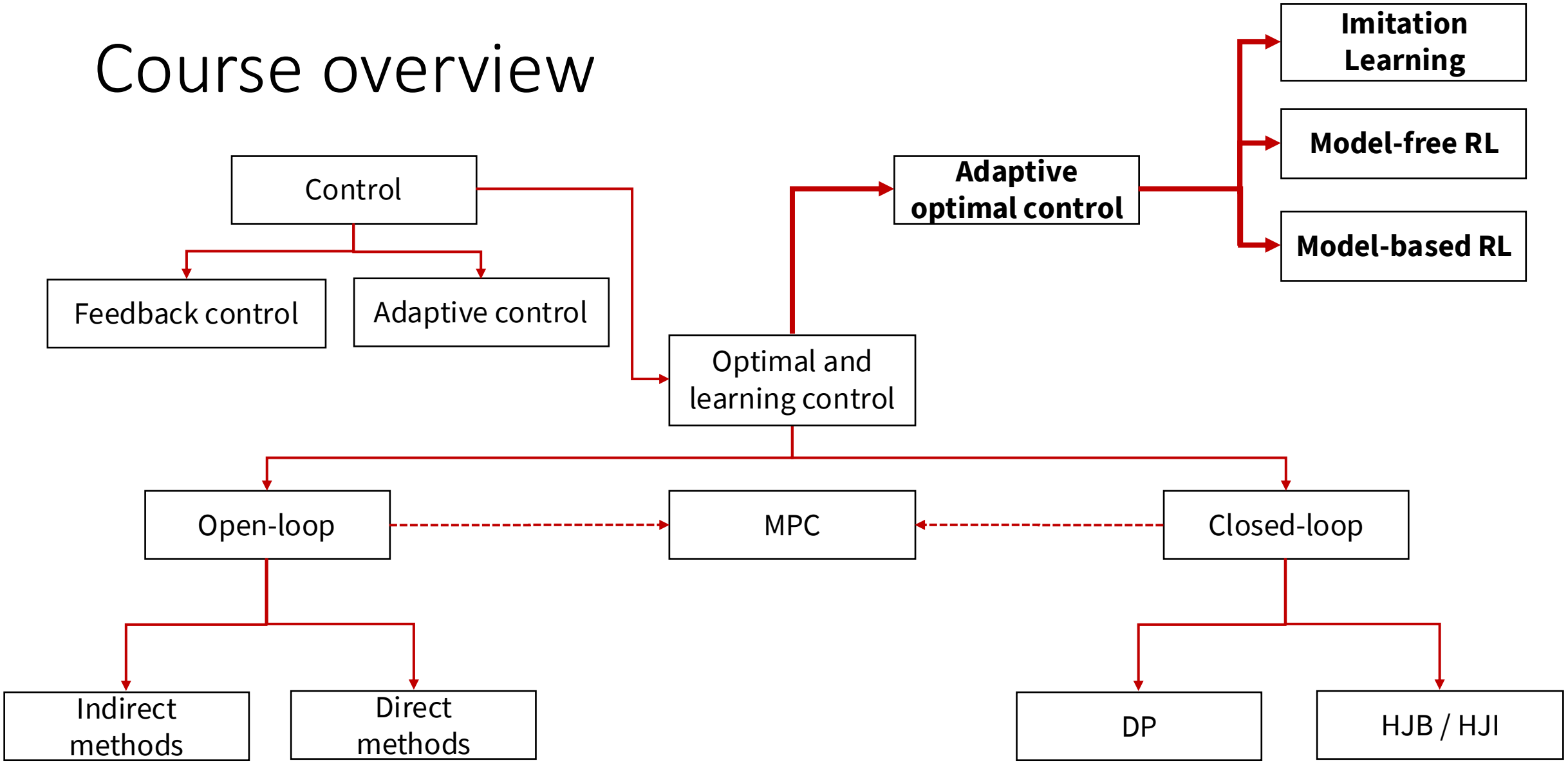
AA203
Optimal and Learning-based Control

Intro to Imitation Learning (IL) and Reinforcement Learning (RL)

Stanford University

ASL
Autonomous Systems Lab

Course overview




all takes are continuous and uncut
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Credits:
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Credits:
Gemini Robotics


