AA203 Optimal and Learning-based Control

Lecture 15

Model-free RL: Value-based methods

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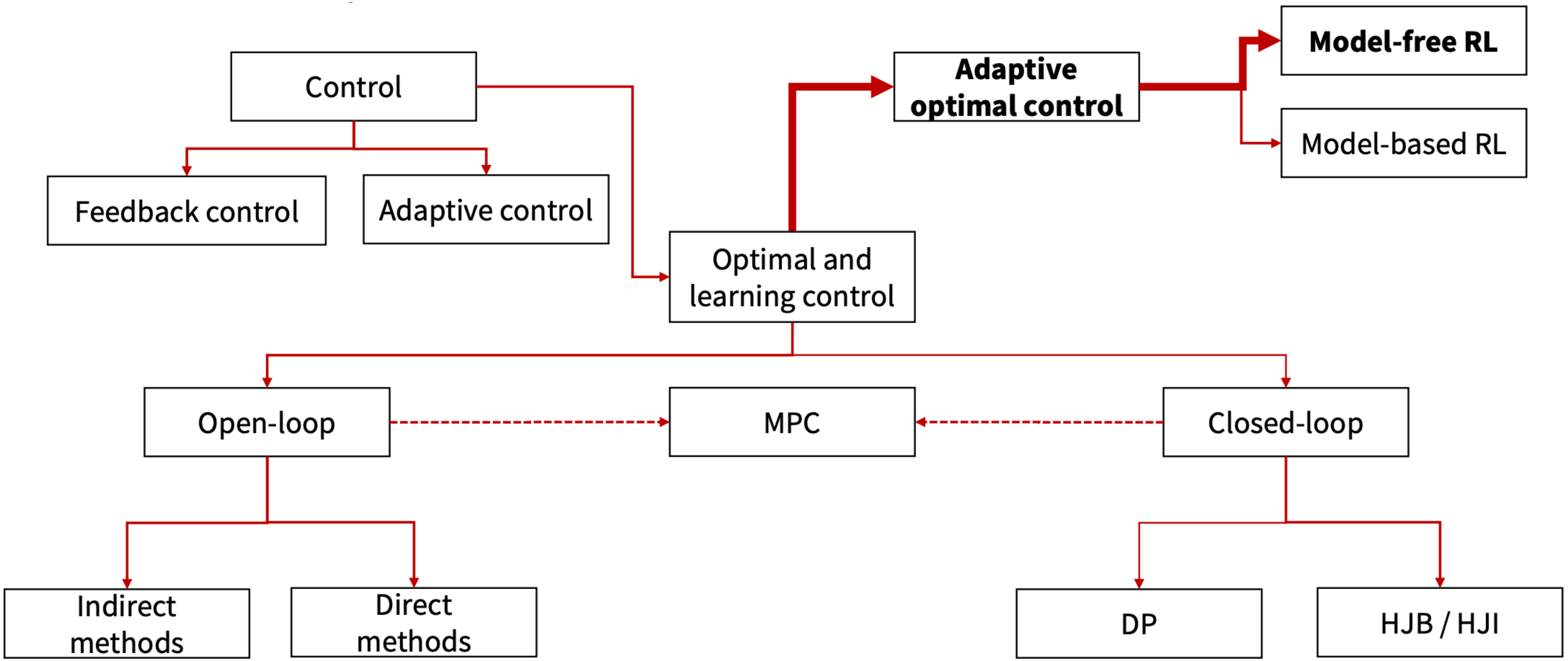
Roadmap



Control
Feedback control
Adaptive control
Optimal and learning control
Adaptive optimal control
Model-free RL
Model-based RL
Open-loop
MPC
Closed-loop
Indirect methods
Direct methods
DP
HJB / HJI

Review

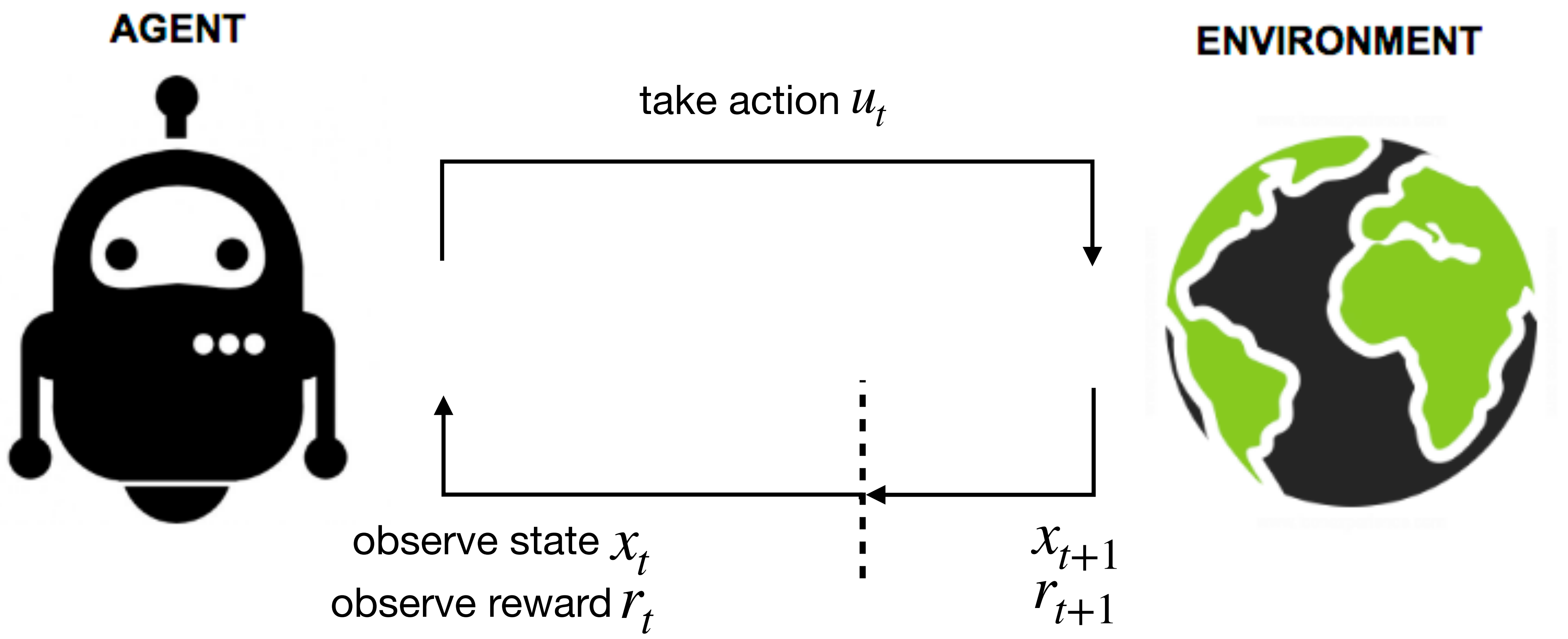
In previous lectures, we made the distinction between *prediction* (given a policy π, estimate V_π, Q_π) and *control* (learn the optimal policy π^*)

Motivated by Dynamic Programming, we discussed *exact methods* for solving MDPs:
- Policy Iteration
- Value Iteration

Problem	Bellman Equation	Algorithm
Prediction	Bellman Expectation Equation	Iterative Policy Evaluation
Control	Bellman Expectation Equation + Greedy Policy Improvement	Policy Iteration
Control	Bellman Optimality Equation	Value Iteration

Limitation: Update equations (i.e., Bellman equations) require access to dynamics model $T\left(x_{t+1} \mid x_t, u_t\right)$

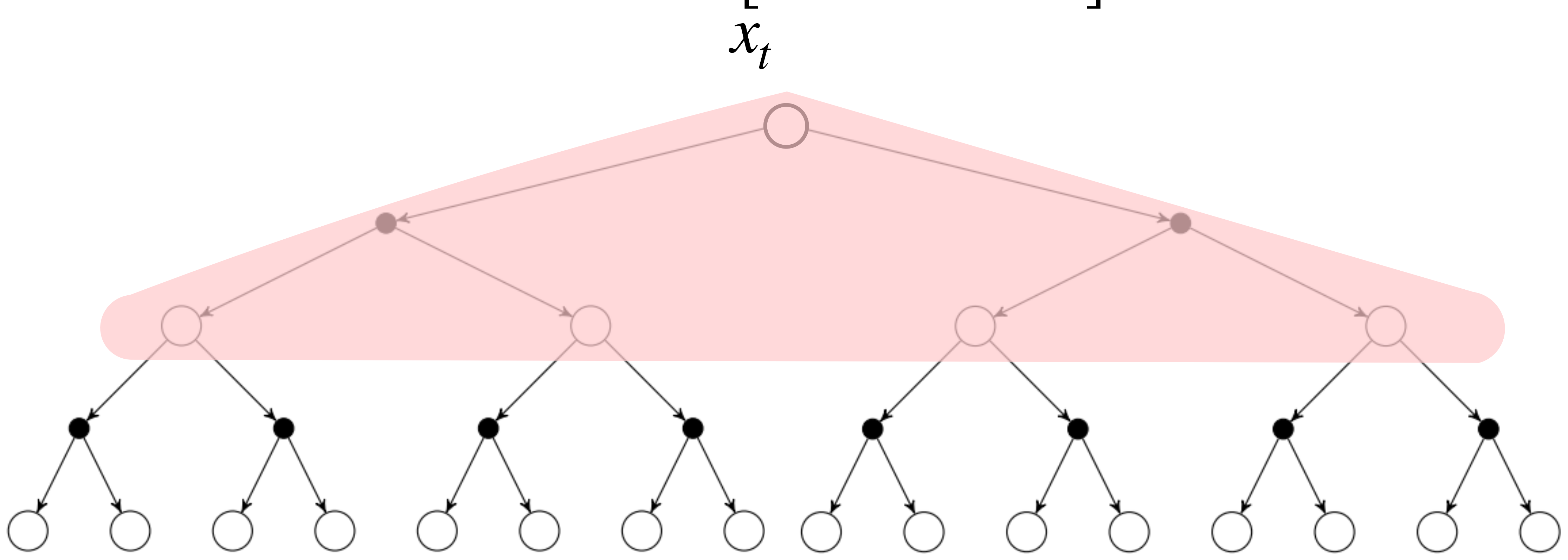
We saw how to use **sampling and bootstrapping** to approximate the expectations in the update equations:
- Monte Carlo (MC) Learning
- Temporal-Difference (TD) Learning



- For prediction:

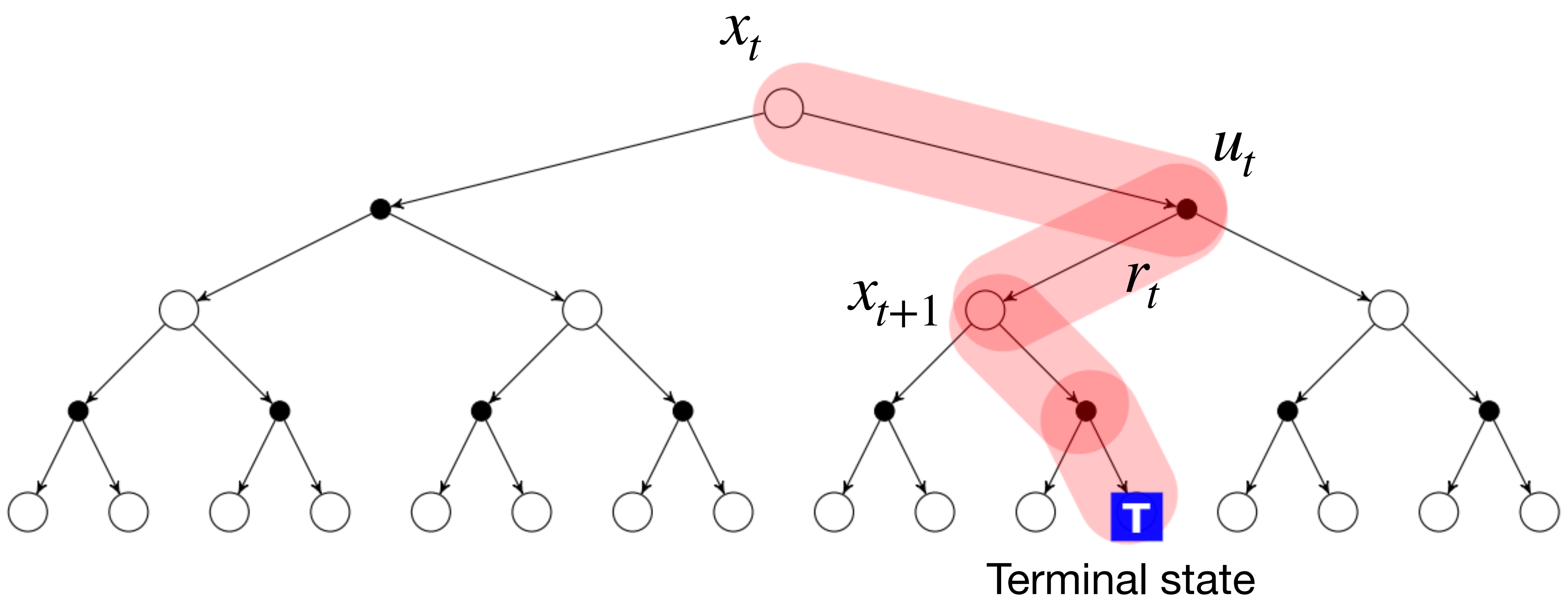
Dynamic Programming backup

$$\hat{V}(x_t) \leftarrow \mathbb{E}\left[R_t + \gamma \hat{V}(x_{t+1})\right]$$



Monte Carlo backup

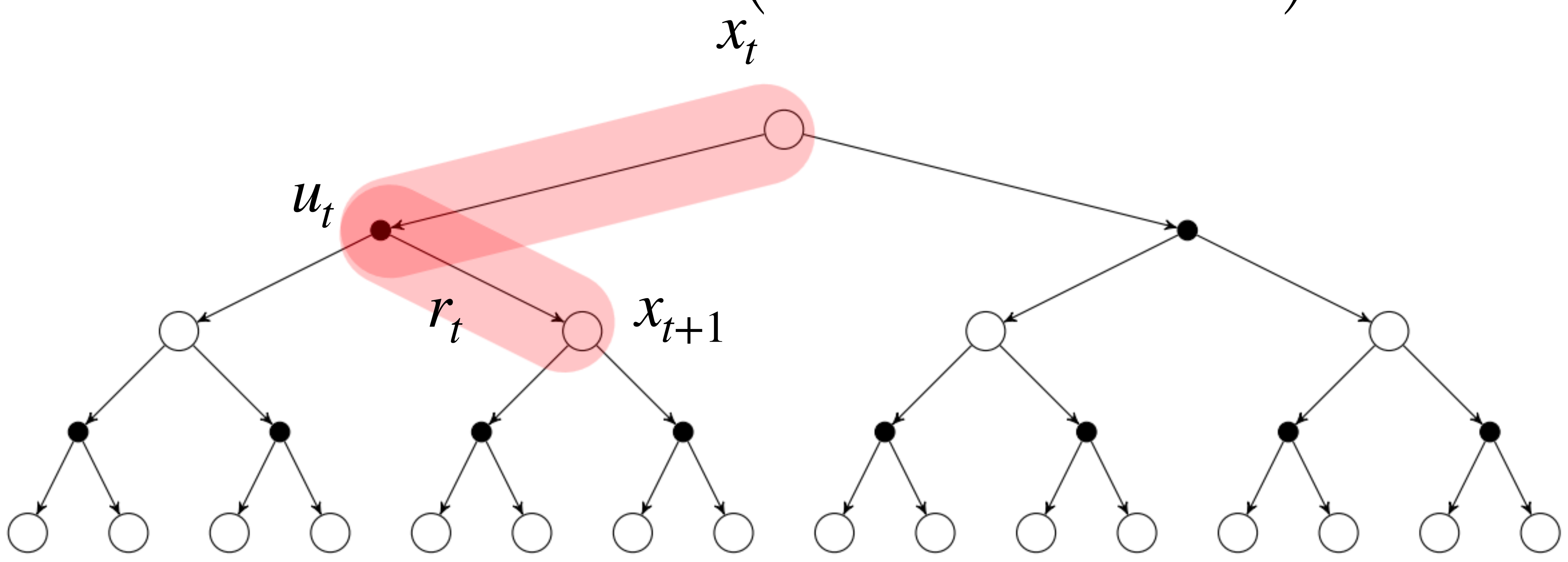
$$\hat{V}(x_t) \leftarrow \hat{V}(x_t) + \alpha\left(G_t - \hat{V}(x_t)\right)$$



- **Sampling:** define the update through samples to approximate expectations
    - MC samples
    - TD samples
    - DP does not sample

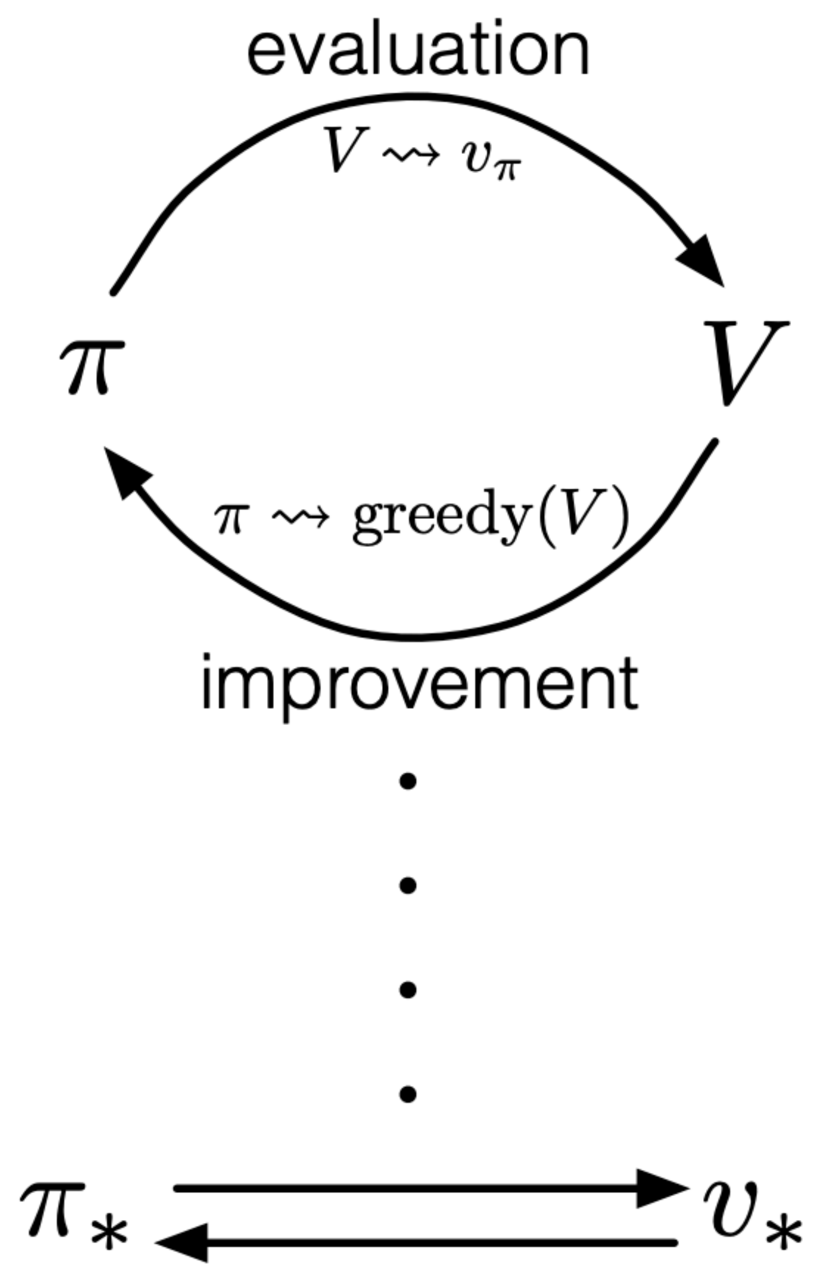
Temporal-Difference backup

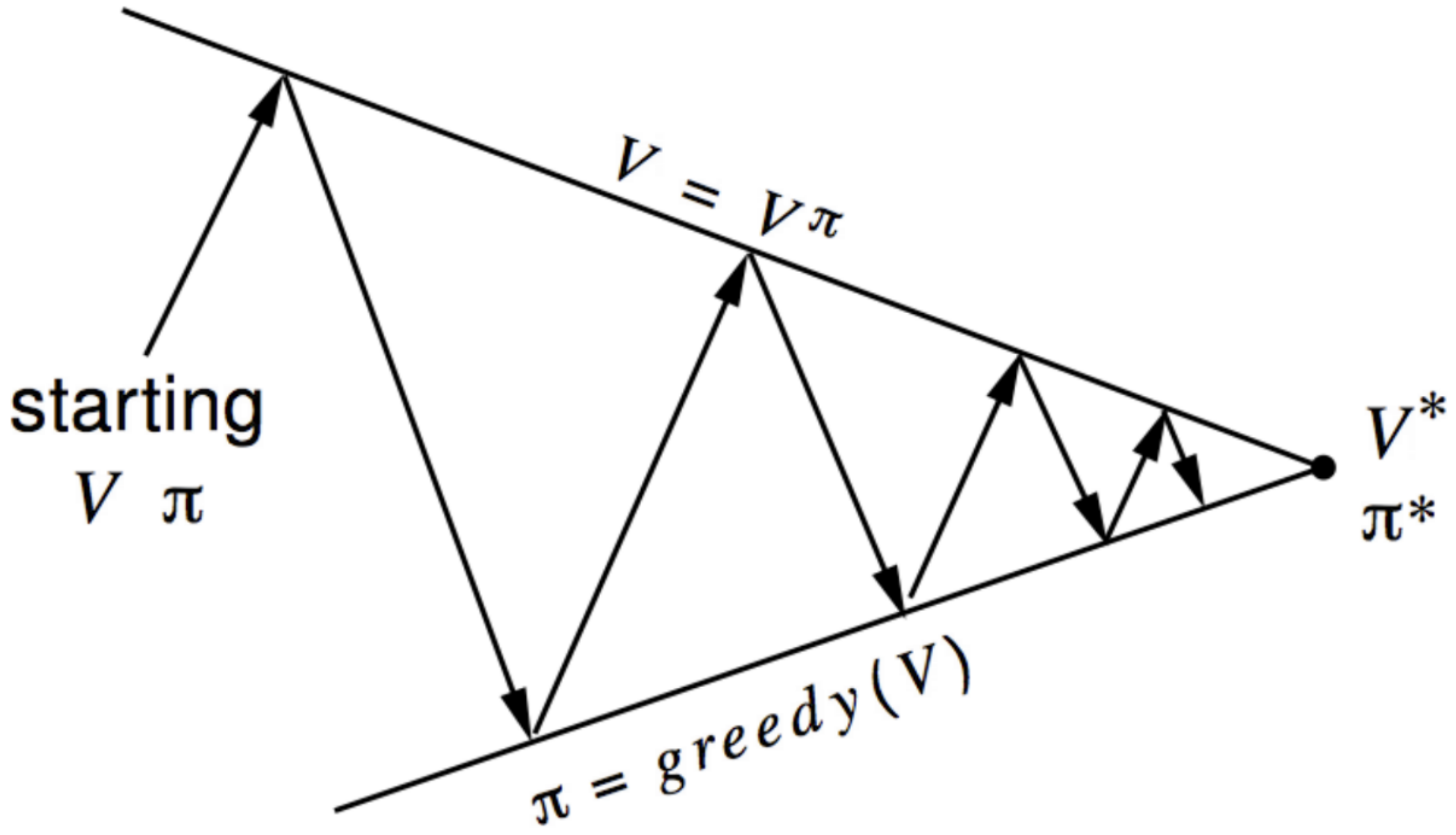
$$\hat{V}(x_t) \leftarrow \hat{V}(x_t) + \alpha\left(R_t + \gamma \hat{V}(x_{t+1}) - \hat{V}(x_t)\right)$$



- **Bootstrapping:** define the update through an estimate
    - MC does not bootstrap
    - TD bootstraps
    - DP bootstraps

- For control: GPI





Problem 1:

Greedy policy improvement over $V(x)$ requires a model of the MDP!

$$\pi_{k+1}(x) = \arg\max_u \left(R(x,u) + \gamma \sum_{x_{t+1} \in \mathcal{X}} T\left(x_{t+1} \mid x_t, u_t\right) V_{k+1}\left(x_{t+1}\right) \right)$$

On the other hand, greedy policy improvement over $Q(x,u)$ does not

$$\pi_{k+1}(x) = \arg\max_u Q(x,u)$$

Problem 2:

Exploration! To estimate state-action values through samples, every state-action pair needs to be visited

- With probability $1-\epsilon$, choose the greedy action
- With probability ϵ, choose a random action
- Ensures that all m actions are tried with non-zero probability

$$\pi(u \mid x) = \begin{cases} \dfrac{\epsilon}{m} + 1 - \epsilon & \text{if } u^* = \operatorname{argmax}_{u \in \mathcal{U}} Q(x,u) \\ \dfrac{\epsilon}{m} & \text{otherwise} \end{cases}$$

