation Research Part C: Emerging Technologies* 40 (2014), pp. 1–13.

[b] Daniel J Fagnant, Kara M Kockelman, and Prateek Bansal. "Operations of Shared Autonomous Vehicle Fleet for Austin, Texas Market". In: *Transportation Research Record: Journal of the Transportation Research Board* 2536 (2015), pp. 98–106.

[c] Kevin Spieser et al. "Toward a systematic approach to the design and evaluation of automated mobility-on-demand systems: A case study in Singapore". In: *Road Vehicle Automation*. NY: Springer, 2014, pp. 229–245.

Agent-based simulation



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[1]Daniel J Fagnant and Kara M Kockelman. "Dynamic ride-sharing and fleet sizing for a system of shared autonomous vehicles in Austin, Texas". In: *Transportation* 45.1 (2018), pp. 143–158.

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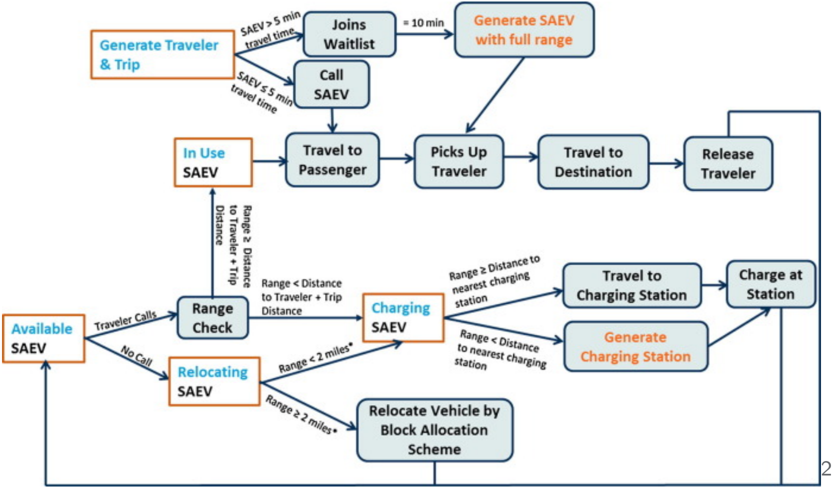

[2]T Donna Chen, Kara M Kockelman, and Josiah P Hanna. "Operations of a shared, autonomous, electric vehicle fleet: Implications of vehicle & charging infrastructure decisions". In: *Transportation Research Part A: Policy and Practice* 94 (2016), pp. 243–254.

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Queueing model

Passenger queueing model

Define a queue of waiting passengers at each zone: $w^{rs}(t)$.

Conservation of waiting passengers:

$$w^{rs}(t+1) = w^{rs}(t) + d^{rs}(t) - \min\left\{\sum_{j \in \mathcal{A}} y^{rs}_{rj}(t), w^{rs}(t)\right\}$$

where $d^{rs}(t)$ are random variables with mean $\bar{d}^{rs}$.

- $y^{rs}_{rj}(t) \leq p_r(t)$ is vehicles departing r for s to link j

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- $y_{rj}^{rs}(t) \leq p_r(t)$ is vehicles departing r for s to link j

This defines a Markov chain on the state space $\mathbb{N}^{|\mathcal{Z}|^2}$.

Vehicle queueing model

- $x_j^{rs}(t)$ is the number of vehicles on link j traveling from r to s
- $p_r(t)$ is the number of vehicles parked at r

$$\sum_{j\in\mathcal{A}} \sum_{(r,z)\in\mathcal{Z}^2} x_j^{rs}(t) + \sum_{r\in\mathcal{Z}} p_r(t) = F$$

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Conservation of link queues:

$$x_j^{rs}(t+1) = x_j^{rs}(t) + \sum_{i\in\mathcal{A}} y_{ij}^{rs}(t) - \sum_{k\in\mathcal{A}} y_{jk}^{rs}(t) \quad \Bigg| \sum_{k\in\Gamma_j^+} y_{jk}^{rs}(t) \leq x_j^{rs}(t)$$

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Conservation of parked queues:

$$p_r(t+1) = p_r(t) + \sum_{i \in \mathcal{A}} \sum_{q \in \mathcal{Z}} y_{ir}^{qr}(t) - \sum_{j \in \mathcal{A}} \sum_{s \in \mathcal{Z}} y_{rj}^{rs}(t)$$

Markov decision process

Queues of passengers and vehicles define a Markov chain.

$$w^{rs}(t+1) = w^{rs}(t) + d^{rs}(t) - \min\left\{\sum_{j\in\mathcal{A}} y_{rj}^{rs}(t), w^{rs}(t)\right\}$$

$$p_r(t+1) = p_r(t) + \sum_{i\in\mathcal{A}}\sum_{q\in\mathcal{Z}} y_{ir}^{qr}(t) - \sum_{j\in\mathcal{A}}\sum_{s\in\mathcal{Z}} y_{rj}^{rs}(t)$$

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Since vehicle movements can be controlled, this is a Markov decision process model.