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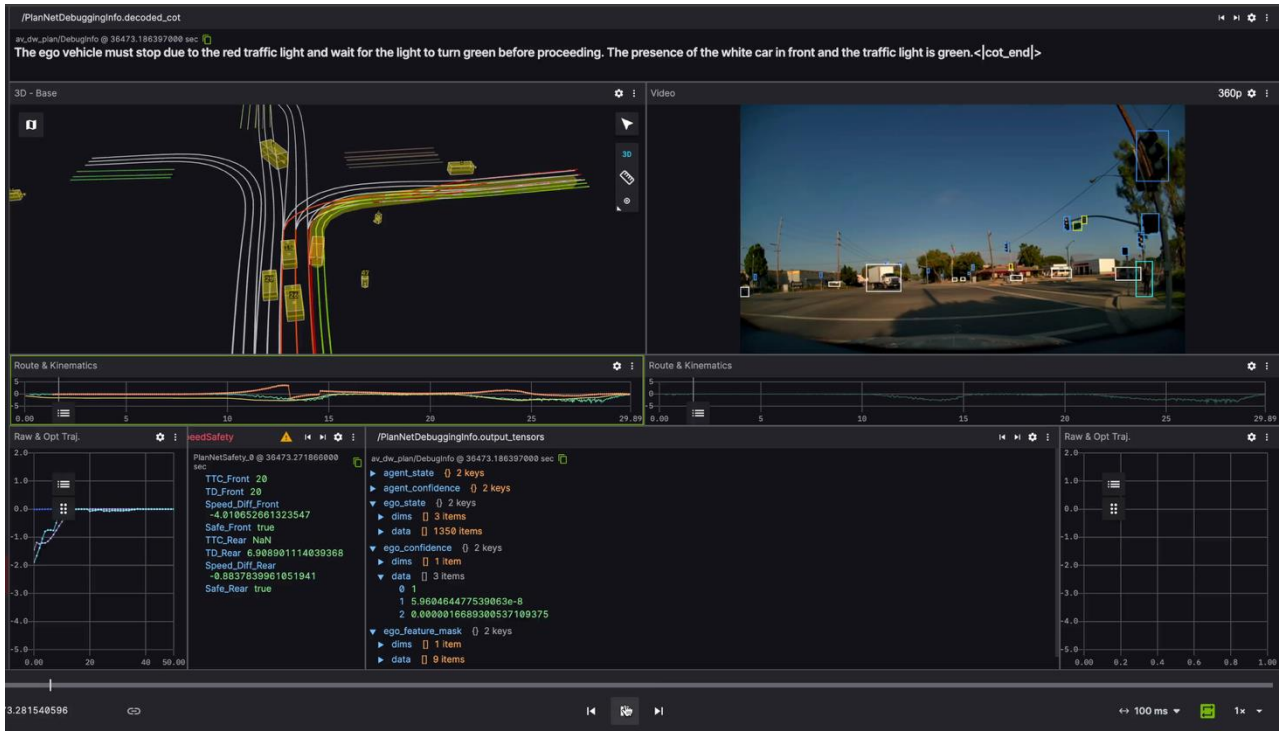
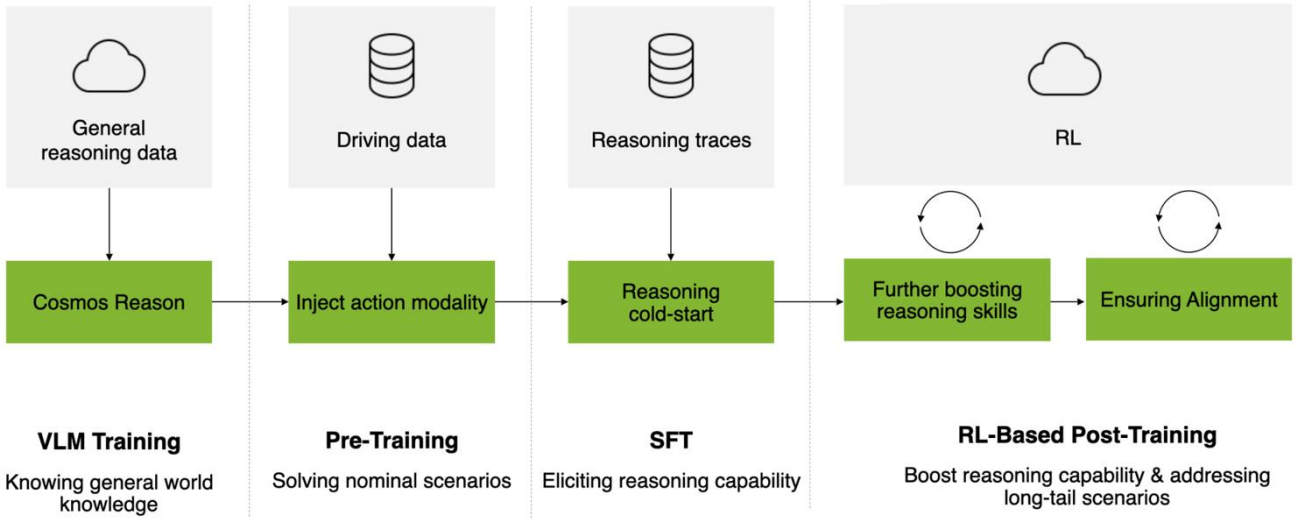
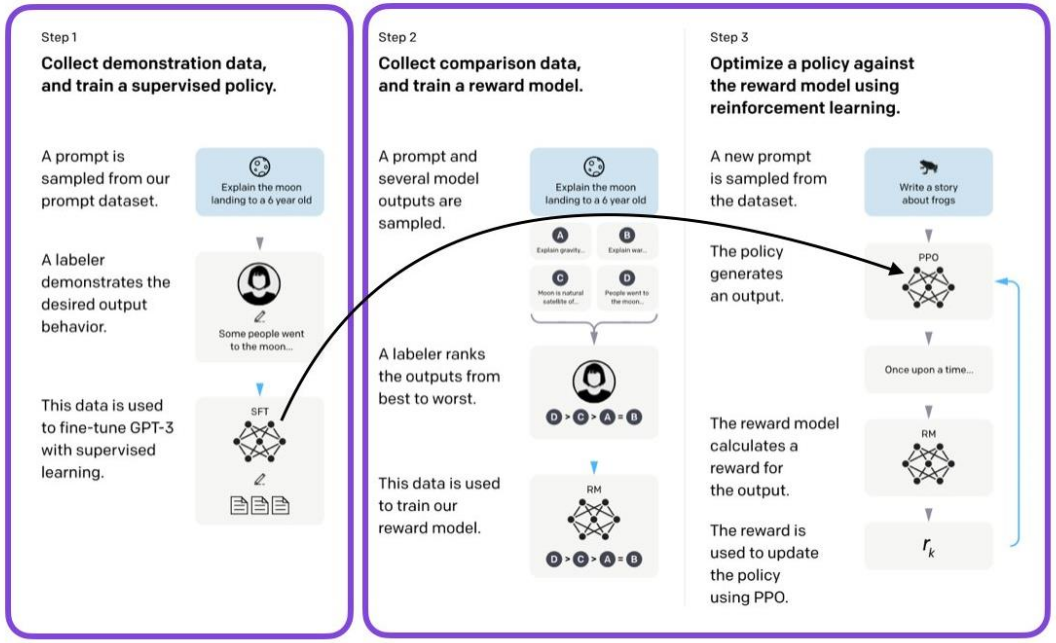
[NVIDIA. Alpamayo-R1]





ChatGPT Gemini Claude deepseek

Finetune the pretrained GPT3 model via SL → RLHF for finetuning the supervised baseline



Learning for control, in practice

12 DoF quadruped

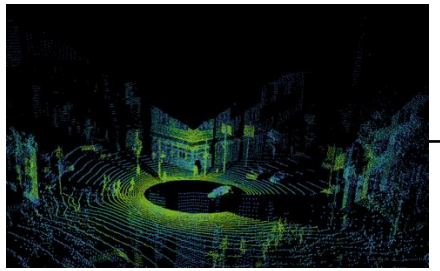
Lidar

RGB front camera

Potentially hazardous terrain

…

How can we develop a reliable control algorithm?



Upcoming weeks: Set it up as a *learning problem*

Option 1

Deeply understand the problem

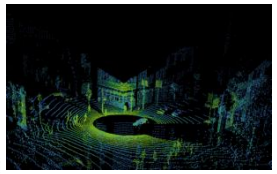
Design a solution

$$\begin{aligned} \min_{\mathbf{u}} \quad & h(\mathbf{x}(t_f), t_f) + \int_{t_0}^{t_f} g(\mathbf{x}(t), \mathbf{u}(t), t)\, dt \\ \text{subject to} \quad & \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) \\ & \mathbf{u}(t) \in \mathcal{U} \end{aligned}$$

Option 2:

Imitation Learning

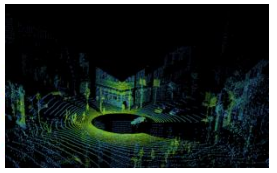
"Learn from demonstrations"

$\{$  ; $\mathbf{u}^*(t)$ $\}$

Option 3:

Reinforcement Learning

"Learn by trial and error"