A taxonomy of RL

RL Algorithms

- Model-free — do not use dynamics $T(x_{t+1} \mid x_t, u_t)$
  - Policy optimization — directly maximize the RL objective
    $$\mathbb{E}_{\tau \sim p_\pi(\tau)}\left[\sum_{t=0}^{H} \gamma^t r\left(x_t, u_t\right)\right]$$
  - Value-based — policy implicitly defined via $V(x)$ or $Q(x, u)$
    set $\pi\left(\mathbf{x}_t\right) = \arg\max_a Q\left(\mathbf{x}_t, \mathbf{u}_t\right)$
- Model-based — use dynamics $T(x_{t+1} \mid x_t, u_t)$
  - Learn the model — estimate $f_\theta \approx T(x_{t+1} \mid x_t, u_t)$
  - Given the model — $T(x_{t+1} \mid x_t, u_t)$ is known

Outline

Tabular methods

- On-policy & Off-policy
  - SARSA
  - Q-learning

Value function approximation

Deep (Value-based) RL Methods & Applications

Temporal-Difference Control

- TD learning has several advantages over MC
  - Lower variance
  - Online
  - Incomplete sequences

- Natural idea: use TD instead of MC in our GPI scheme
  - Apply TD to estimate $Q(x, u)$
  - Use ϵ-greedy policy improvement
  - Update every time-step

Updating action-value functions with Sarsa

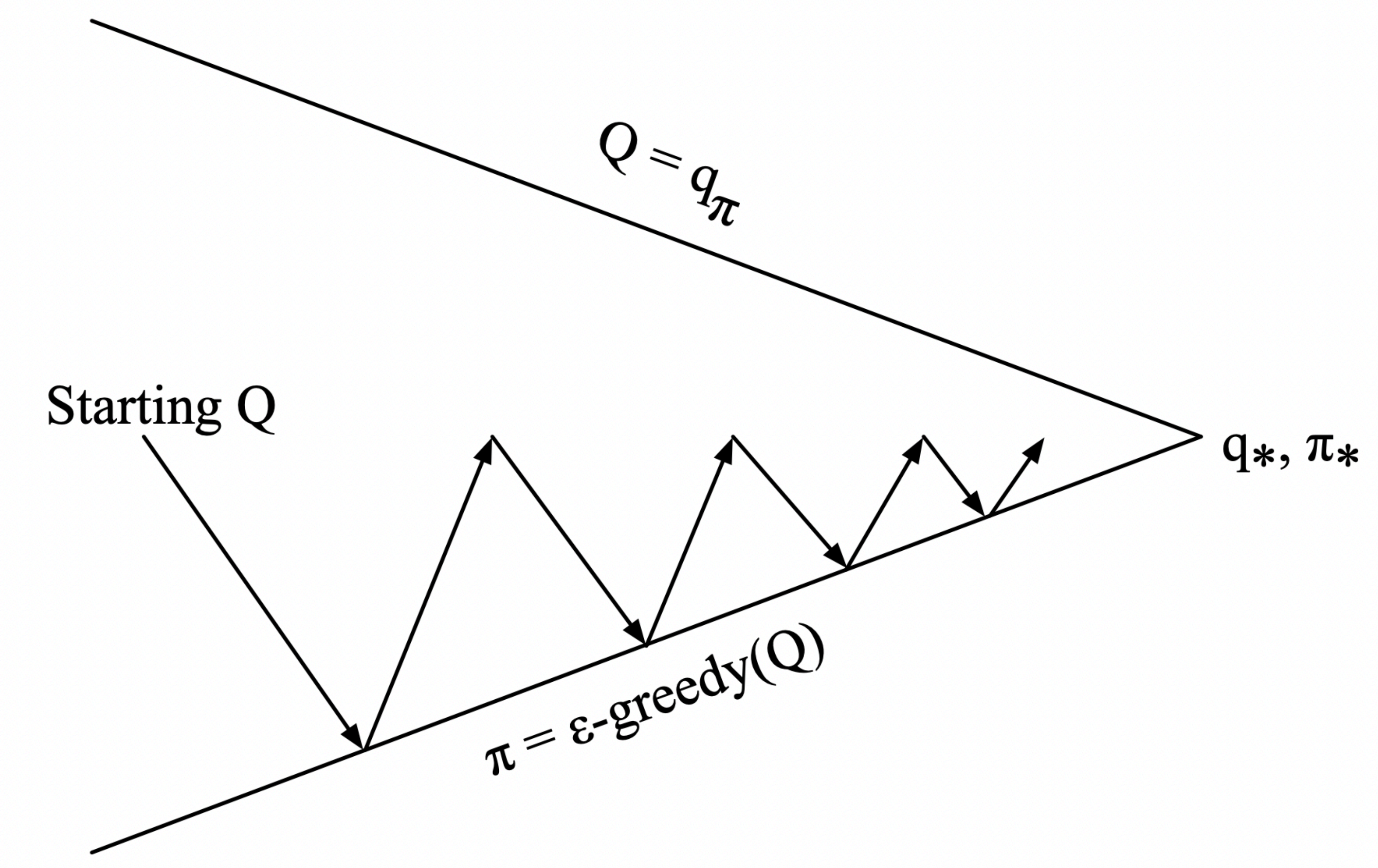
- Uses every element of the quintuple of events, $(x_t, u_t, r_t, x_{t+1}, u_{t+1})$, that make up a transition from one state–action pair to the next through the following update rule

$$Q(x_t, u_t) \leftarrow Q(x_t, u_t) + \alpha \left(r_t + \gamma Q\left(x_{t+1}, u_{t+1}\right) - Q(x_t, u_t) \right)$$

Temporal-Difference backup

$$\hat{V}\left(x_t\right) \leftarrow \hat{V}\left(x_t\right) + \alpha \left(R_t + \gamma \hat{V}\left(x_{t+1}\right) - \hat{V}\left(x_t\right) \right)$$

- In RL literature, $(x_t, u_t, r_t, x_{t+1}, u_{t+1})$ is often expressed as $(s_t, a_t, r_t, s_{t+1}, a_{t+1})$: hence the name



Every time-step:

Policy evaluation Sarsa, $Q \approx q_\pi$

Policy improvement ϵ-greedy policy improvement

Sarsa algorithm

Initialize $Q(x, u), \forall x \in X, \forall u \in U$, arbitrarily, and $Q(\text{terminal-state}, \cdot\,) = 0$
Repeat (for each episode):
  Initialize x_t
  Choose u_t from x_t using policy derived from Q (e.g., ϵ-greedy)
  Repeat (for each step of episode):
    Take action u_t, observe r_t, x_{t+1}
    Choose u_{t+1} from x_{t+1}using policy derived from Q (e.g., ϵ-greedy)
    $Q(x_t, u_t) \leftarrow Q(x_t, u_t) + \alpha \left(r_t + \gamma Q\left(x_{t+1}, u_{t+1}\right) - Q(x_t, u_t)\right)$
    $x_t \leftarrow x_{t+1}; u_t \leftarrow u_{t+1}$
  until x_t is terminal

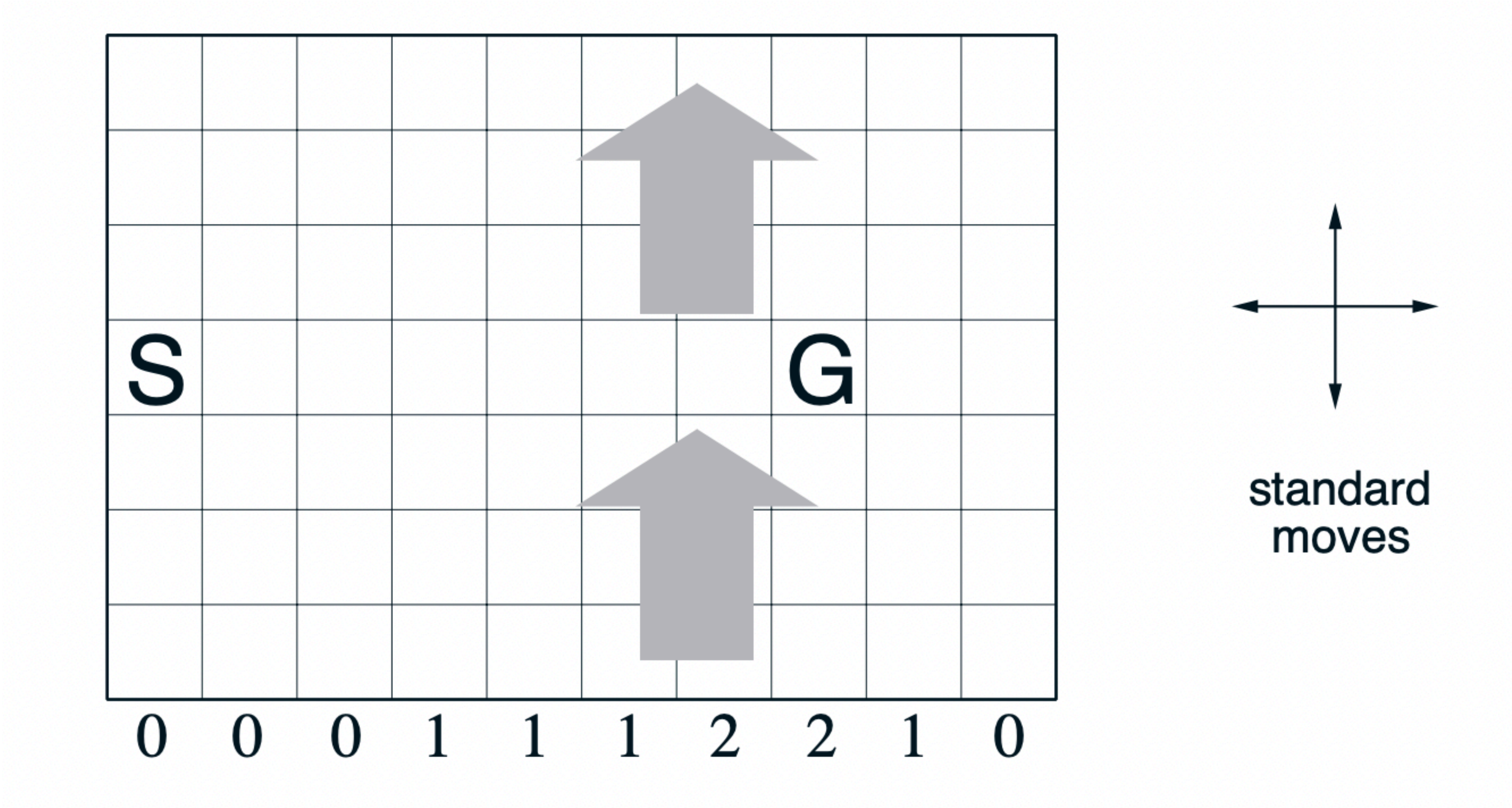
Sarsa algorithm for ?-policy control

Initialize $Q(x, u), \forall x \in X, \forall u \in U$, arbitrarily, and $Q(\text{terminal-state}, \cdot\,) = 0$
Repeat (for each episode):
    Initialize x_t
    Choose u_t from x_t using policy derived from Q (e.g., ϵ-greedy)
    Repeat (for each step of episode):
        Take action u_t, observe r_t, x_{t+1}
        Choose u_{t+1} from u_{t+1}using policy derived from Q (e.g., ϵ-greedy)
        $Q(x_t, u_t) \leftarrow Q(x_t, u_t) + \alpha \left(r_t + \gamma Q\left(x_{t+1}, u_{t+1}\right) - Q(x_t, u_t) \right)$
        $x_t \leftarrow x_{t+1}; u_t \leftarrow u_{t+1}$
    until x_t is terminal