Stability and passenger service

$\sum_{(r,s)\in\mathcal{Z}^2} w^{rs}(t)$ is the number of waiting passengers at time t

- If demand is unserved, then $\sum_{(r,s)\in\mathcal{Z}^2} w^{rs}(t)$ will increase over time

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Definition

The stochastic queueing model is stable if there exists some $K < \infty$ s.t.

$$\frac{1}{T}\sum_{t=1}^{T}\sum_{(r,s)\in\mathcal{Z}^2}\mathbb{E}\left[w^{rs}(t)\right] \leq K \qquad \forall T \in \mathbb{N}$$

Equivalently, $\exists$ Lyapunov function $\nu(\mathbf{w}(t)) \geq 0$ s.t.

$$\mathbb{E}\left[\nu(\mathbf{w}(t+1)) - \nu(\mathbf{w}(t))|\mathbf{w}(t)\right] \leq \kappa - \epsilon|\mathbf{w}(t)|$$

for all $\mathbf{w}(t)$ for $\kappa < \infty$, $\epsilon > 0$.

Assumptions

- SAV travelers wait in the system until served
  - If SAV travelers exited, the concept of stability would need to be redefined.
- Constant travel times for vehicles
- Entire SAV fleet can be centrally dispatched

Maximum-stability policy

Max-pressure policy with maximum stability

$$
\begin{aligned}
\max \quad & \frac{1}{T}\sum_{\tau=1}^{T}\sum_{(r,s)\in\mathcal{Z}^2} w^{rs}(t) f^{rs}(t+\tau) \\
\text{s.t.} \quad & \sum_{s\in\mathcal{Z}} f^{rs}(t+\tau) \leq p_r(t+\tau) \\
& p_r(t+\tau+1) = \quad p_r(t+\tau) + \sum_{q\in\mathcal{Z}} f^{qr}\left(t+\tau-\Phi_q^r\right) - \\
& \qquad \sum_{s\in\mathcal{Z}} f^{rs}(t+\tau) + \sum_{q\in\mathcal{Z}}\sum_{i\in\mathcal{A}} x_i^{qr}(t+\tau-\Phi_i^r) \\
& f^{rs}(t+\tau) \geq 0
\end{aligned}
$$

- T is the planning horizon — how far we look ahead
- $f^{rs}(t+\tau)$ anticipates future vehicle dispatch
- $p_r(t+\tau)$ anticipates future vehicle availability

Planning horizon analysis

$$
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\end{aligned}
$$

- T is the planning horizon — how far we look ahead

T must be large enough to dispatch vehicles across the network. At least $\max_r \left\{\Phi_q^r\right\}$.

Stability region

What demand rates $\bar{\mathbf{d}} \in \mathcal{D}$ could be served by *any* SAV dispatch policy?

- We want to serve any $\bar{\mathbf{d}} \in \mathcal{D}$ with the max-pressure policy.

Stability region

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- We want to serve any $\bar{\mathbf{d}} \in \mathcal{D}$ with the max-pressure policy.

Average SAV flow rates from r to s are enough to serve average demand:

$$\sum_{i \in \Gamma_r^+} \bar{y}_{ri}^{rs} \geq \bar{d}^{rs} \qquad \forall (r,s) \in \mathcal{Z}^2$$

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Constraints on average SAV flow rates:

$$\sum_{q \in \mathcal{Z}} \sum_{i \in \Gamma_r^-} \bar{y}_{ir}^{qr} = \sum_{s \in \mathcal{Z}} \sum_{j \in \Gamma_r^+} \bar{y}_{jr}^{rs} \qquad \forall q \in \mathcal{Z}$$

$$\sum_{i \in \Gamma_j^-} \bar{y}_{ij}^{rs} = \sum_{j \in \Gamma_j^+} \bar{y}_{jk}^{rs} \qquad \forall (r,s) \in \mathcal{Z}^2, \forall j \in \mathcal{A}_o$$

$$\sum_{(r,s) \in \mathcal{Z}^2} \sum_{(i,j) \in \mathcal{A}^2} \bar{y}_{ij}^{rs} \leq F$$

Stability region — $\mathcal{D}$

Proposition

If $\mathbf{d} \notin \mathcal{D}$, then the system cannot be stabilized by some $\bar{\mathbf{y}} \in \mathcal{Y}$.