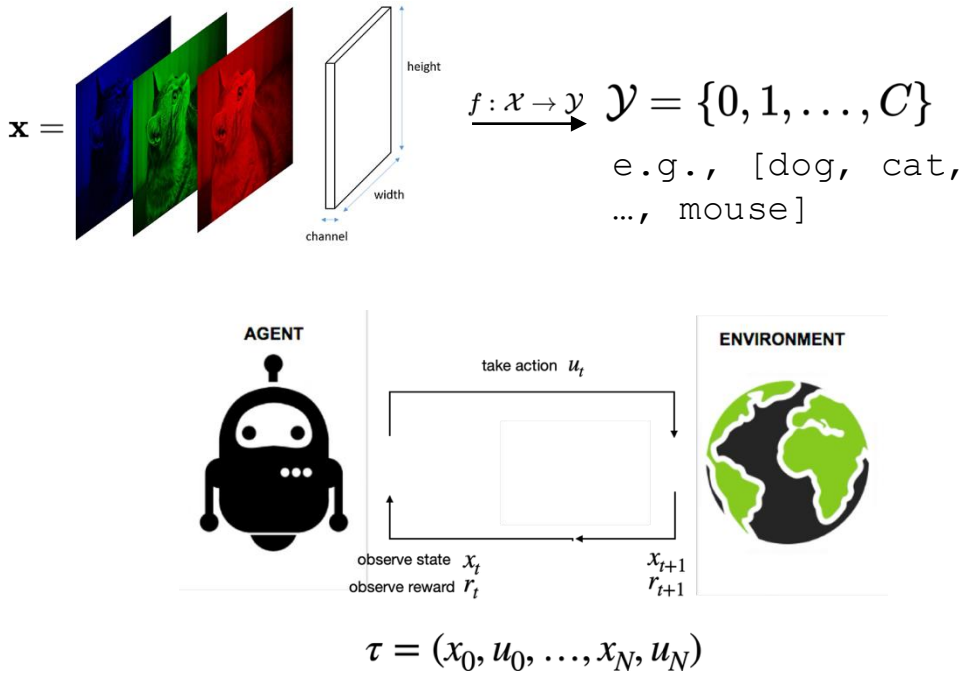
$\{$  ; Success, Failure $\}$

Outline

Primer on Supervised Learning

Imitation Learning

Reinforcement Learning



Key learning goals:

- An overview of the fundamentals of IL and RL
- Identify the distinctions and similarities between these paradigms

Imitation Learning and Reinforcement Learning

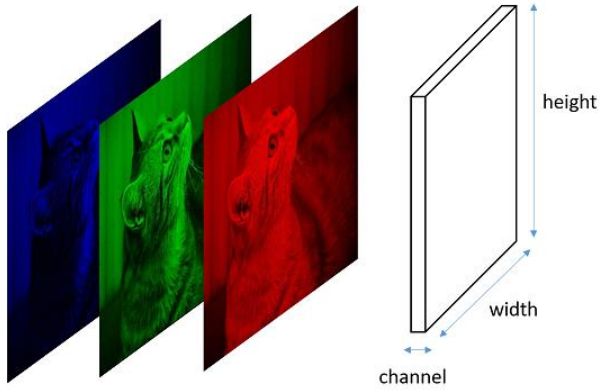
Resources:

- PoRA book Chapters 17-19, and references therein, and also:
  - Sutton and Barto: “Reinforcement Learning: An Introduction”
- Classes:
  - Stanford CS 224R Deep Reinforcement Learning
  - Berkeley CS 185/285 Deep Reinforcement Learning

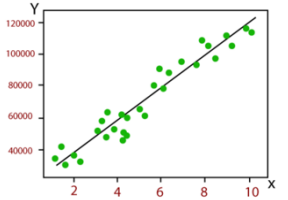
A primer on supervised learning (SL)

- In SL, the task is to learn a mapping $f : \mathcal{X} \to \mathcal{Y}$

  from inputs $\mathbf{x} \in \mathcal{X}$ to outputs $\mathbf{y} \in \mathcal{Y}$

$\mathbf{x} =$ 

$f : \mathcal{X} \to \mathcal{Y}$

$\mathcal{Y} = \mathbb{R}^n$ 

$\mathcal{Y} = \{0, 1, \ldots, C\}$

`e.g., [dog, cat, …, mouse]`

$f : \mathcal{X} \to \mathcal{Y}$

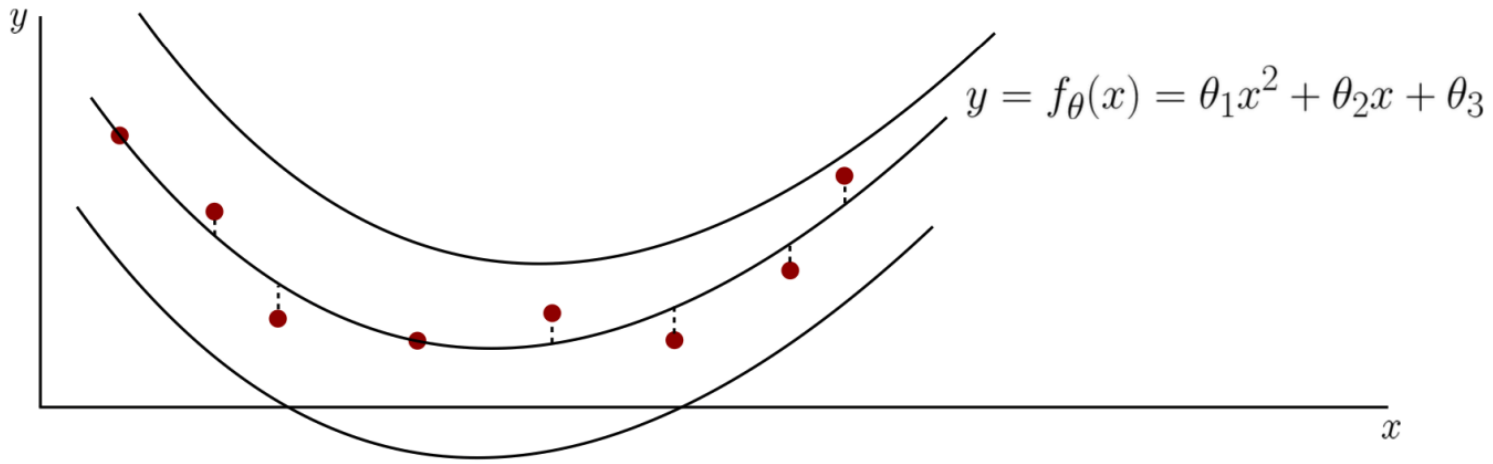
$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_D \end{bmatrix}$$

- Learning involves data in the form of input-output pairs $\mathcal{D} = \{(\mathbf{x}_n, \mathbf{y}_n)\}_{n=1}^N$ e.g., Maximum Likelihood Estimation

$$\text{NLL}(\theta) = -\frac{1}{N} \sum_{n=1}^{N} \log p(\mathbf{y} | f_\theta(\mathbf{x})) \quad \hat{\boldsymbol{\theta}}_{\text{mle}} = \underset{\boldsymbol{\theta}}{\text{argmin}}\, \text{NLL}(\boldsymbol{\theta})$$