On-policy: evaluate or improve the policy that is used to make decisions

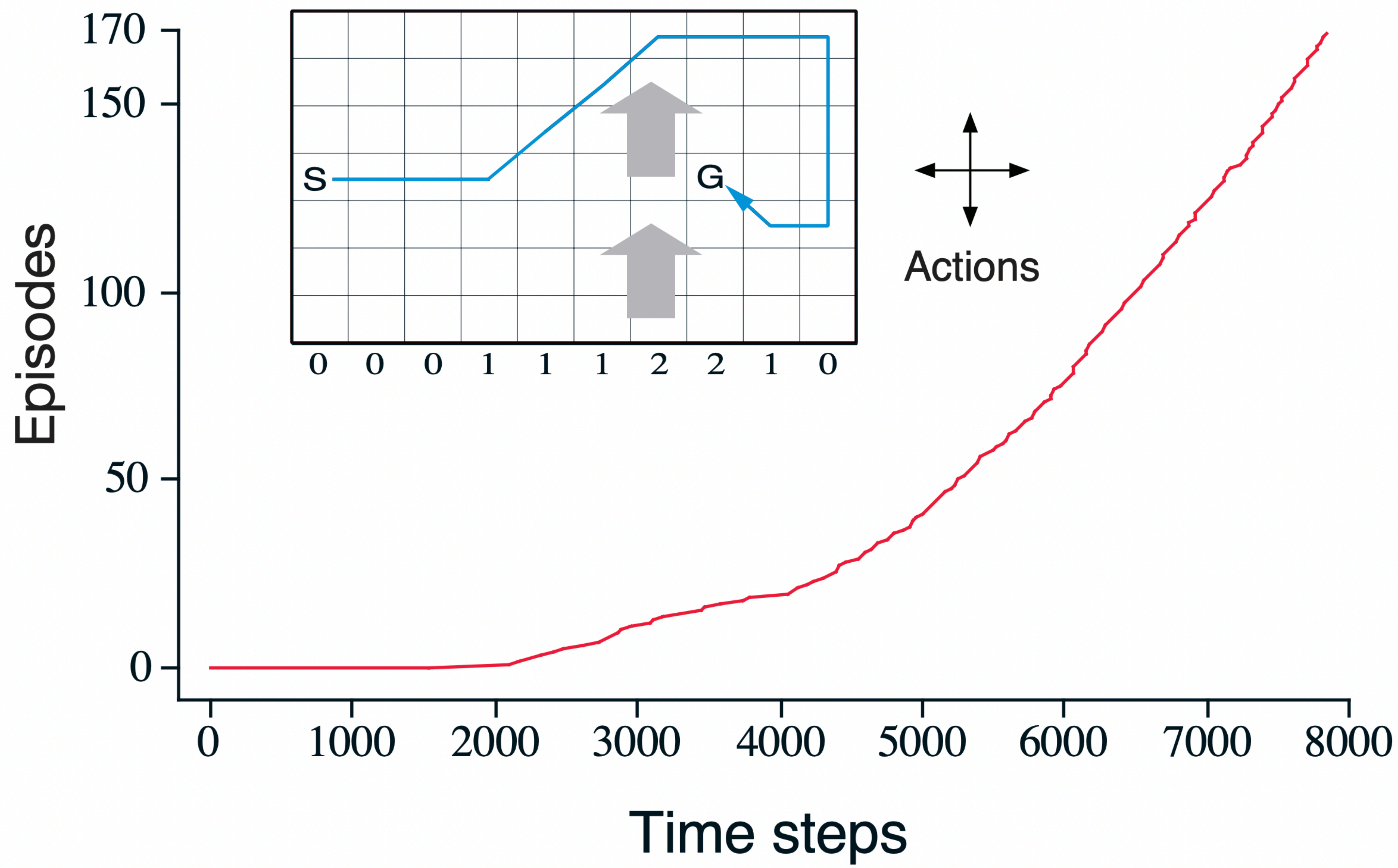
Off-policy: evaluate or improve a policy different from that used to generate the data

Windy Gridworld example



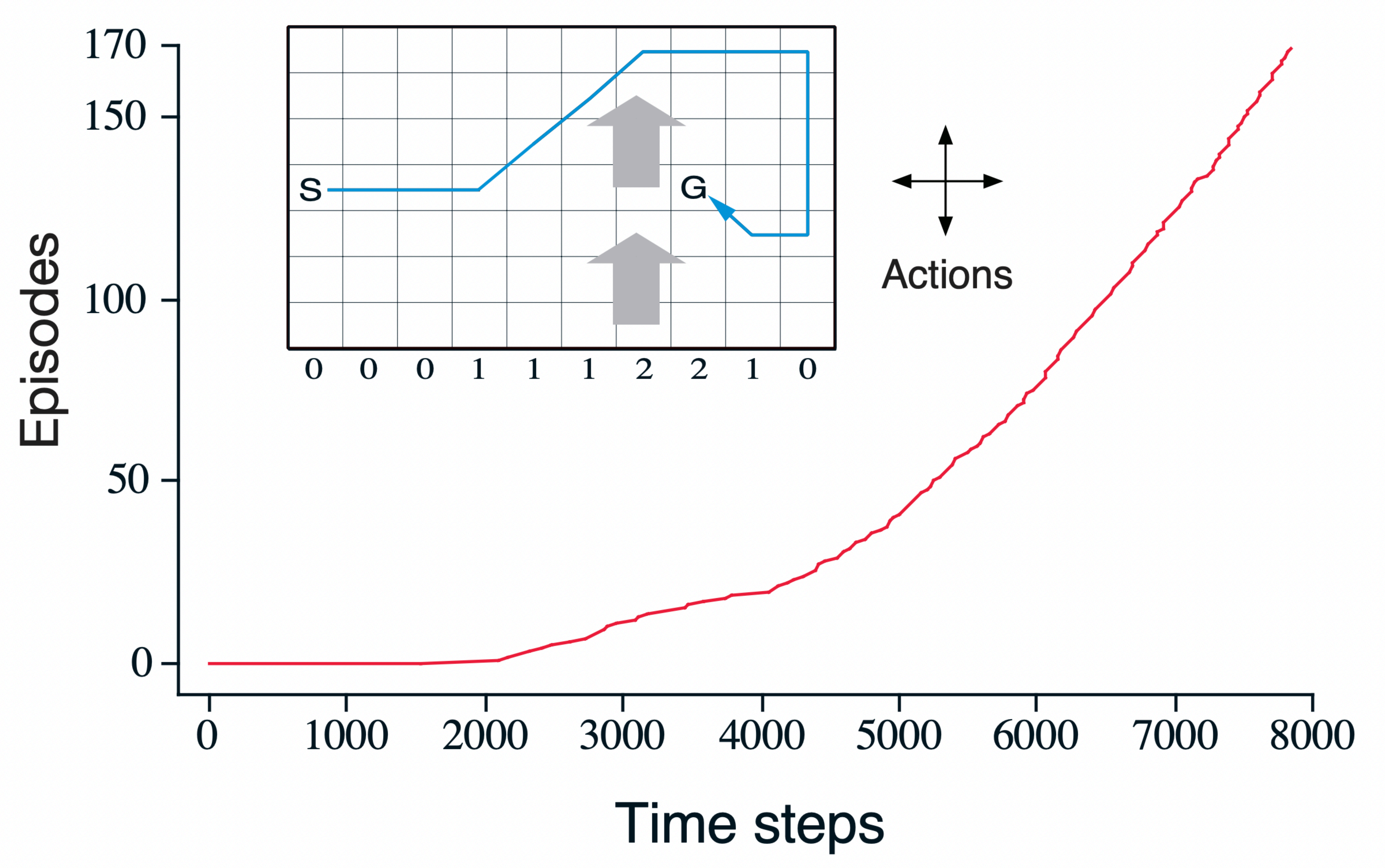
- Reward -1 until goal is reached
- $\epsilon = 0.1$
- $\alpha = 0.5$
- $\gamma = 1$

Windy Gridworld example



Question:
Would MC methods easily apply to this problem? And why?

Windy Gridworld example



Question:
Would MC methods easily apply to this problem? And why?

No, because termination is not guaranteed for all policies. If a policy was ever found that caused the agent to stay in the same state, then the next episode would never end.

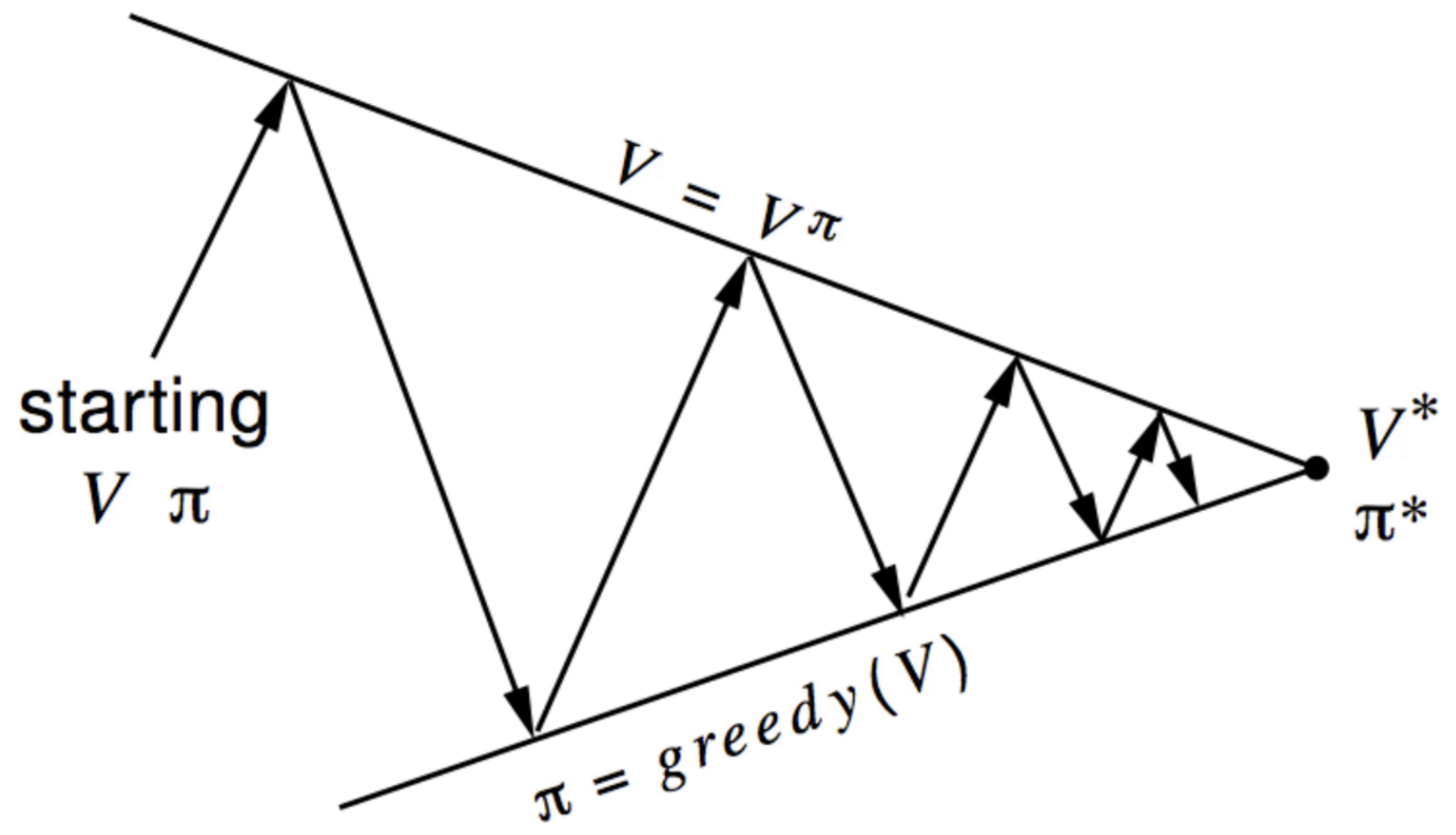
Off-policy learning

- Evaluate target policy $\pi(u \mid x)$ to compute $V_\pi(x)$ or $Q_\pi(x, u)$ while following behavior policy $\mu(u \mid x)$, i.e.,

$$\{x_1, u_1, r_1, \ldots, x_T\} \sim \mu, \text{ “the data we observe is obtained under policy } \mu\text{”}$$

Why is this important?