Proof. For any SAV dispatch policy $\exists$ an (r,s) with an $\eta > 0$ s.t. $\sum_{i \in \Gamma_r^+} \bar{y}_{ri}^{rs} - \bar{d}^{rs} \geq \eta$. Then on average $w^{rs}(t)$ will increase by η each time step. □

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$$\sum_{(r,s) \in \mathcal{Z}^2} \sum_{(i,j) \in \mathcal{A}^2} \bar{y}_{ij}^{rs} \leq F$$

so if $\bar{\mathbf{d}} \notin \mathcal{D}$, then a larger fleet size is needed to serve $\bar{\mathbf{d}}$.

Passenger service rates

Proposition

The boundary of $\mathcal{D}$ is linear wrt F, i.e. if the fleet size increases to αF then demand of $\alpha \bar{\mathbf{d}}$ can be stabilized.

Proof. αF admits a linear increase of α in all other constraints defining the stable region. □

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Proof. αF admits a linear increase of α in all other constraints defining the stable region. □

An increase in the SAV fleet size should result in a proportional increase in the number of passengers that can be served.

Stability region — maximize service rate

$$\max \sum_{(r,s)\in\mathcal{Z}^2} \bar{d}^{rs} \qquad \text{s.t.}$$

$$\sum_{i\in\Gamma_r^+} \bar{y}_{ri}^{rs} \geq \bar{d}^{rs} \qquad \forall (r,s)\in\mathcal{Z}^2$$

$$\sum_{q\in\mathcal{Z}} \sum_{i\in\Gamma_r^-} \bar{y}_{ir}^{qr} = \sum_{s\in\mathcal{Z}} \sum_{j\in\Gamma_r^+} \bar{y}_{jr}^{rs} \qquad \forall q\in\mathcal{Z}$$

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$$\sum_{(r,s)\in\mathcal{Z}^2} \sum_{(i,j)\in\mathcal{A}^2} \bar{y}_{ij}^{rs} \leq F$$

Analytical method to find the theoretical maximum service rate.

Stability region — let $\mathcal{D}^0$ be the interior of $\mathcal{D}$

Average SAV flow rates from r to s are enough to serve average demand:

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$$\sum_{(r,s) \in \mathcal{Z}^2} \sum_{(i,j) \in \mathcal{A}^2} \bar{y}_{ij}^{rs} \leq F$$

Stability proof sketch

If $\bar{\mathbf{d}} \in \mathcal{D}^0$, then there exists some $\bar{\mathbf{y}}$ such that

$$\bar{d}^{rs} - \sum_{i \in \mathcal{A}} \bar{y}_{ri}^{rs} \leq -\epsilon$$

Stability proof sketch

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$$\bar{d}^{rs} - \sum_{i \in \mathcal{A}} \bar{y}^{rs}_{ri} \leq -\epsilon$$

Proposition

There exists a sequence $(\mathbf{y}(t))$ *such that*

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T} \mathbf{y}(t) = \bar{\mathbf{y}}$$

The max-pressure policy constructs a sequence $\hat{\mathbf{y}}(t+\tau)$ with limit $\bar{\mathbf{y}}$.

Lemma

Suppose that there exists a $T \in \mathbb{N}$ *and a* $\kappa_1, \kappa_2 < \infty$ *such that*

$$\mathbb{E}\left[\nu(\mathbf{w}(t+T)) - \nu(\mathbf{w}(t+T+1) + \nu(\mathbf{w}(t+1)) - \nu(\mathbf{w}(t))|\mathbf{w}(t)\right] \leq \kappa_1$$

$$\mathbb{E}\left[\nu(\mathbf{w}(t+T+1) - \nu(\mathbf{w}(t+T))|\mathbf{w}(t)\right] \leq \kappa_2 - \epsilon|\mathbf{w}(t)|$$

then the system is stable.

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Lemma

Suppose that there exists a $T \in \mathbb{N}$ *and a function* $\nu(\mathbf{w}(t))$ *such that*

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$$\mathbb{E}\left[\frac{1}{T}\sum_{\tau=1}^{T}\left(\nu(\mathbf{w}(t+\tau+1) - \nu(\mathbf{w}(t+\tau))\right) | \mathbf{w}(t)\right] \leq \kappa_2 - \epsilon|\mathbf{w}(t)|$$

then the system is stable.