Learning from a probabilistic standpoint

- Let's consider regression as an example

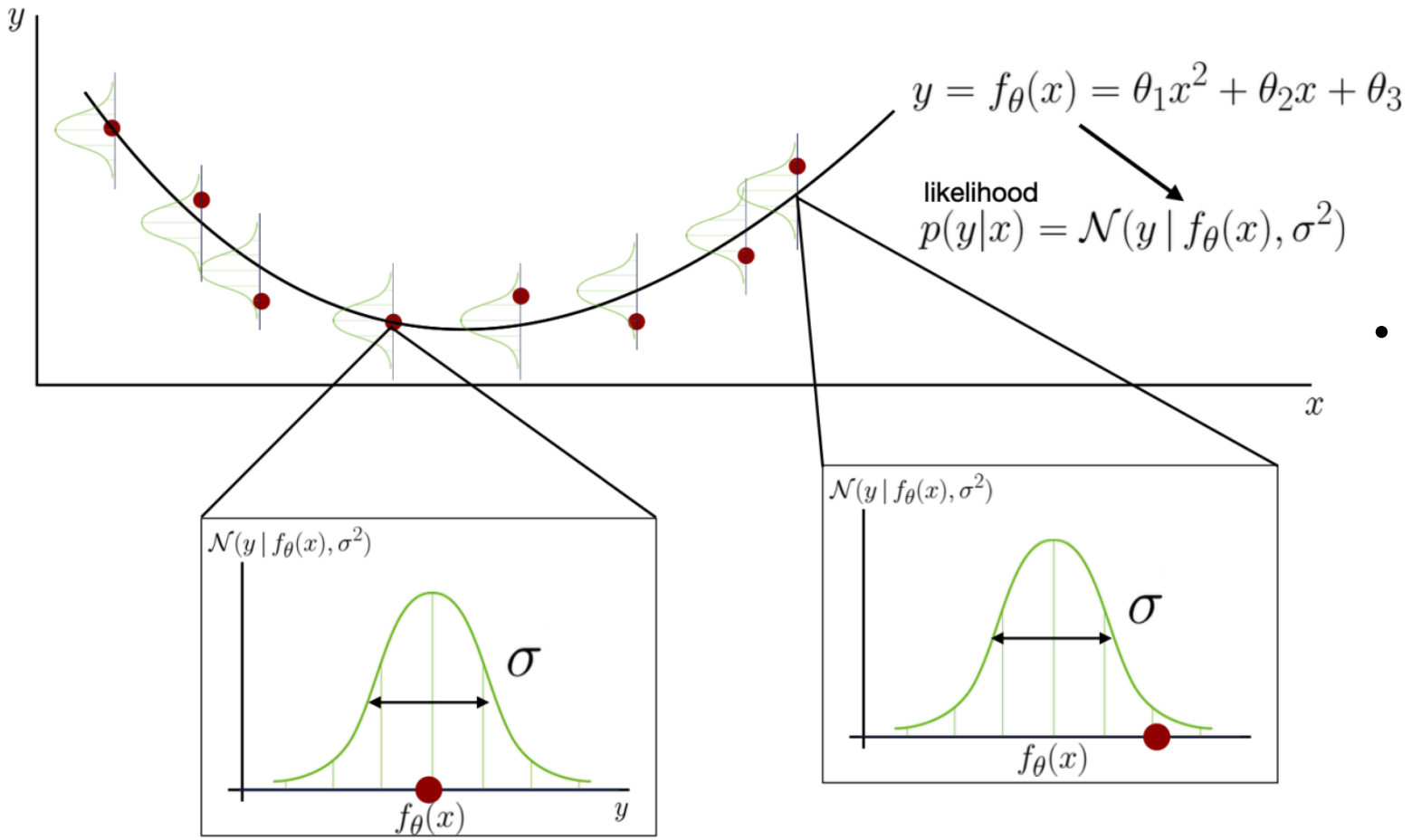


- Learning through minimization of squared error

$$\theta^* = \arg\min_\theta \frac{1}{n} \sum_{i=1}^{n} (y_i - f_\theta(x_i))^2$$

Learning from a probabilistic standpoint

- Let's consider regression as an example



- Learning through likelihood maximization:

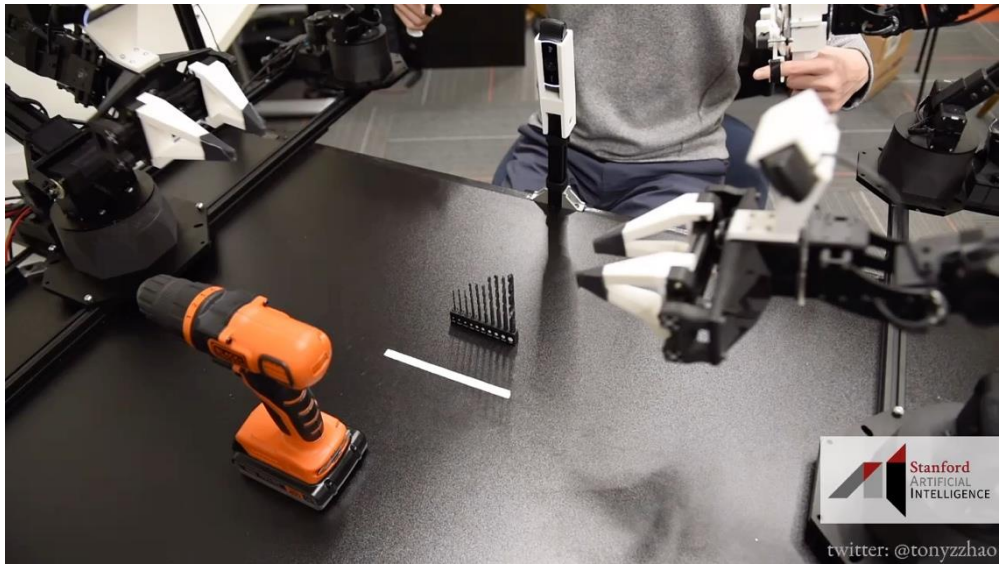
$$\theta^* = \arg\max_\theta \prod_{i=1}^{n} \mathcal{N}(y_i \mid f_\theta(x_i), \sigma^2)$$

$$\mathbf{NLL}(\theta) = -\frac{1}{N} \sum_{n=1}^{N} \log p(\mathbf{y}|f_\theta(\mathbf{x}))$$

$$\hat{\boldsymbol{\theta}}_{\mathbf{mle}} = \operatorname*{argmin}_{\boldsymbol{\theta}} \mathbf{NLL}(\boldsymbol{\theta})$$

The basics of Imitation Learning

- Imitation learning refers to a class of methods that enable skills to be transferred from an *expert* to a *learner*
- In the context of robotics, the expert is typically a human operator or a pre-existing control policy and the learner is the robot that aims to mimic the expert's behavior
- *Example:*



The basics of Imitation Learning

- Assume access to a dataset $\mathcal{D} = \{(\mathbf{x}_n, \mathbf{u}_n)\}_{n=1}^N$ of state-control pairs collected by an expert
- At a high-level, approaches to IL belong to two main categories:
  - *Behavior Cloning* — *"learn the policy used by expert"*
  - *Inverse Reinforcement Learning (a.k.a. Inverse Optimal Control)* — *"learn the objective optimized by the expert"*