- Learn from observing humans or other agents
- Re-use experience generated from old policies $\pi_1, \pi_2, \ldots, \pi_{t-1}$
- Learn about **optimal** policy while following **exploratory** policy
- Learn about *multiple* policies while following *one* policy



Off-policy learning of action-values

- We consider off-policy learning of action-values $Q(x, u)$
- As in Sarsa, we use the behavior policy μ to obtain $(x_t, u_t, r_t, x_{t+1}, u'_{t+1})$, but we consider an alternative successor action $u'_{t+1} \sim \pi(u'_{t+1} \,|\, x_{t+1})$
- And update $Q(x, u)$ towards value of alternative action

$$Q(x_t, u_t) \leftarrow Q(x_t, u_t) + \alpha \left(r_t + \gamma Q\left(x_{t+1}, u'_{t+1}\right) - Q(x_t, u_t) \right)$$

Q-learning

Specifically, in Q-learning

- The target policy π is chosen as the **greedy** policy w.r.t. $Q(x, u)$

$$\pi(x_{t+1}) = \underset{u'_{t+1}}{\operatorname{argmax}}\, Q\left(x_{t+1}, u'_{t+1}\right)$$

- The behavior policy μ is chosen as the **ϵ-greedy** policy w.r.t. $Q(x, u)$

Which leads to the following Q-learning target and update:

$$\begin{aligned} & r_{t+1} + \gamma Q\left(x_{t+1}, u'_{t+1}\right) \\ = {} & r_{t+1} + \gamma Q\left(x_{t+1}, \underset{u'_{t+1}}{\operatorname{argmax}}\, Q\left(x_{t+1}, u'_{t+1}\right)\right) \\ = {} & r_{t+1} + \gamma \max_{u'_{t+1}} Q\left(x_{t+1}, u'_{t+1}\right) \end{aligned}$$

$$Q(x_t, u_t) \leftarrow Q(x_t, u_t) + \alpha\left(r_t + \gamma \max_{u'_{t+1}} Q\left(x_{t+1}, u'_{t+1}\right) - Q(x_t, u_t)\right)$$

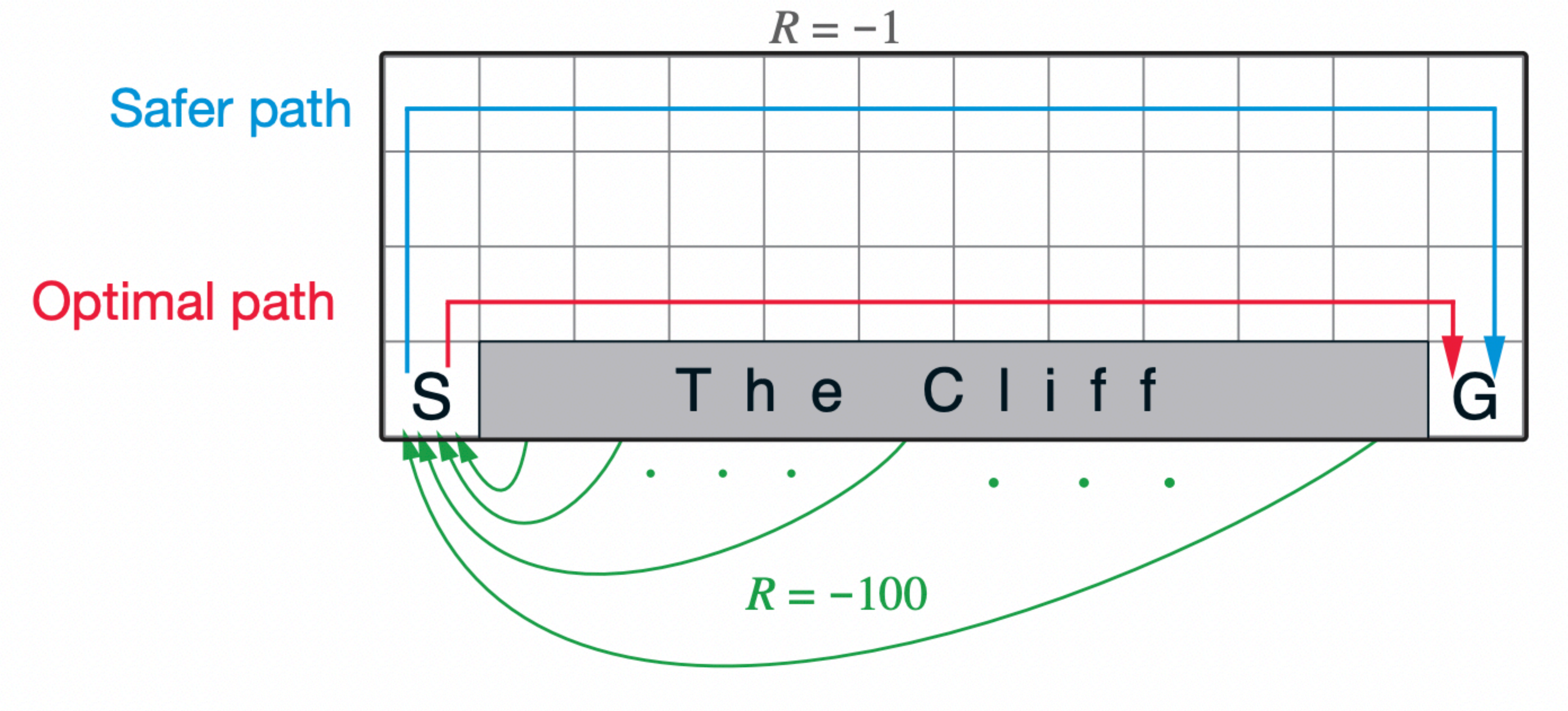
Q-learning algorithm for off-policy control

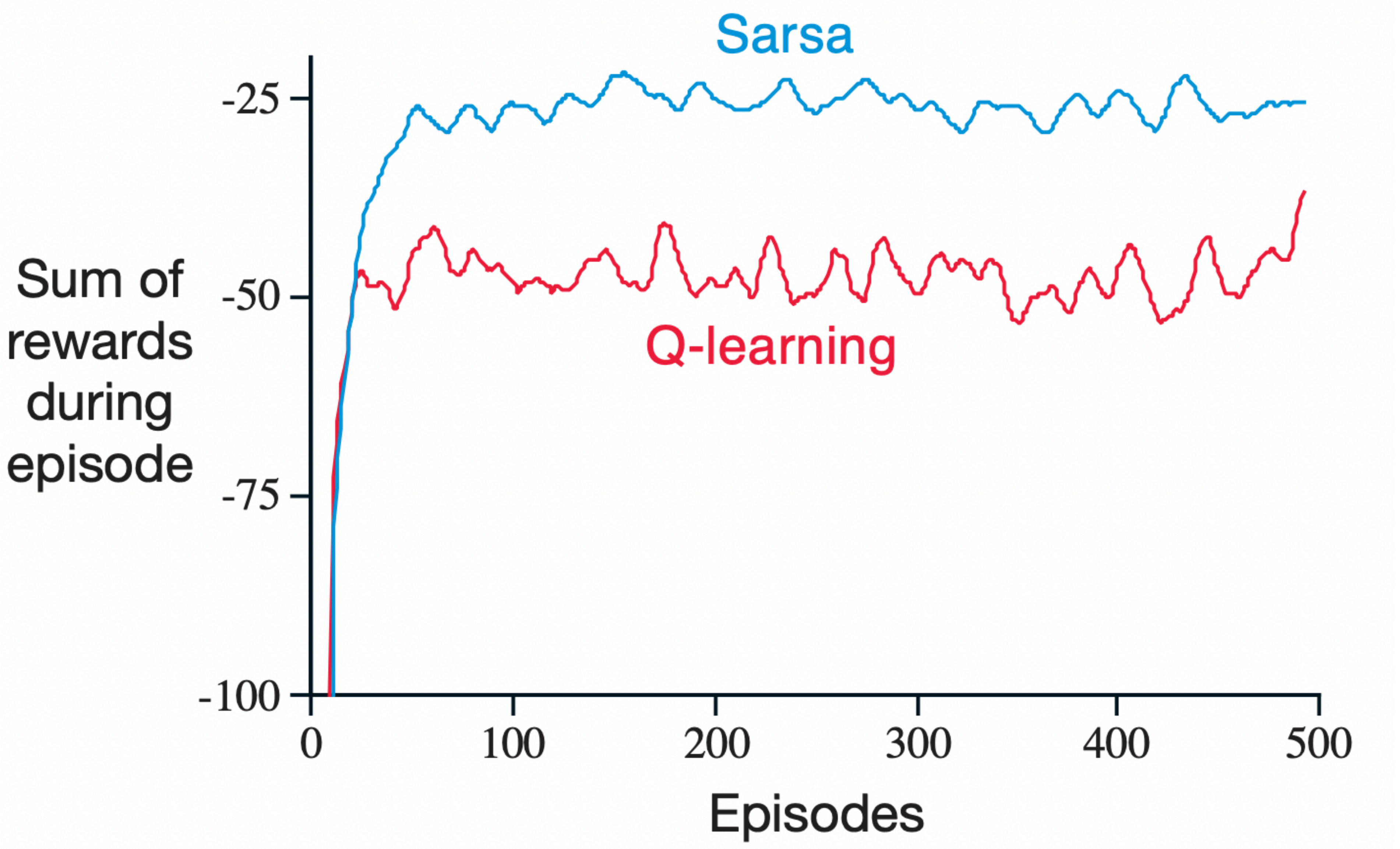
Initialize $Q(x, u), \forall x \in X, \forall u \in U$, arbitrarily, and $Q(\text{terminal-state}, \cdot\,) = 0$
Repeat (for each episode):
    Initialize x_t
    Repeat (for each step of episode):
        Choose u_t from x_t using policy derived from Q (e.g., ϵ-greedy)
        Take action u_t, observe r_t, x_{t+1}
        $Q(x_t, u_t) \leftarrow Q(x_t, u_t) + \alpha \left(r_t + \gamma \max_{u'_{t+1}} Q\left(x_{t+1}, u'_{t+1}\right) - Q(x_t, u_t) \right)$
    until x_t is terminal

Theorem

Q-learning control converges to the optimal action-value function, $Q(s, a) \to q_*(s, a)$

Differences between Sarsa and Q-learning

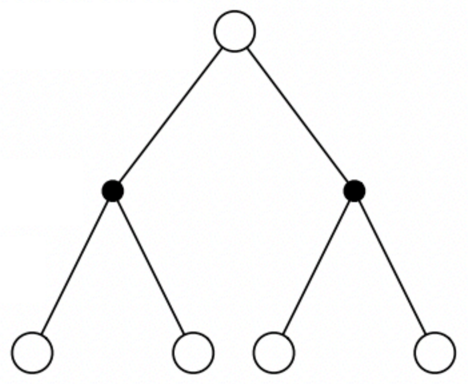
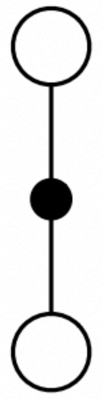
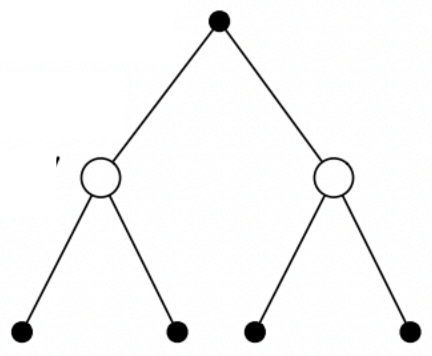
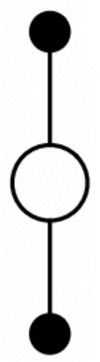
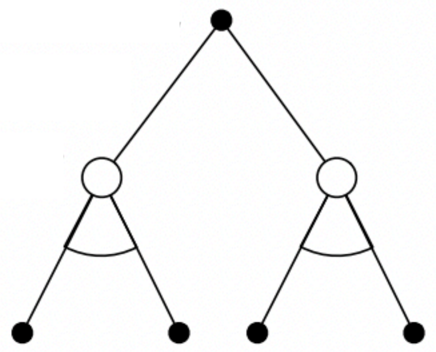
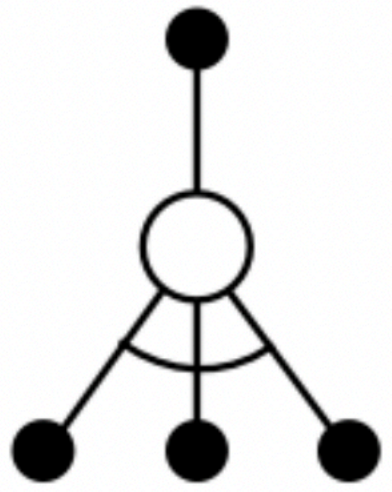




- Reward -1 until goal is reached, -100 if on "The Cliff"
- $\epsilon = 0.1$
- $\alpha = 0.5$
- $\gamma = 1$

- Sarsa converges to the **optimal ϵ-greedy policy**
- Q-learning converges to the **optimal policy π^*** / value function Q^*

Relationship between DP and TD

	Full Backup (DP)	*Sample Backup (TD)*
Bellman Expectation Equation for $V_\pi(x)$	 Iterative Policy Evaluation	 TD Learning
Bellman Expectation Equation for $Q_\pi(x, u)$	 Q-Policy Iteration	 Sarsa
Bellman Optimality Equation for $Q^*(x, u)$	 Q-Value Iteration	 Q-Learning