Lyapunov function $\nu(\mathbf{w}(t)) = \sum_{(r,s)\in\mathcal{Z}^2} (w^{rs}(t))^2$

Proposition

$\forall \bar{\mathbf{d}} \in \mathcal{D}^0$ $\exists M < \infty$ such that if $T > M$ then the max-pressure control using the planning horizon T yields

$$\mathbb{E}\left[\frac{1}{T}\sum_{\tau=1}^{T}\sum_{(r,s)\in\mathcal{Z}^2}(w^{rs}(t+\tau+1))^2 - (w^{rs}(t+\tau))^2 \,|\mathbf{w}(t)\right] \leq \kappa - \epsilon|\mathbf{w}(t)|$$

- For any $\eta > 0$, there exists a M s.t. if $T > M$ then
$$\frac{1}{T}\sum_{\tau=1}^{T}\hat{\mathbf{y}}(t+\tau) \leq |\bar{\mathbf{y}} - \eta\mathbf{1}|$$
- If $\eta < \epsilon$, $\exists \epsilon_2 > 0 = \epsilon - \eta$ such that
$$\mathbb{E}\left[\nu(\mathbf{w}(t+1) - \mathbf{w}(t))|\mathbf{w}(t)\right] \leq \kappa - \epsilon_2|\mathbf{w}(t)|$$

Planning horizon analysis

- For any $\eta > 0$, there exists a M s.t. if $T > M$ then

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We need $\eta < \epsilon$.

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We need $\eta < \epsilon$.

$$\sum_{i\in\Gamma_r^+}\bar{y}_{ri}^{rs} > \bar{d}^{rs} \Rightarrow \sum_{i\in\Gamma_r^+}\bar{y}_{ri}^{rs} - \bar{d}_{rs} > \epsilon$$

Planning horizon analysis

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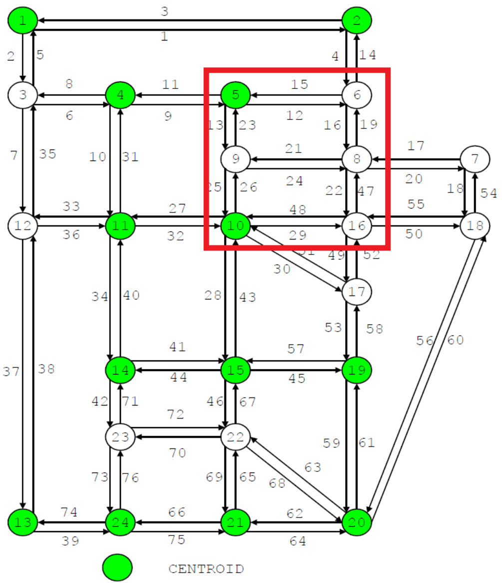
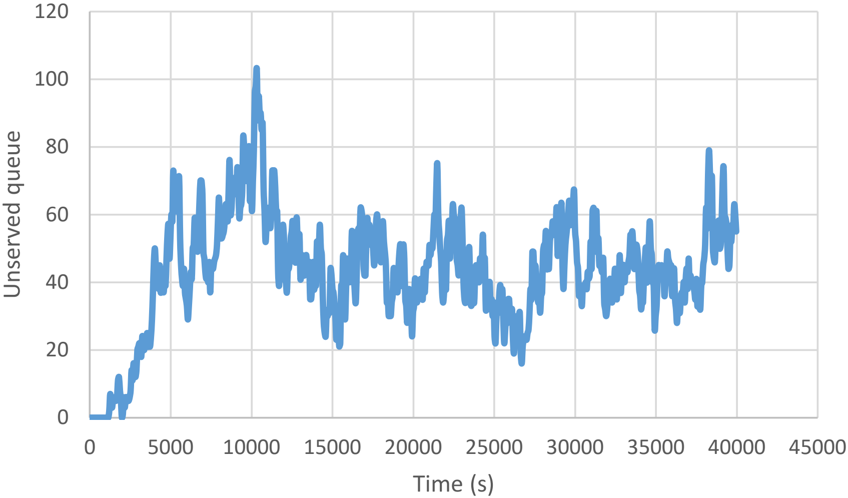
The larger the time horizon, the closer demand can get to the boundary of the stable region.