Behavior Cloning

$$\pi_\theta(\mathbf{u}|\mathbf{x})$$

Inverse Reinforcement Learning

$$R : \mathcal{X} \times \mathcal{U} \to \mathbb{R}$$

Behavior Cloning (BC)

Main idea: Train policy via supervised learning

Skeleton of an IL algorithm:

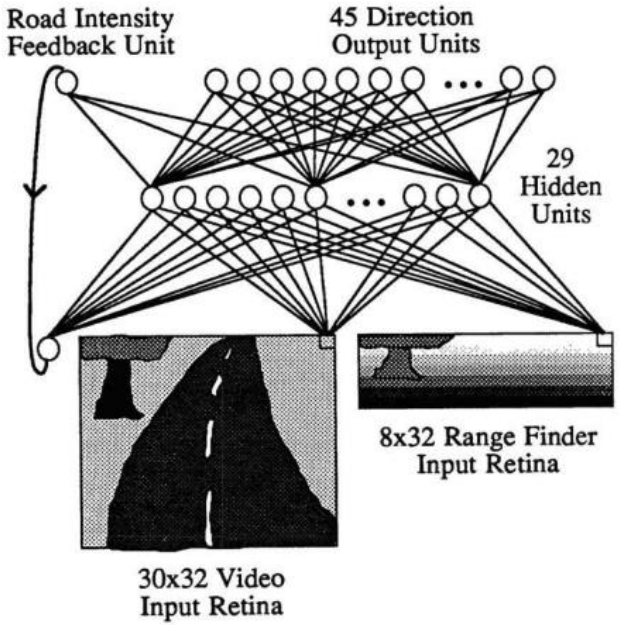
1) Collect a dataset of "expert" demonstrations  $\mathcal{D} = \{(\mathbf{x}_n, \mathbf{u}_n)\}_{n=1}^N$

2) Train policy $\pi_\theta(\mathbf{u}|\mathbf{x})$ to mimic expert: $\text{NLL}(\theta) = -\frac{1}{N}\sum_{n=1}^{N} \log \pi_\theta(\mathbf{u}|\mathbf{x})$

e.g., MLE, cross-entropy, L2 / MSE, etc. $\hat{\boldsymbol{\theta}}_{\text{mle}} = \underset{\boldsymbol{\theta}}{\text{argmin}}\, \text{NLL}(\boldsymbol{\theta})$

A first (deep) imitation learning system

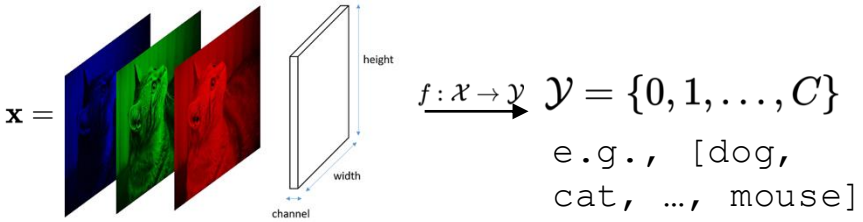
ALVINN: **A**n **A**utonomous **L**and **V**ehicle **I**n a **N**eural **N**etwork (1989)



Is this guaranteed to work?

Spoiler: NO - Imitation learning is different from "plain" supervised learning

- Supervised learning: learn a mapping from input data to output labels based on a dataset of input-output pairs



- IL builds heavily on SL, however with some key differences:
  - Even small prediction errors will compound over time
  - The solution may have inherent structural properties, such as physical constraints or temporal dependencies, that are not present in standard supervised learning tasks
  - In IL, the source domain may differ from the one at deployment (e.g., humanoid control by learning from videos of humans)
  - Covariate shift, where the distribution of the expert's data may differ from the distribution of the learner's data
  - Obtaining expert demonstrations may be expensive or time consuming

Inverse Reinforcement Learning (IRL)

IRL takes an orthogonal approach to IL, by attempting to recover a reward function from a policy, or from demonstrations of a policy

Example: (BC vs IRL)
A warehouse robot must navigate from a starting point to a goal while avoiding obstacles.
Behavior Cloning:
- Collect demonstration data, train a policy that imitates the demonstrations
- Effective in familiar scenarios, however, it may struggle if the layout changes substantially

Inverse RL:
- Collect the same demonstration data, infer the reward function (e.g., positive for moving closer to the goal, negative for moving close to obstacles)
- The reward function encapsulates the underlying principles of the task

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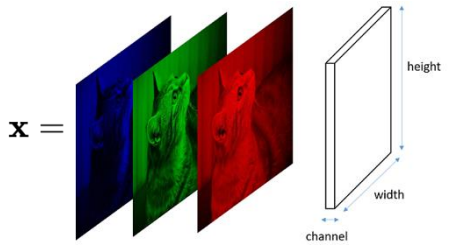
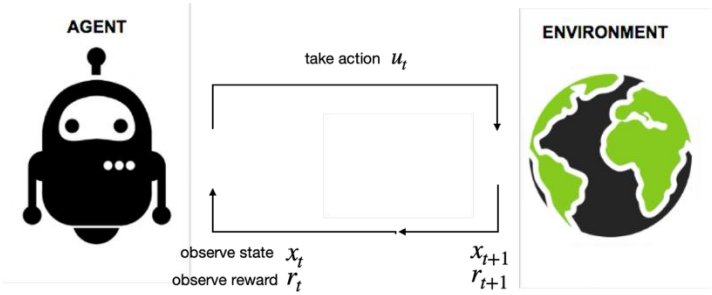


$f : \mathcal{X} \to \mathcal{Y}$ $\mathcal{Y} = \{0, 1, \ldots, C\}$

`e.g., [dog, cat, …, mouse]`



$$\tau = (x_0, u_0, \ldots, x_N, u_N)$$

Key learning goals:

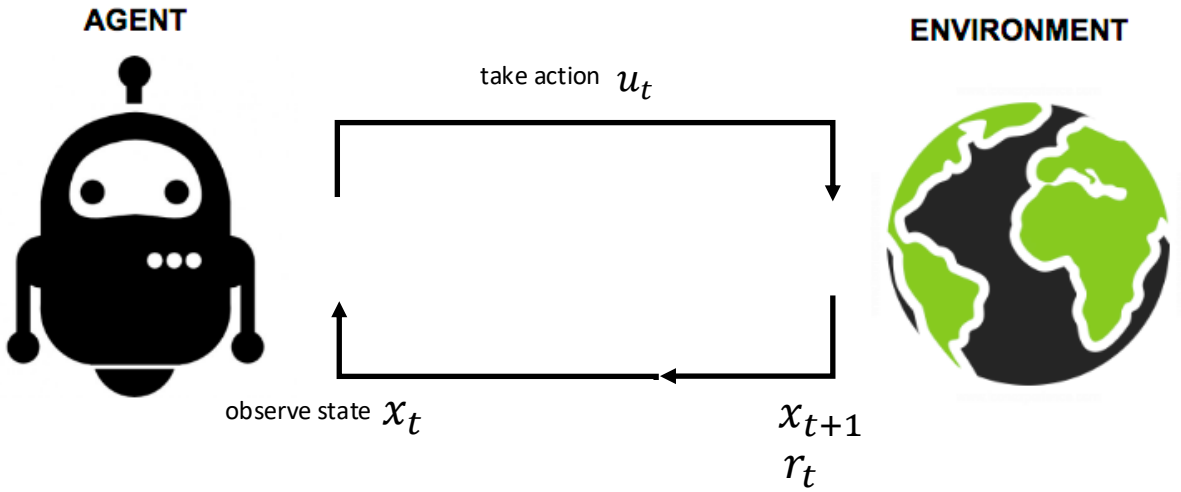
- An overview of the fundamentals of IL and RL
- Identify the distinctions and similarities between these paradigms

What is reinforcement learning?

Fundamentally:

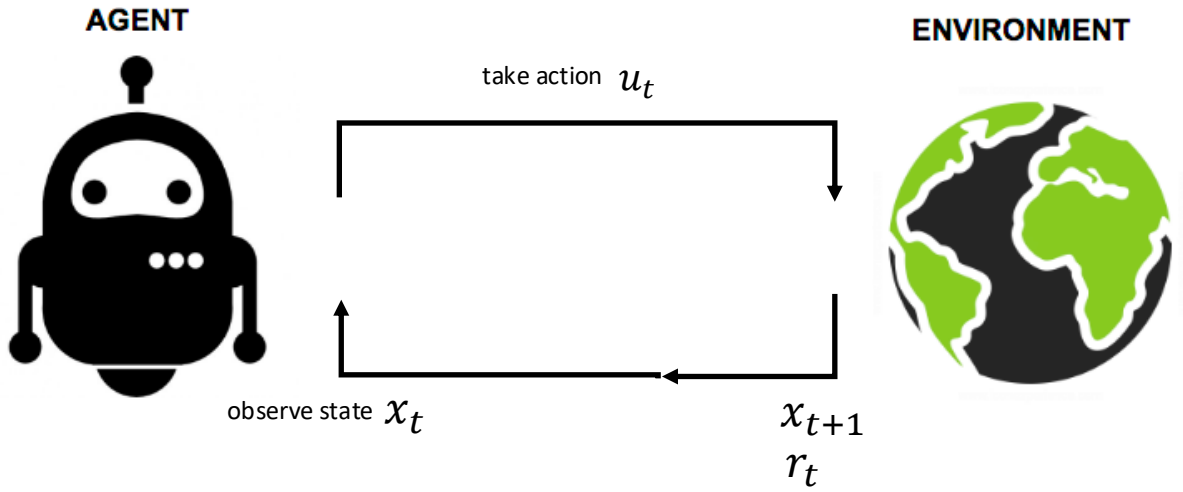
- A mathematical formalism for **learning-based** decision making
- An approach for learning decision making and control **from experience**

Most RL algorithms follow the same basic learning loop:



$$\tau = (x_0, u_0, \ldots, x_N, u_N)$$

What is reinforcement learning?

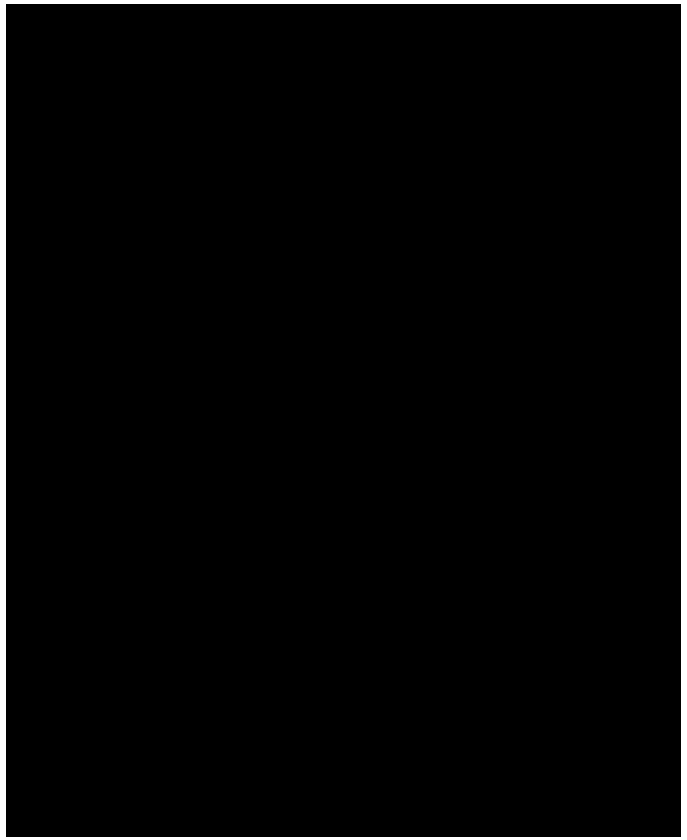


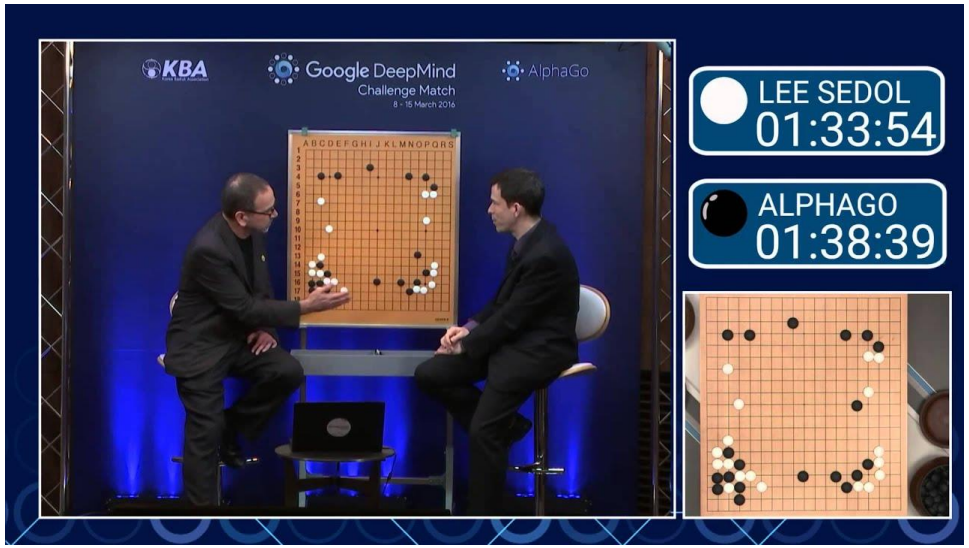
Elements of RL:

- **Policy** $\pi(u_t \mid x_t)$:a mapping from states to actions that defines the agent's behavior
- **Environment:** the system the agent interacts with. Mathematically, we represent it as $\mathrm{p}(x_{t+1} \mid x_t, u_t)$
- **Reward** $\mathrm{R}(x_t, u_t)$:scalar measure of (immediate) success. Defines the goal of the agent
- **Value function:** measures performance in the long run. Defines how much the agent can expect to achieve in the future
- **Model (optional):** represents the agent's understanding of the environment. Its goal is to mimic the behavior of the environment. Mathematically, we represent it as $\mathrm{p}(\hat{x}_{t+1} \mid x_t, u_t)$

Why reinforcement learning?