Outline

Tabular methods

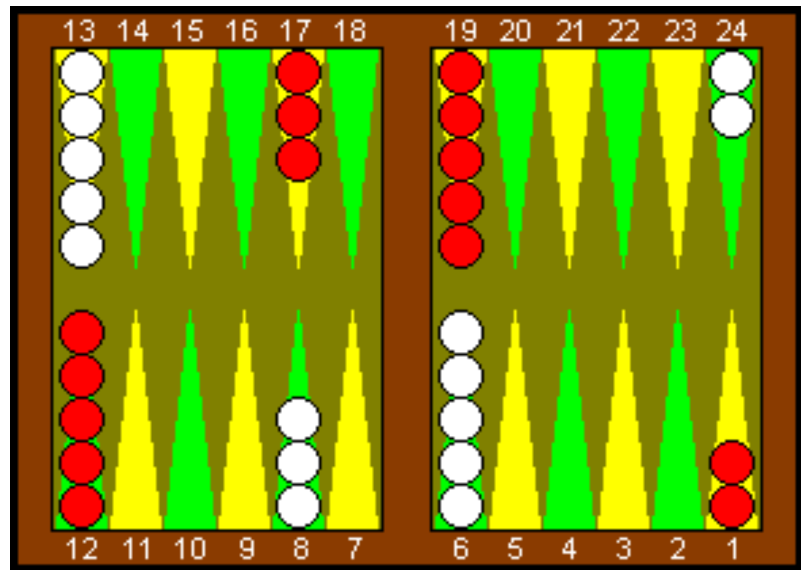
- On-policy & Off-policy
  - SARSA
  - Q-learning

Value function approximation

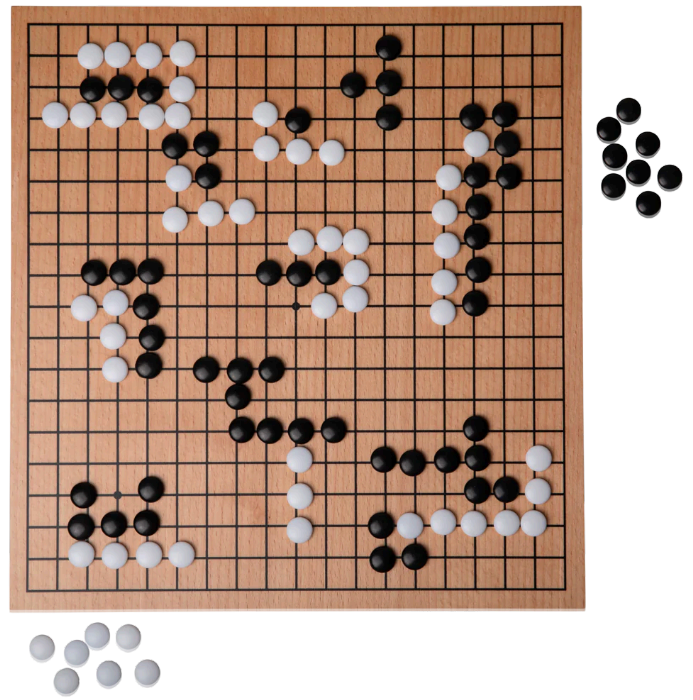
Deep (Value-based) RL Methods & Applications

Solving large-scale problems with RL

- Reinforcement learning can be used to solve *large* problems, e.g.,



Backgammon: 10^{20} states



Go: 10^{170} states

All those problems where we have a continuous state space

How can we scale the methods for model-free RL we developed over the last lectures?

Value function approximation

- So far we used **lookup tables** to represent value functions:
  - One entry for every state x in $V(x)$
  - One entry for every state-action pair (x, u) in $Q(x, u)$

- In large MDPs, lookup table might be prohibitive. For two main reasons:
  - Memory: too many actions/states to store
  - Sparsity/Curse of dimensionality: learning the value of each state/action pair individually might take too long

Solution:

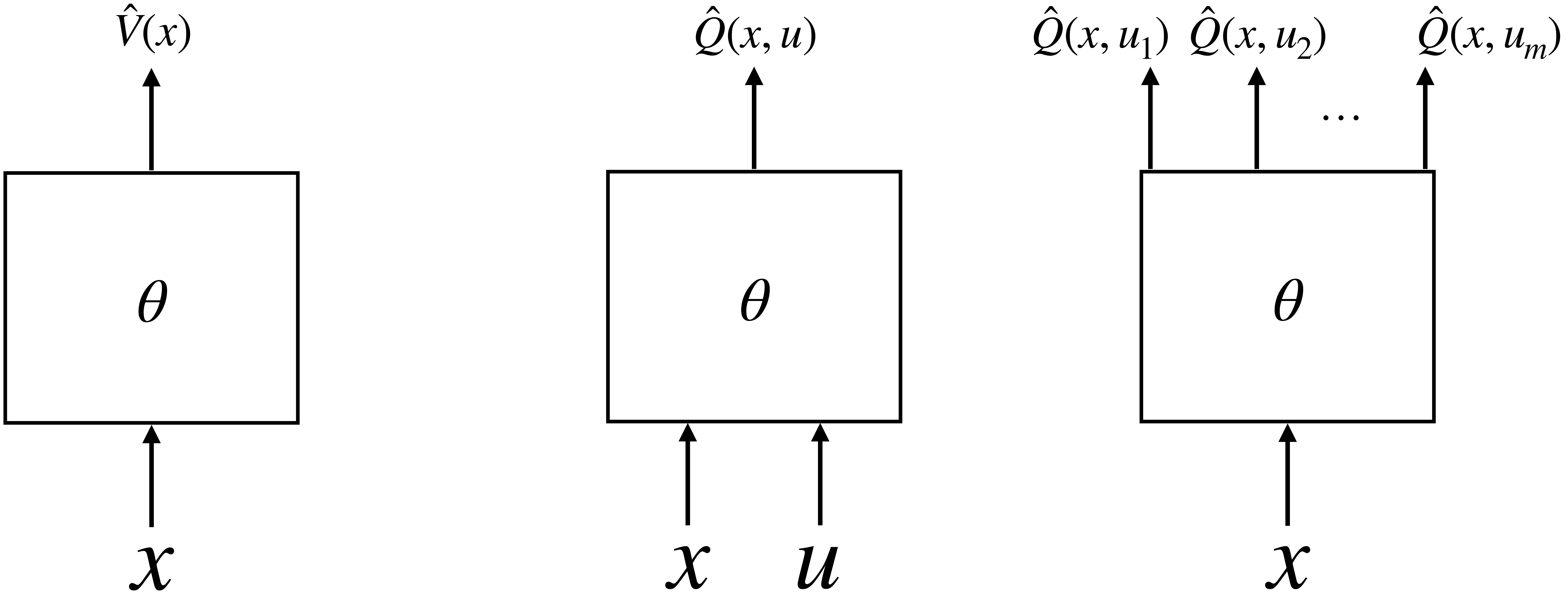
- Estimate the value function through function approximation, i.e., define a parametric function with parameters θ

$$\hat{Q}_\theta(x, u) \approx Q(x, u)$$
$$\hat{V}_\theta(x) \approx V(x)$$

→ Represent the value function compactly (depends only on parameters θ)

→ Generalize *across states* (avoid having to visit the entire state-action space by generalizing from seen to unseen states)

Different types of value function approximations



There are many possible function approximators

- Linear regression, Neural network, Random forest, Nearest neighbor, etc.

Approximating value fn. by (stochastic) gradient descent

Goal: find the parameter vector θ that minimizes the mean-squared error between the estimated value $\hat{V}_\theta(x)$ and the true value $V_\pi(x)$

$$J(\theta) = \mathbb{E}_\pi \left[V_\pi(x) - \hat{V}_\theta(x) \right]$$

Gradient descent converges to a local minimum

$$\begin{aligned} \Delta\theta &= -\frac{1}{2}\alpha \nabla_\theta J(\theta) \\ &= \alpha \mathbb{E}_\pi \left[\left(V_\pi(x) - \hat{V}_\theta(x) \right) \nabla_\theta \hat{V}_\theta(x) \right] \end{aligned}$$

Stochastic GD samples the gradient

$$\Delta\theta = \alpha \left(V_\pi(x) - \hat{V}_\theta(x) \right) \nabla_\theta \hat{V}_\theta(x)$$

Approximating value fn. by (stochastic) gradient descent

In the previous slide, we assumed to know the true value function V_π → in RL there is no supervisor, only reward

In practice, we use a *target* for V_π

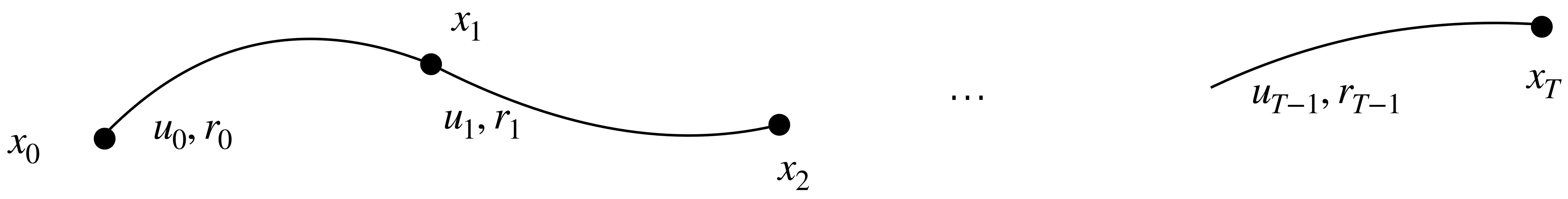
- Monte-Carlo: the target is the return

$$\Delta\theta = \alpha\left(G_t - \hat{V}_\theta(x_t)\right)\nabla_\theta \hat{V}_\theta(x_t)$$

- Temporal-Difference: the target is the TD target

$$\Delta\theta = \alpha\left(r_t + \gamma\hat{V}_\theta(x_{t+1}) - \hat{V}_\theta(x_t)\right)\nabla_\theta \hat{V}_\theta(x_t)$$

Intuition



Tabular $\hat{V}(x)$

Tabular $\hat{V}(x)$

MC

1) Compute return $G_t = r_t + \gamma r_{t+1} + \ldots, \forall t$

2) Update estimate

$$\hat{V}(x) = \begin{bmatrix} \hat{V}(x_t) + \alpha(G_t - \hat{V}(x_t)) \\ \cdots \end{bmatrix}$$

1) Collect dataset $\mathscr{D} = \{(x_t, G_t)\}$

2) Update θ

$$\theta = \theta + \alpha \left(G_t - \hat{V}_\theta(x_t) \right) \nabla_\theta \hat{V}_\theta(x_t)$$