Sioux Falls

Example — stable demand



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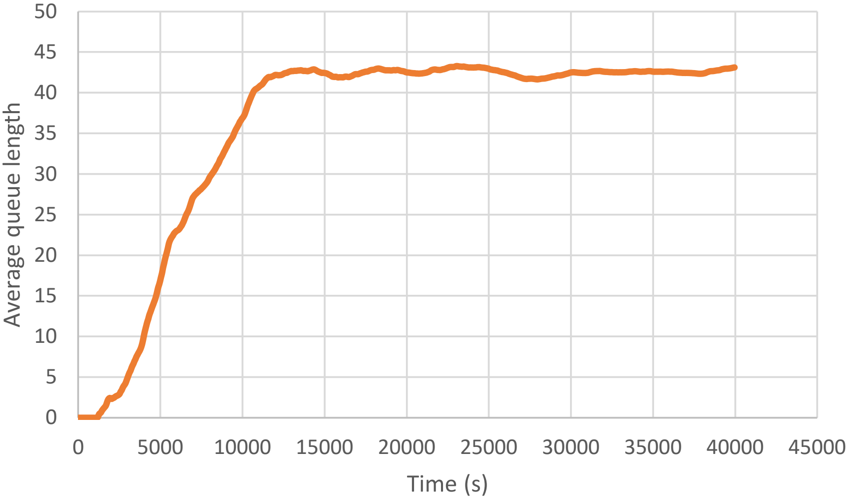

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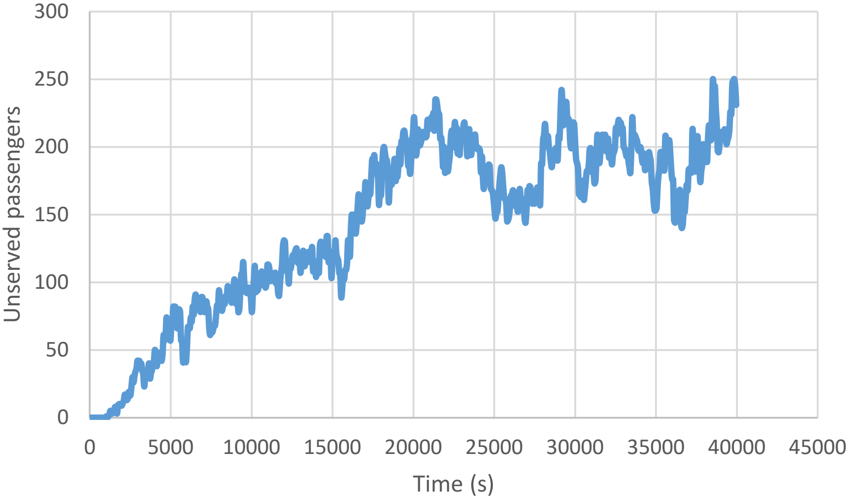