- RL can discover new, unexpected solutions

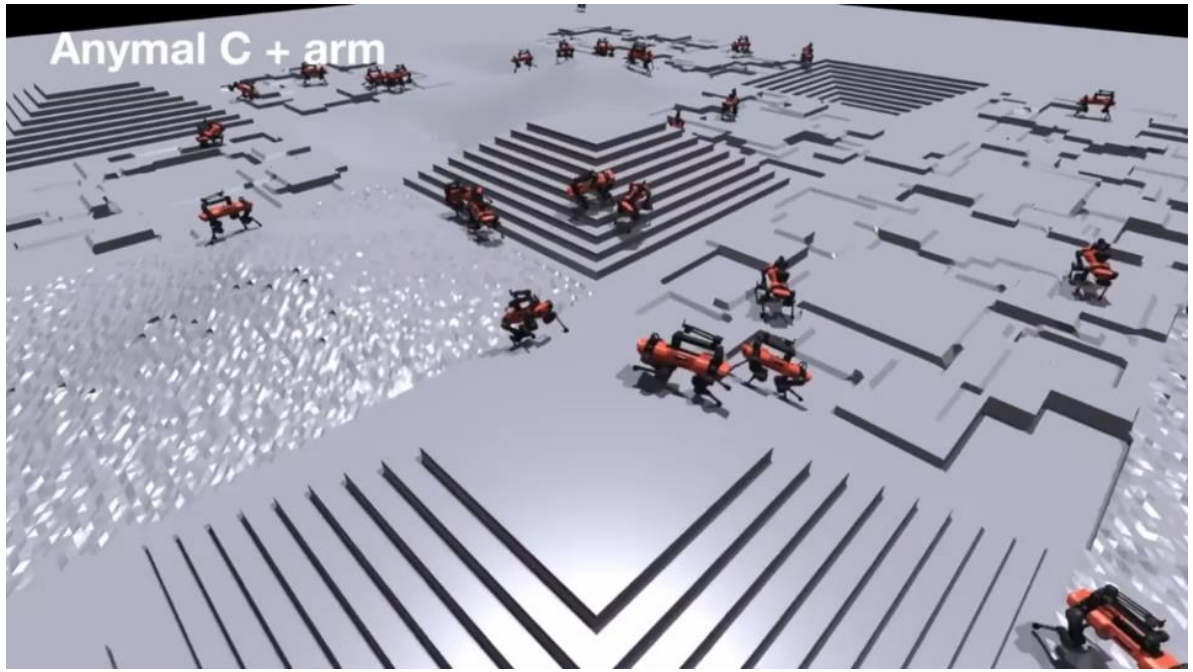




"Move 37" in Lee Sedol AlphaGo match

Why reinforcement learning?

- RL algorithms are very effective at optimizing complex objectives, with virtually no assumptions on the reward structure
  - This makes RL appealing for **complex physical tasks**



Characteristics of reinforcement learning?

How does RL differ from other machine learning paradigms?

- No supervision, only a reward signal
- Feedback can be **delayed**, not instantaneous (i.e., credit assignment)
- Data is **not** i.i.d., earlier decisions affect later interactions (tension between exploration and exploitation)

Markov Decision Process

State: $x \in \mathcal{X}$

Action: $u \in \mathcal{U}$

Transition function / Dynamics: $T(x_t \mid x_{t-1}, u_{t-1}) = p(x_t \mid x_{t-1}, u_{t-1})$

Reward function: $r_t = R(x_t, u_t): \mathcal{X} \times \mathcal{U} \to \mathbb{R}$

Discount factor: $\gamma \in (0,1)$

Typically represented as a tuple

$$\mathcal{M} = (\mathcal{X}, \mathcal{U}, T, R, \gamma)$$

- The agent interacts with the environment to generate trajectories $\tau = (x_0, u_0, x_1, u_1, \ldots x_T)$
- We define the trajectory distribution

$$p(x_0, u_0, \ldots, x_T) = p(\tau) = p(x_0) \prod_{t=1}^{T} \pi(u_t \mid x_t) p(x_{t+1} \mid x_t, u_t)$$

- We can express the RL objective as an expectation under the trajectory distribution

$$\pi^* = \arg\max_{\pi} \mathbb{E}_{\tau \sim p(\tau)} \left[\sum_{t \geq 0} \gamma^t R\left(x_t, u_t\right) \right]$$

Some examples

Chess

State:	$x \in \mathcal{X}$	*position of pieces*
Action:	$u \in \mathcal{U}$	*next move*
Dynamics	$p(x_t \mid x_{t-1}, u_{t-1})$	*given a move, defines the next configuration of the board (deterministic)*
Reward function:	$r_t = R(x_t, u_t)$	*-1/+1 for lost/won game*

Quadruped

State:	$x \in \mathcal{X}$	*robot's pose (e.g., foot positions, joint angles) and its position on the terrain*
Action:	$u \in \mathcal{U}$	*joint torque commands for each leg*
Dynamics	$p(x_t \mid x_{t-1}, u_{t-1})$	*the robot applies the chosen torque commands to its joints, deterministically resulting in the next pose and position based on the physics of the terrain*
Reward function:	$r_t = R(x_t, u_t)$	*+10 forward step / -5 falling / 0 standing still*

Value functions

State-value function: "*the expected total reward if we start in that state and act accordingly to a particular policy*"

$$\mathrm{V}_{\pi}(x_t) = \mathbb{E}_p \left[\sum_{t' \geq t} \gamma^{t'} R\left(x_{t'}, \pi\left(x_{t'}\right)\right) \right]$$

Action-state value function: "*the expected total reward if we start in that state, take an action, and act accordingly to a particular policy*"

