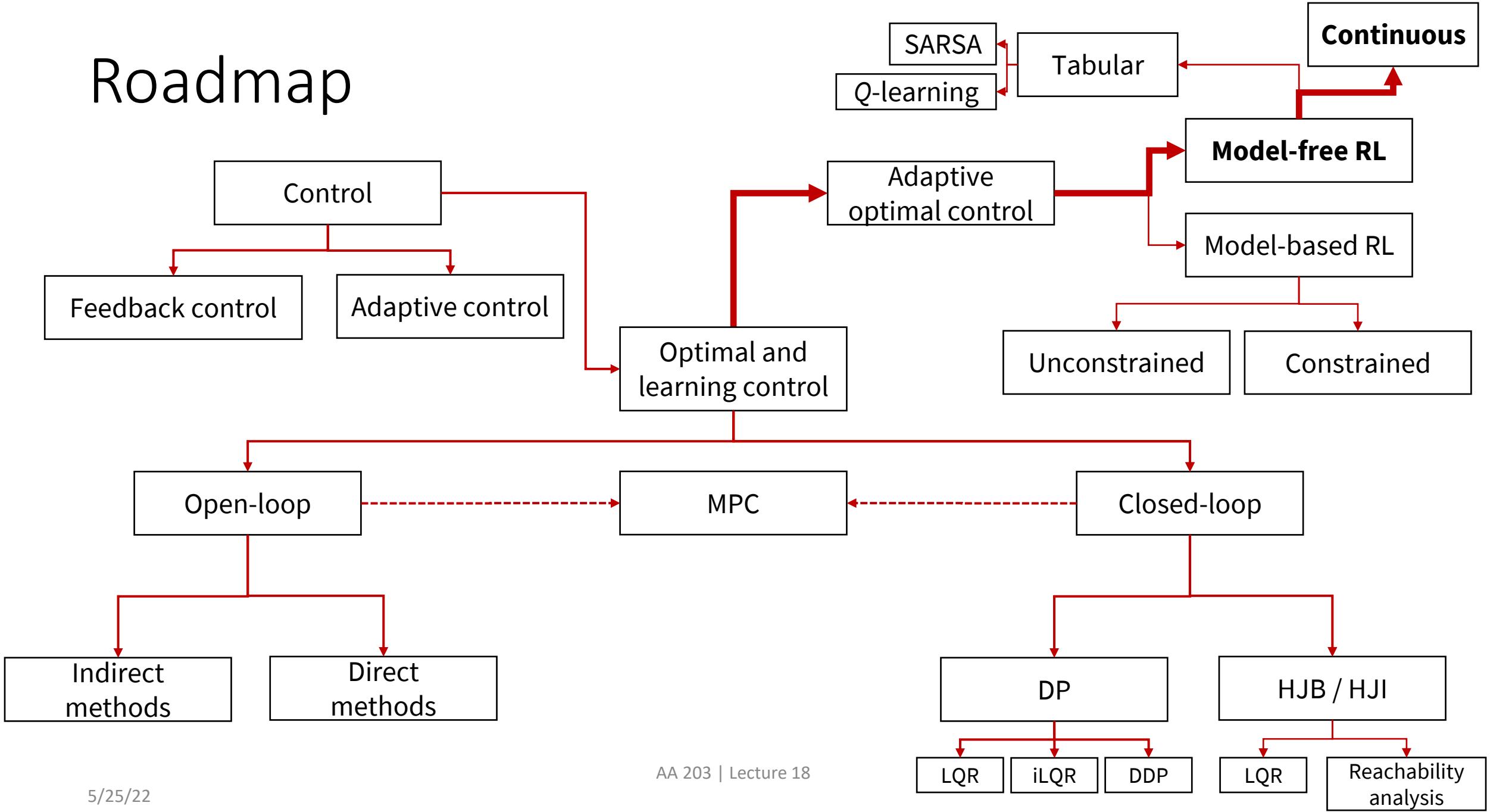
AA203
Optimal and Learning-based Control