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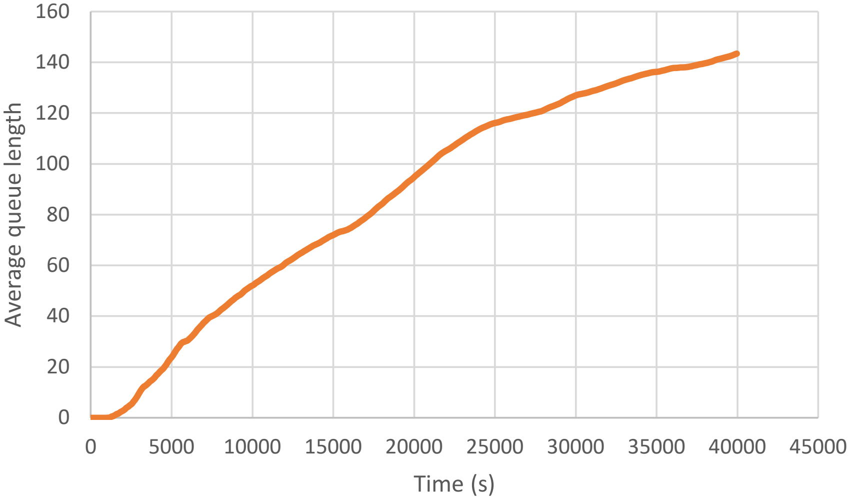
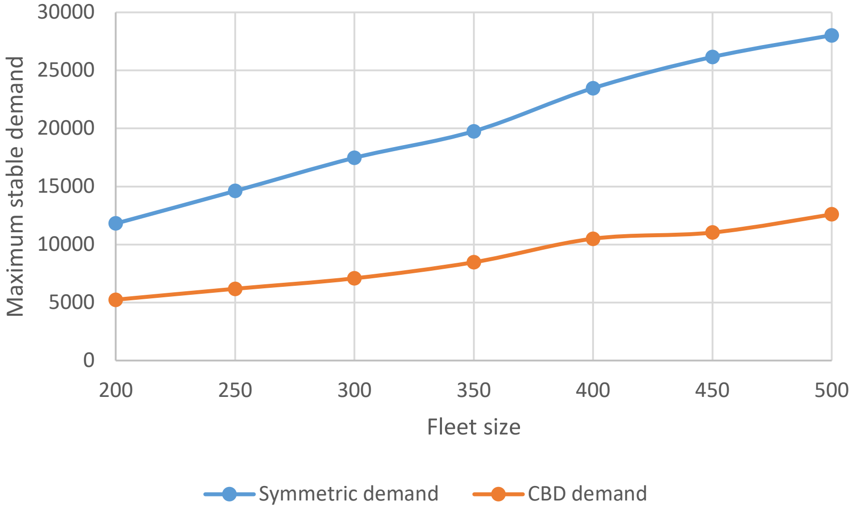
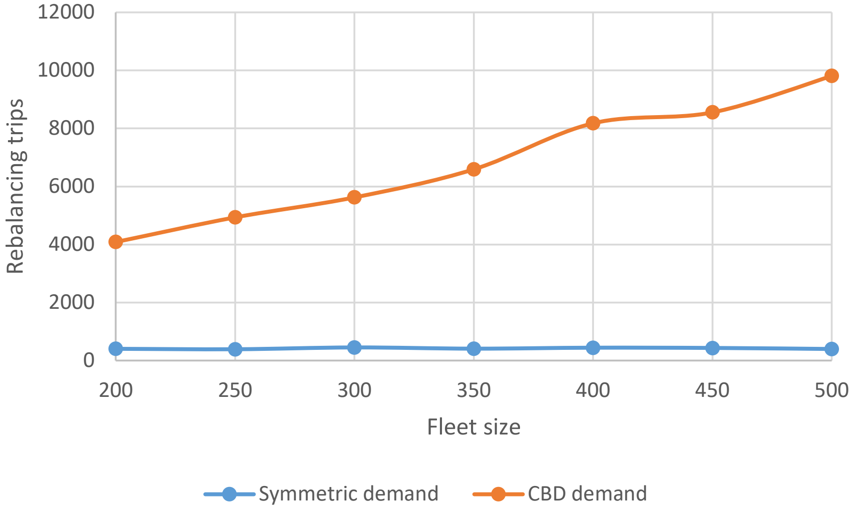


Maximum stable demand vs. fleet size



OD demand proportions are constant.

Maximum stable demand vs. fleet size



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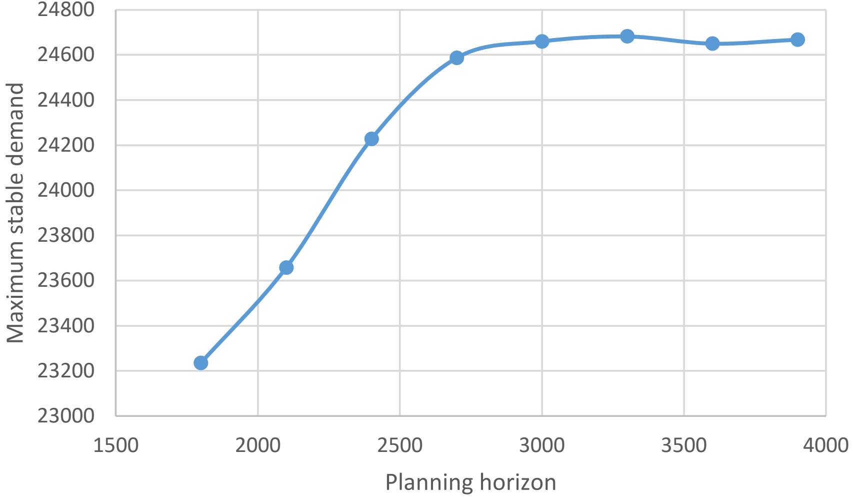
Effect of planning horizon T on maximum stable demand



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Conclusions

- Stability analysis of SAVs
- Maximum-stability policy with proof
- Numerical results evaluating stable region

Future work:

- Decentralized policy
- Ridesharing, electric vehicles
- Efficient heuristics, or evaluate stability of heuristic policies

Thank you

- Questions?

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