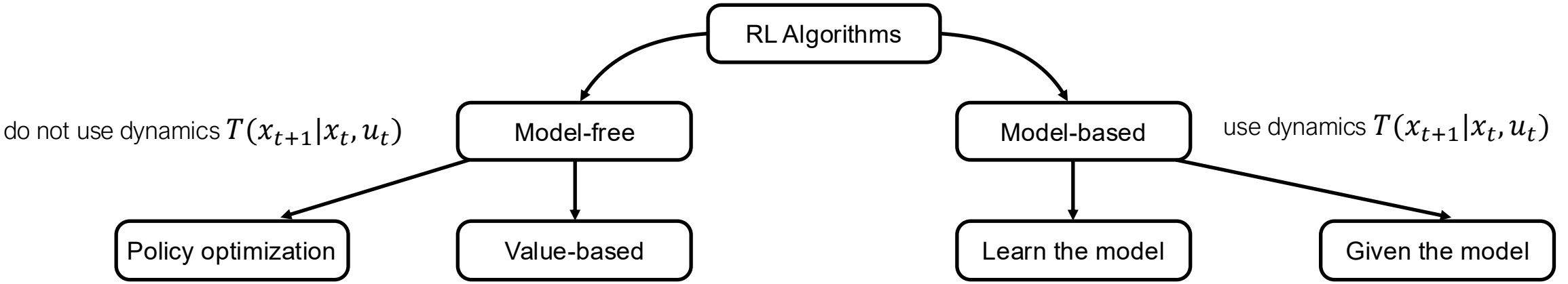
$$\mathrm{Q}_{\pi}(x_t, u_t) = \mathbb{E}_p \left[\sum_{t' \geq t} \gamma^{t'} R\left(x_{t'}, u_{t'}\right) \right]$$

Optimal state-value function: $V^*(x) = \max_{\pi} V_{\pi}(x)$

Optimal action-state value function: $Q^*(x, u) = \max_{\pi} Q_{\pi}(x, u)$

The majority of RL algorithms are defined via manipulations of these concepts

A taxonomy of RL

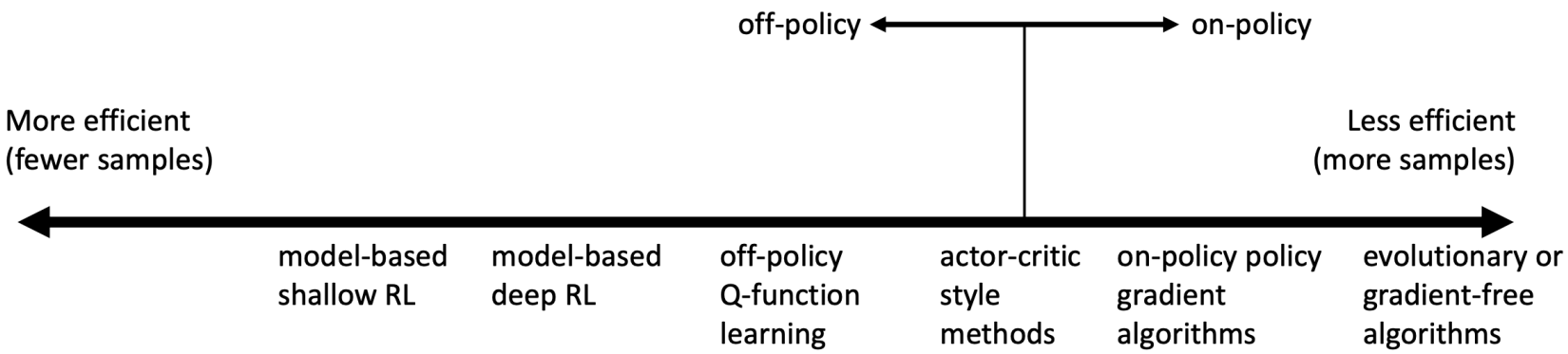


Why so many RL algorithms?

- Different tradeoffs:
  - Sample efficiency
  - Stability & easy of use
- Different assumptions:
  - Stochastic or deterministic
  - Continuous or discrete
  - Episodic or infinite horizon
- Different things are easy or hard in different settings:
  - Easier to represent the policy?
  - Easier to represent the model?

Comparison: sample efficiency

- Sample efficiency = how many samples do we need to get a good policy?
- Crucial question: is the algorithm *off policy?*
  - Off policy: able to improve the policy without generating new samples from the current policy
  - On policy: each time the policy is changed, even a little bit, we need to generate new samples



Why even bother using less efficient algorithms? Wall-clock time is not the same as efficiency!

Comparison: stability and ease of use

- Does it converge?
- And if it does, to what?
- Does it *always* converge?

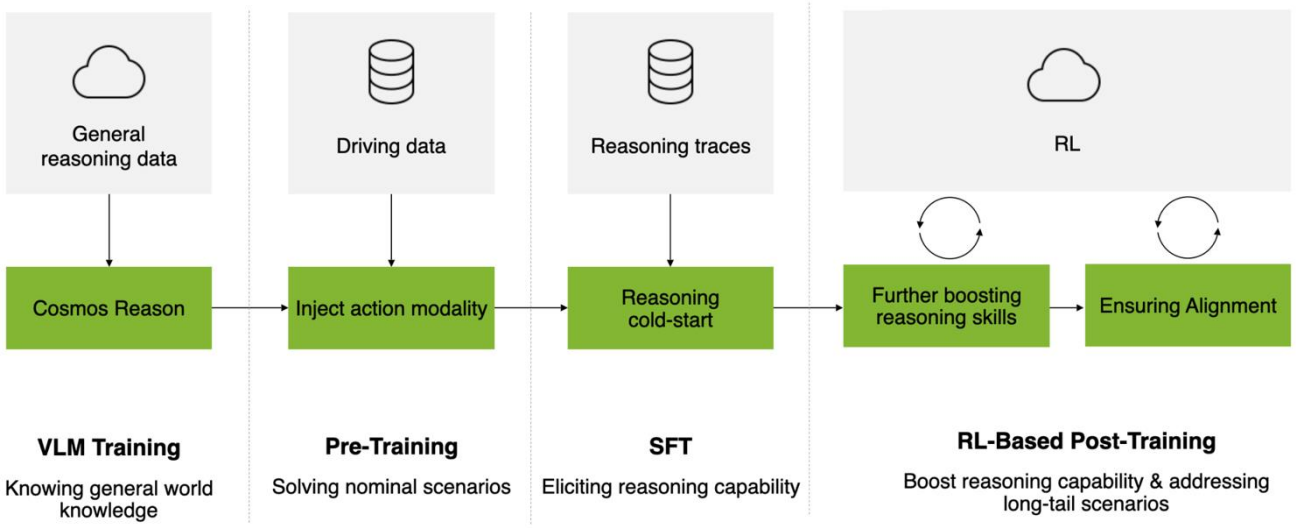
- Supervised learning: almost always gradient descent
- Reinforcement learning: often not gradient descent
  - Q-learning: fixed point iteration
  - Model-based RL: model estimator is not optimized for expected reward

Some remarks

- So far, it may be tempting to view IL and RL as “alternatives”

Imitation Learning:	Pros: Simplicity, Efficiency, No need for explicit reward signal Cons: Dependence on demonstration quality, generalization, no exploration
Reinforcement Learning:	Pros: Autonomous discovery, long-term optimization in complex tasks, no labels Cons: Training instability, sample inefficiency, reward function design

- Today, IL and RL are widely used in scaffolded training pipelines, where both are essential for achieving the desired behavior, robustness, and overall performance.



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Next class

- Imitation Learning (IL)