Policy gradient and actor-critic

Stanford University

ASL
Autonomous Systems Lab

Roadmap


SARSA
Q-learning
Tabular
Continuous
Model-free RL
Control
Adaptive optimal control
Model-based RL
Feedback control
Adaptive control
Optimal and learning control
Unconstrained
Constrained
Open-loop
MPC
Closed-loop
Indirect methods
Direct methods
DP
HJB / HJI
LQR
iLQR
DDP
LQR
Reachability analysis

Model-free RL: deep RL and policy gradient

- Review Q-learning
- Policy gradient
- Introduce variance reduction methods for policy gradient estimation
- Brief survey of the modern model-free RL landscape

- Readings:
  - R. Sutton and A. Barto, *Reinforcement Learning: An Introduction,* 2018.
  - J. Achiam,“Spinning Up in Deep RL,” https://spinningup.openai.com/.

Review: Q-Learning

Recall *generalized policy iteration*; loop:

1. Perform *policy evaluation* step to estimate Q_π
2. Perform *policy improvement* step using Q_π to yield π'
3. Set $\pi \leftarrow \pi'$

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Policy evaluation for Q^* via

$$\min_\theta \left(r_t + \gamma \max_u Q_{\theta'}(x_{t+1}, u) - Q_\theta(x_t, u_t) \right)^2$$

with a greedy policy improvement step, $\pi(x) = \operatorname{argmax}_u Q_\theta(x, u)$.

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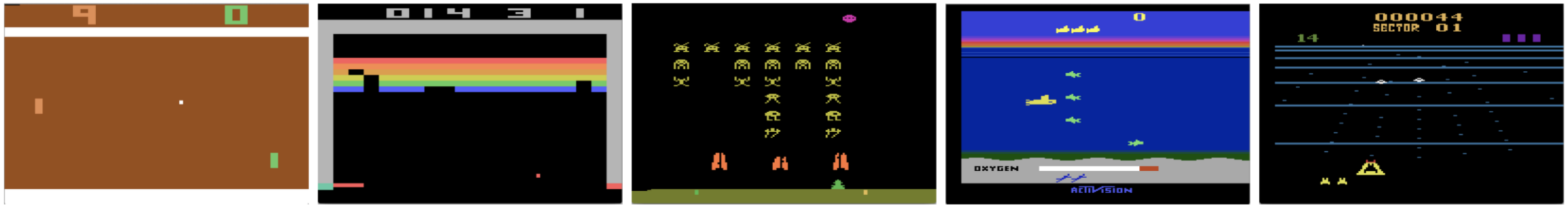
Side note on *maximization bias*: even if state-action value (Q function) estimates are unbiased, may still have biased value estimates due to max.

Deep Q-Learning

- In continuous setting, many possible function approximators for Q
  - Linear, nearest neighbors, aggregation
- Recent success: neural networks with loss function

$$\left(r_t + \gamma \max_u Q_{\theta'}(x_{t+1}, u) - Q_\theta(x_t, u_t)\right)^2$$

- Deep Q Network (DQN; Mnih et al. 2013)
  - Key idea: experience replay, i.e., keep a buffer of recent transitions to train on
    - Middle ground between on-policy (e.g., SARSA) and off-policy (e.g., basic Q-learning)
    - Benefits: better data efficiency, stabilizes training, mitigates NN overfitting



Model-free, policy based: Policy Gradient

Alternative: instead of learning the Q function, learn the policy directly!

Define a class of policies π_θ where θ are the parameters of the policy.

Can we learn the optimal θ from interaction?

Goal: use trajectories to estimate a gradient of policy performance w.r.t. parameters θ

Policy Gradient

A particular value of θ induces a distribution $p(\tau;\theta)$ over possible trajectories.

- Distribution comes from stochastic dynamics $T(x'|\, x,u)$ as well as stochastic policy $u \sim \pi(\cdot \mid x;\theta)$.

