Principles of Robot Autonomy I

Camera models and camera calibration
Camera models and camera calibration

• Aim
  • Learn how to calibrate a camera
  • Learn about 3D reconstruction

• Readings
  • SNS: 4.2.3
Step 3

• In previous lecture, we have derived a mapping between a point $P$ in the 3D camera reference frame to a point $p$ in the 2D image plane.

• Last step is to include in our mapping an additional transformation to account for the difference between the world frame and the 3D camera reference frame.
Rigid transformations

\[ P_C = t + q \]

\[ q = R P_W \]

where \( R \) is the rotation matrix relating camera and world frames

\[ R = \begin{bmatrix} i_W \cdot i & j_W \cdot i & k_W \cdot i \\ i_W \cdot j & j_W \cdot j & k_W \cdot j \\ i_W \cdot k & j_W \cdot k & k_W \cdot k \end{bmatrix} \]

\[ \Rightarrow P_C = t + R P_W \]
Rigid transformations in homogeneous coordinates

\[
\begin{pmatrix}
P_C \\
1
\end{pmatrix}
= \begin{bmatrix}
R & t \\
0_{1 \times 3} & 1
\end{bmatrix}
\begin{pmatrix}
P_W \\
1
\end{pmatrix}
\]

Point $P_C$ in homogeneous coordinates

Point $P_W$ in homogeneous coordinates
Perspective projection equation

- Collecting all results

\[ p^h = [K \ 0_{3 \times 1}] P_C^h = K [I_{3 \times 3} \ 0_{3 \times 1}] \begin{bmatrix} R & t \\ 0_{3 \times 1} & 1 \end{bmatrix} P_W^h \]

- Hence

Intrinsic parameters

Extrinsic parameters

- Degrees of freedom: 4 for \( K \) (or 5 if we also include skewness), 3 for \( R \), and 3 for \( t \). Total is 10 (or 11 if we include skewness)
Camera calibration: direct linear transformation method

- **Goal:** find the intrinsic and extrinsic parameters of the camera

**Strategy:** given known correspondences \( p_i \leftrightarrow P_{W,i} \), compute unknown parameters \( K, R, t \) by applying perspective projection

\( P_{W,1}, P_{W,2}, \ldots, P_{W,n} \) with **known** positions in world frame
\( p_1, p_2, \ldots, p_n \) with **known** positions in image frame
Step 1

• First consider combined parameters

\[ p_h^i = M P_h^{W,i} , \ i = 1, \ldots, n, \]  
where \( M = K[R \ t] = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} \)

• This gives rise to \( 2n \) component-wise equations, for \( i = 1, \ldots, n \)

\[
\begin{align*}
  u_i &= \frac{m_1 \cdot P_h^{W,i}}{m_3 \cdot P_h^{W,i}} \\
  v_i &= \frac{m_2 \cdot P_h^{W,i}}{m_3 \cdot P_h^{W,i}}
\end{align*}
\]

or

\[
\begin{align*}
  u_i (m_3 \cdot P_h^{W,i}) - m_1 \cdot P_h^{W,i} &= 0 \\
  v_i (m_3 \cdot P_h^{W,i}) - m_2 \cdot P_h^{W,i} &= 0
\end{align*}
\]
Calibration problem

• Stacking all equations together

\[ \tilde{P} m = 0, \quad \text{where } m = \begin{bmatrix} m_1^T \\ m_2^T \\ m_3^T \end{bmatrix} \]

- \( \tilde{P} \) contains in block form the known coefficients stemming from the given correspondences
- To estimate 11 coefficients, we need at least 6 correspondences
Solution

• To find non-zero solution

\[ \min_{m \in \mathbb{R}^{12}} \| \tilde{P} m \|^2 \]
subject to \[ \| m \|^2 = 1 \]

• Solution: select eigenvector of \( \tilde{P}^T \tilde{P} \) with the smallest eigenvalue

• Readily computed via SVD (singular value decomposition)
Step 2

• Next, we need to extract the camera parameters, i.e., we want to factorize $M$ as

$$M = \begin{bmatrix} KR & Kt \end{bmatrix}$$

• This can be done efficiently (indeed, explicitly) by using RQ factorization, whereby the submatrix $M_{1:3,1:3}$ is decomposed into the product of an upper triangular matrix $K$ and a rotation matrix $R$

• Calibration will be investigated in Problem 1 in HW3
Radial distortion

• So far, we have assumed that a linear model is an accurate model of the imaging process
• For real (non-pinhole) lenses this assumption will not hold
Once the camera is calibrated, can we measure the location of a point $P$ in 3D given its known observation $p$?

- **No**: one can only say that $P$ is located *somewhere* along the line joining $p$ and $O$!
Issues with recovering structure
Recovering structure

- **Structure**: 3D scene to be reconstructed by having access to 2D images

- **Common methods**
  1. Through recognition of landmarks (e.g., orthogonal walls)
  2. Depth from focus: determines distance to one point by taking multiple images with better and better focus
  3. Stereo vision: processes two distinct images taken at the *same time* and assumes that the relative pose between the two cameras is *known*
  4. Structure from motion: processes two images taken with the same or different cameras at *different times* and from different *unknown* positions
Depth from focus

• Take several images until the projection of point $P$ is in focus; let $z$ denote the distance at which the image is in focus
• Since we know $z$ and $f$, through the thin lens equation we obtain $Z$

\[ \frac{1}{z} + \frac{1}{Z} = \frac{1}{f} \]
Stereopsis, or why we have two eyes
Binocular reconstruction

- **Given**: calibrated stereo rig and two image matching points $p$ and $p'$
- **Find** corresponding scene point by intersecting the two rays $\overrightarrow{Op}$ and $\overrightarrow{O'p'}$ (process known as **triangulation**
Approximate triangulation

- Due to noise, triangulation problem is often solved as finding the point \( Q \) with images \( q \) and \( q' \) that minimizes

\[
d^2(p, q) + d^2(p', q')
\]

Re-projection error
Next time: image processing, feature detection & description