TD

1) Compute target $r_t + \gamma \hat{V}(x_{t+1}), \forall t$

2) Update estimate

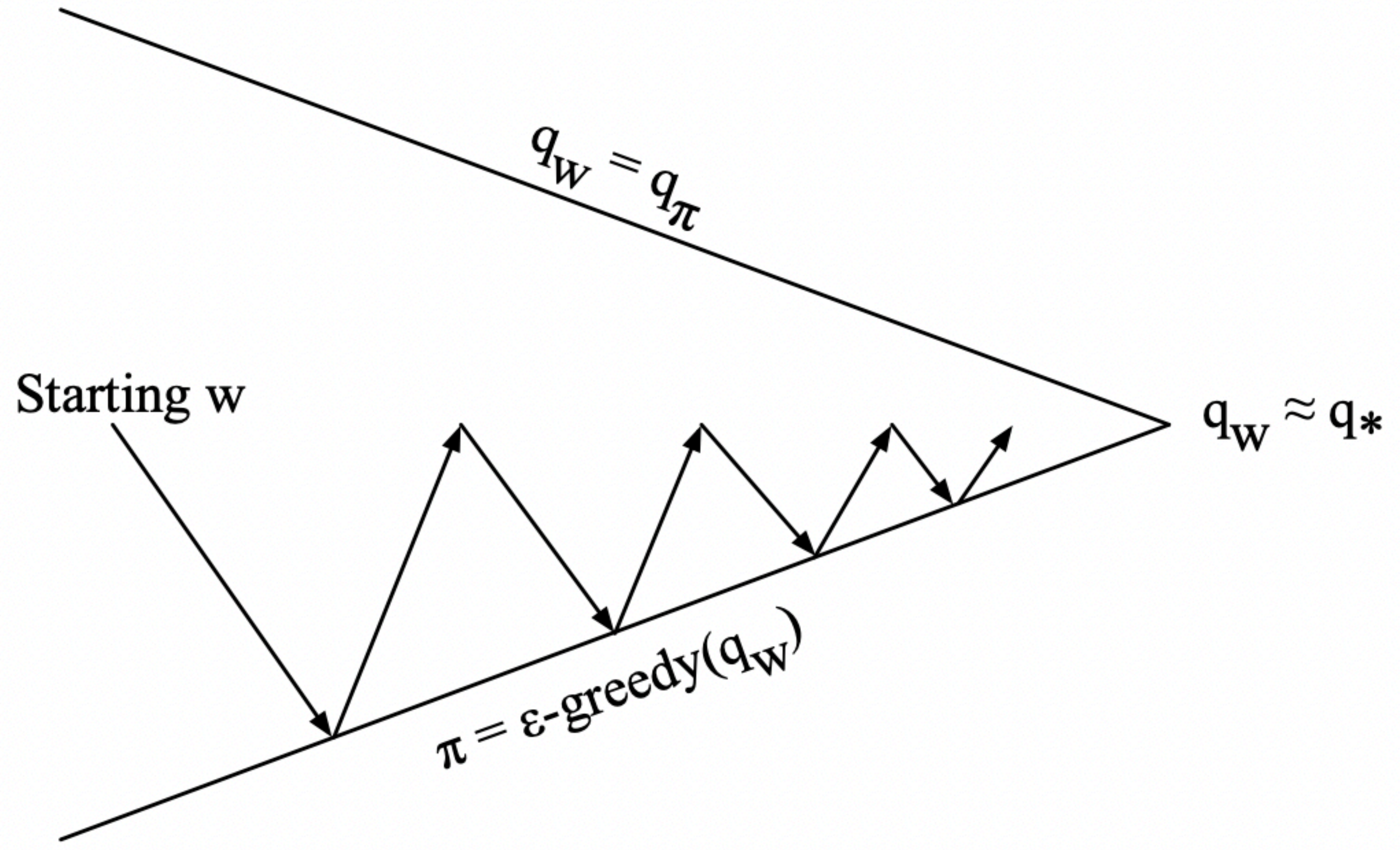
$$\hat{V}(x) = \begin{bmatrix} \hat{V}(x_t) + \alpha(r_t + \gamma \hat{V}(x_{t+1}) - \hat{V}(x_t)) \\ \cdots \end{bmatrix}$$

1) Collect dataset $\mathscr{D} = \{(x_t, r_t + \gamma \hat{V}_\theta(x_t))\}$

2) Update estimate

$$\theta = \theta + \alpha \left(r_t + \gamma \hat{V}_\theta(x_t) - \hat{V}_\theta(x_t) \right) \nabla_\theta \hat{V}_\theta(x_t)$$

Control with function approximation



Policy evaluation **Approximate** policy evaluation, $\hat{q}(\cdot, \cdot, \mathbf{w}) \approx q_\pi$

Policy improvement ϵ-greedy policy improvement

Action-value function approximation

Exactly the same intuitions apply when we try to approximate the action value function:

- Minimize the mean-squared error between the estimated value $\hat{Q}_\theta(x,u)$ and the true value $Q_\pi(x,u)$

$$J(\theta) = \mathbb{E}_\pi\left[Q_\pi(x,u) - \hat{Q}_\theta(x,u)\right]$$

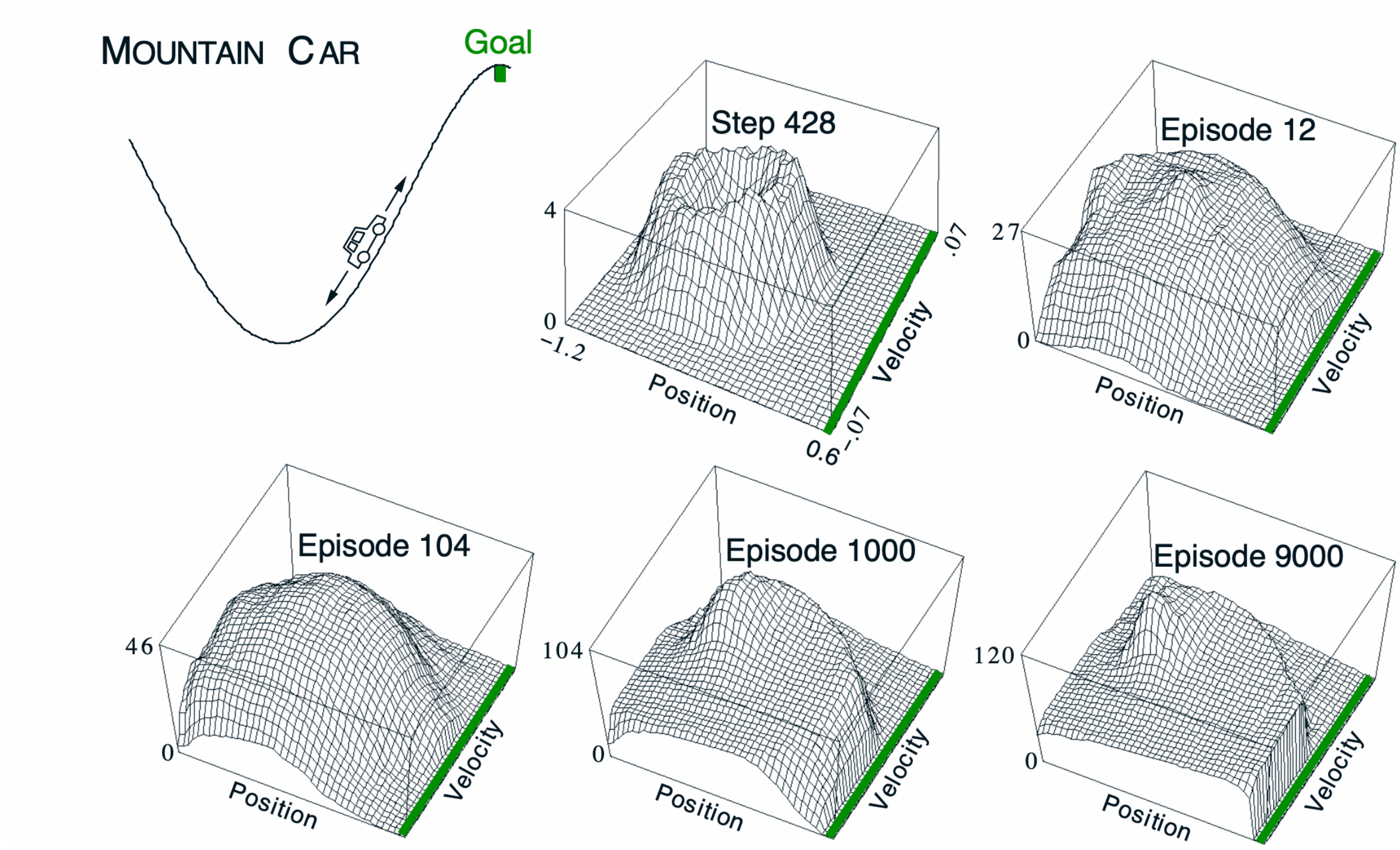
- Use stochastic gradient descent to find a local minimum

$$\Delta\theta = \alpha\left(Q_\pi(x,u) - \hat{Q}_\theta(x,u)\right)\nabla_\theta \hat{Q}_\theta(x,u)$$

Fitted Q-Iteration: update θ via stochastic gradient descent on TD target

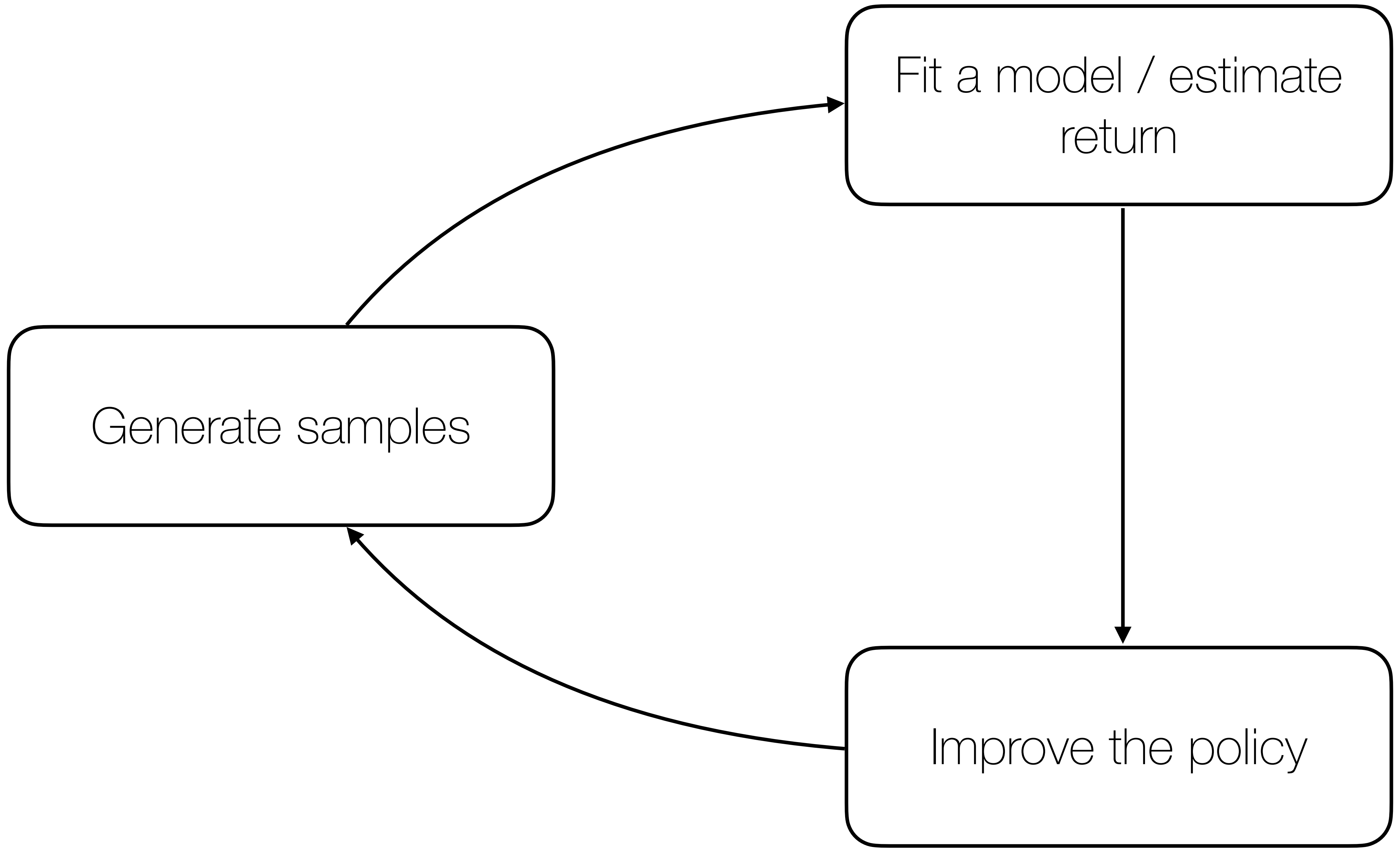
$$\Delta\theta = \alpha\left(r_t + \gamma \max_{u'_{t+1}} Q_\theta\left(x_{t+1}, u'_{t+1}\right) - \hat{Q}_\theta(x_t,u_t)\right)\nabla_\theta \hat{Q}_\theta(x_t,u_t)$$

Example: Sarsa with fn. approximation



The skeleton of fitted Q-learning

Run the policy and observe
(x_t, u_t, r_t, x_{t+1})