Objective function:

$$J(\theta) = E_{\tau\sim p(\tau;\theta)}[r(\tau)]$$

i.e.,

$$J(\theta) = \int_{\tau} r(\tau)p(\tau;\theta)d\tau$$

where $r(\tau)$ is the total discounted cumulative reward of a trajectory τ.

Policy Gradient

Gradient of objective w.r.t. parameters:

$$\nabla_\theta J(\theta) = \int_\tau r(\tau) \nabla_\theta p(\tau;\theta) d\tau$$

Trick: $\nabla_\theta p(\tau;\theta) = p(\tau;\theta) \frac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta) \nabla_\theta \log p(\tau;\theta)$

$$\nabla_\theta J(\theta) = \int_\tau (r(\tau) \nabla_\theta \log p(\tau;\theta)) p(\tau;\theta)\, d\tau$$

$$\nabla_\theta J(\theta) = E_{\tau \sim p(\tau;\theta)} [r(\tau) \nabla_\theta \log p(\tau;\theta)]$$

Policy Gradient

$$\nabla_\theta J(\theta) = E_{\tau \sim p(\tau;\theta)}[r(\tau)\nabla_\theta \log p(\tau;\theta)]$$

$$\log p(\tau;\theta) = \log\left(\prod_{t\geq 0} T(x_{t+1}|x_t,u_t)\pi_\theta(u_t|x_t)\right)$$

$$= \sum_{t\geq 0} \log T(x_{t+1}|x_t,u_t) + \log \pi_\theta(u_t|x_t)$$

$$\Rightarrow \nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \nabla_\theta \log \pi_\theta(u_t|x_t)$$

Policy Gradient

$$\nabla_\theta J(\theta) = E_{\tau \sim p(\tau;\theta)}[r(\tau)\nabla_\theta \log p(\tau;\theta)]$$

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$$\Rightarrow \nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \nabla_\theta \log \pi_\theta(u_t|x_t)$$

We don't need to know the transition model to compute this gradient!

Policy Gradient

If we use π_θ to sample a trajectory, we can approximate the gradient via N Monte Carlo samples:

$$\nabla_\theta J(\theta) = E_{\tau \sim p(\tau;\theta)}[r(\tau)\nabla_\theta \log p(\tau;\theta)]$$
$$\approx \frac{1}{N}\sum_{i=1}^{N}\left(r\left(\tau^{(i)}\right)\sum_{t\geq 0}\nabla_\theta \log \pi_\theta(u_t^{(i)}|x_t^{(i)})\right)$$

Intuition: adjust θ to:

- Boost probability of actions taken if reward is high
- Lower probability of actions taken if reward is low

Learning by trial and error

Policy Gradient Recap

Pros:

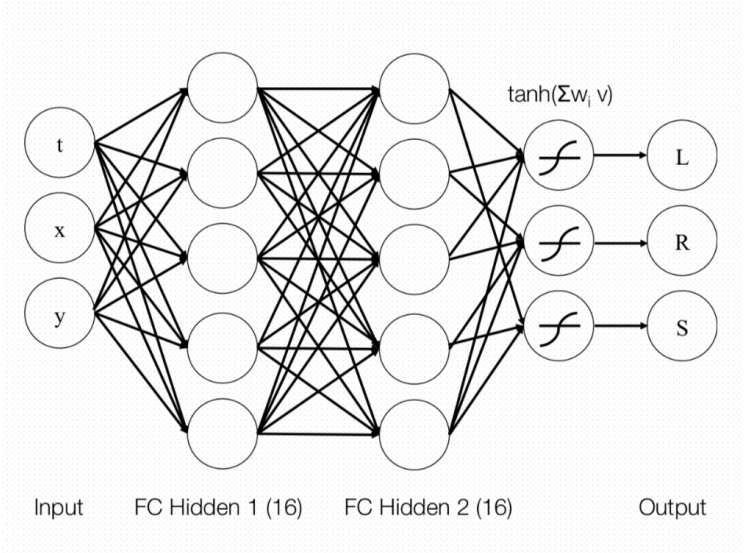
- Learns policy directly – can be more stable (less moving parts than *Q*-learning*)*
- Works for continuous action spaces (no need to "argmax" *Q*)
- Converges to local maximum of $J(\theta)$