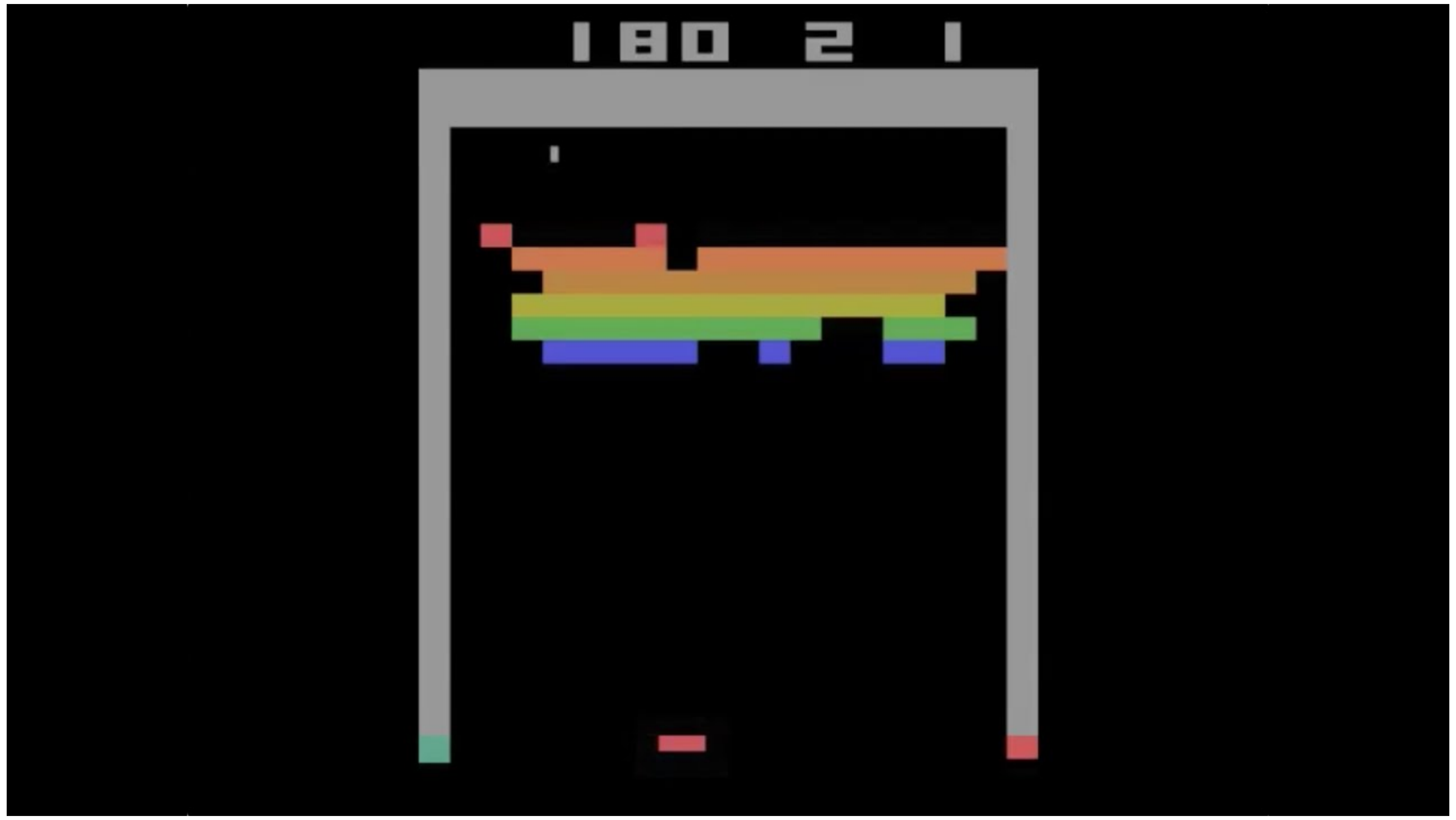
Set target
$y_t \leftarrow r_t + \gamma \max_u Q_\theta(x_{t+1}, u)$

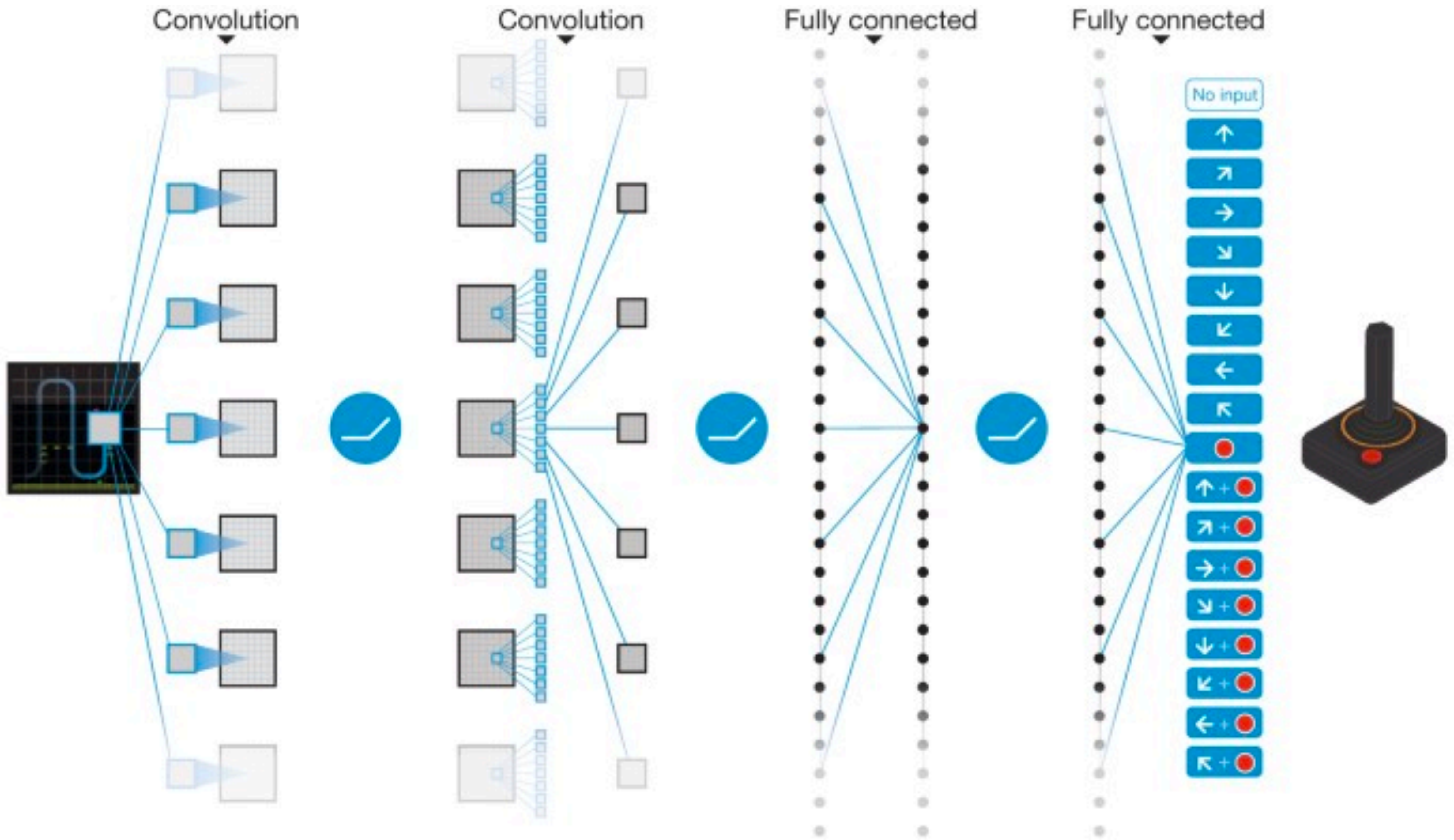
Update θ to minimize
$J(\theta) = \mathbb{E}_\pi \left[y_t - Q_\theta(x_t, u_t) \right]$

set $\pi\left(x_t\right) = \arg\max_a Q_\theta\left(x_t, u_t\right)$

Deep Q-Networks (DQN)

One of the most popular Deep RL algorithms and arguably one of the first successes of RL with neural networks





(1) Use **deep neural nets** to represent Q_θ in Q-learning

(2) Uses **experience replay** and **fixed Q-targets**

Deep Q-Networks (DQN)

(2) Uses **experience replay** and **fixed Q-targets**

- These two ideas turned out to be very important to stabilize training. Specifically, these concepts attempt to solve two issues:

  i) Samples within a trajectory are highly correlated → makes supervised learning unstable

  ii) The target $r_t + \gamma \max_{u'_{t+1}} Q_\theta\left(x_{t+1}, u'_{t+1}\right)$ is a moving target (i.e., as we update θ, the target of our regression also changes)

Intuitively:

- Take action u_t according to ϵ-greedy policy
- Store transition (x_t, u_t, r_t, x_{t+1}) in replay memory $\mathscr{D}$
- Sample batch of transitions $\{(x_t, u_t, r_t, x_{t+1})_i\}_{i=1}^B$ from $\mathscr{D}$ (**Experience replay decorrelates data**)
- Compute Q-learning targets w.r.t. old, fixed parameters ϕ
- Optimize MSE between Q-network prediction and Q-learning targets (**Fixed targets stabilize the objective**)

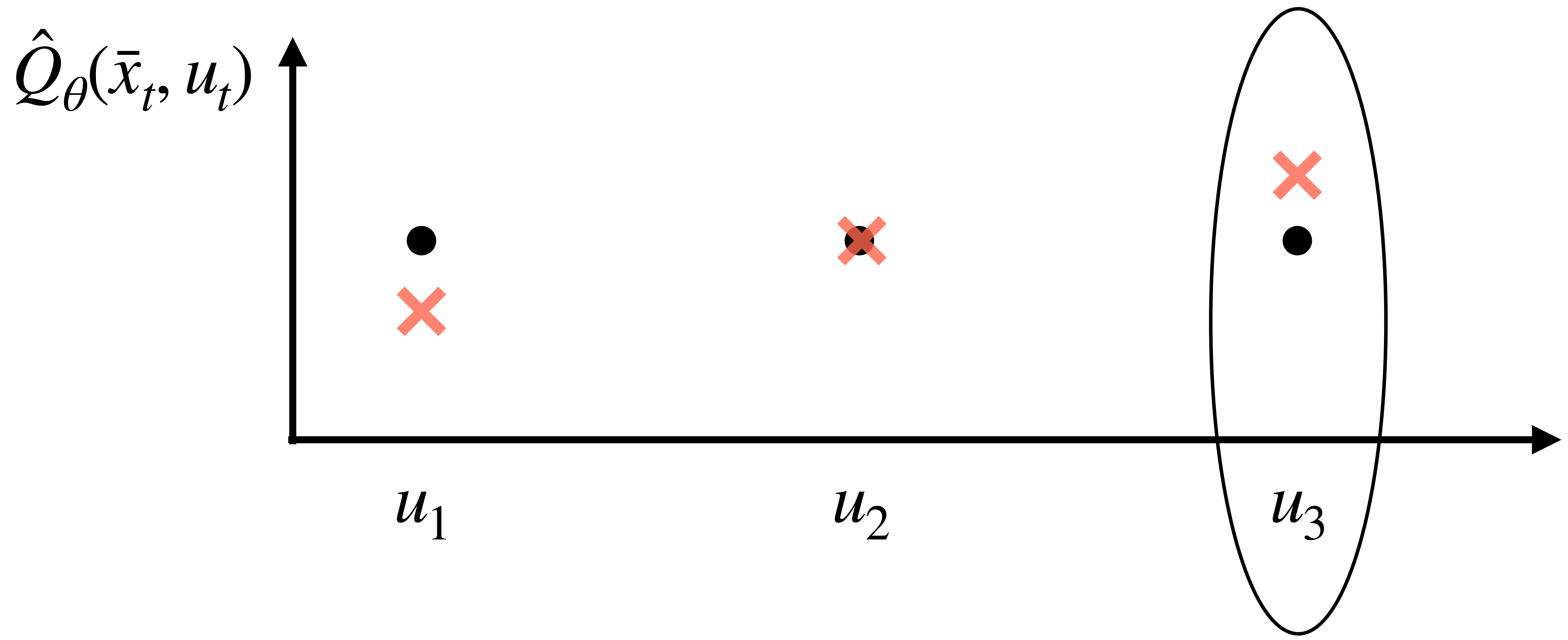
$$J(\theta) = \mathbb{E}_{(x_t,u_t,r_t,x_{t+1})\sim\mathscr{D}}\left[r_t + \gamma \max_u Q_\phi(x_{t+1}, u) - \hat{Q}_\theta(x_t, u_t)\right]$$

Maximization bias

- In the algorithms we covered so far, a maximum over estimated values is used implicitly as an estimate of the maximum value

$$\max_u \hat{Q}_\theta(x_t, u_t) \approx \max_u Q_\pi(x_t, u_t)$$

- This can lead to a significant positive bias. For example:



Double Q-learning

- Several possible solutions; in general, want to avoid using *max of estimates as estimate of max*
- Double Q-learning [van Hasselt, NeurIPS 2010]: use two independent estimates Q_1, Q_2
  - Use one estimate to determine the maximizing action $u* = \arg\max_u Q_1(x, u)$
  - And the other to provide the estimate of its value $Q_2(x, u*) = Q_2(x, \arg\max_u Q_1(x, u))$
  - This estimate will be unbiased
- Alternative approach: maintain two independent q-networks, always use $\min(Q_1, Q_2)$ [Fujimoto et al, ICML 2018]

Next time

- Model-free RL: policy optimization methods

References

- Mnih et al. *Playing Atari with Deep Reinforcement Learning*. 2013
- van Hasselt et al. *Double Q*-learning. NeurIPS 2010
- Fujimoto et al. *Addressing Function Approximation Error in Actor-Critic Methods*. ICML 2018