Cons:

- Needs data from current policy to compute gradient – data inefficient
- Gradient estimates can be very noisy

Deep policy gradient

- Parametrize policy as deep neural network



NN output is parameters of distribution:

- For discrete action space, logits of a categorical distribution
- For a continuous action space, e.g., parameters of a normal distribution

$$\pi_\theta(u|x) = \mathcal{N}\big(\mu_\theta(x), \Sigma_\theta(x)\big)$$

- In practice, very unstable
  - Need to reduce variance of gradient estimator: baselines and actor-critic

Time dependency of policy gradient theorem

- Previous estimator for policy gradient was

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(r(\tau^{(i)})\sum_{t\geq 0}\nabla_\theta \log \pi_\theta(u_t^{(i)}|x_t^{(i)})\right)$$

Action $u_{t'}$ can not change reward r_t for $t < t'$ (i.e., previous timesteps):

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{t\geq 0}\nabla_\theta \log \pi_\theta(u_t^{(i)}|x_t^{(i)})\sum_{\tau\geq t} r(x_\tau^{(i)}, u_\tau^{(i)})\right)$$

(caveat: this is not a rigorous argument we're presenting here)

REINFORCE

Loop forever:

Generate episode $x_0, u_0, r_0, x_1, u_1, r_1$ … with π_θ

Loop for all $t = 0, \ldots, N-1$:

$G_t \leftarrow \sum_{k=t}^{N} r_k$

$\theta \leftarrow \theta + \alpha \, G_t \, \nabla_\theta \log \pi_\theta(u_t | x_t)$

Cumulative tail reward, the tail "return"

Adding baselines to policy evaluation

- Monte Carlo policy gradient estimator has **extremely high variance.**
- We want to search for gradient estimators that have lower variance
- Add in state-dependent **baseline**

$$\tilde{G}_t = G_t - b(x_t)$$
$$J(\theta) = \mathrm{E}_{x_t,u_t,\ldots}[\tilde{G}_t]$$

Policy gradient theorem yields

$$\nabla_\theta J(\theta) = \mathrm{E}_{x_0,u_0,\ldots}[\sum_{t\geq 0} \tilde{G}_t \nabla_\theta \log \pi(u_t|x_t,\theta)]$$

A closer look at the baseline

Claim: adding baseline does not change the value of the expected gradient

$$\nabla_\theta J(\theta) = \mathrm{E}[\sum_{t\geq 0}(G_t - b(x_t))\nabla_\theta \log \pi(u_t|x_t,\theta)]$$

$$= \mathrm{E}[\sum_{t\geq 0} G_t \nabla_\theta \log \pi(u_t|x_t,\theta)] - \mathrm{E}[\sum_{t\geq 0} b(x_t)\nabla_\theta \log \pi(u_t|x_t,\theta)]$$

$$\mathrm{E}[b(x_t)\nabla_\theta \log \pi(u_t|x_t,\theta)] = \mathrm{E}_{x_t}[b(x_t)\underbrace{\mathrm{E}_{u_t}[\nabla_\theta \log \pi(u_t|x_t,\theta)]}_{=0 \text{ for all } t}]$$

Any state-dependent function, independent of action, works.

Toy example

Consider a one-step problem with

$$r(x, [u_1, u_2]) = 100 + u_1$$

and suppose that our policy is parameterized as

$$\pi_\theta(u|x) = \mathcal{N}(\theta, I)$$
$$\Rightarrow \nabla_\theta \log \pi_\theta(u|x) = u - \theta$$

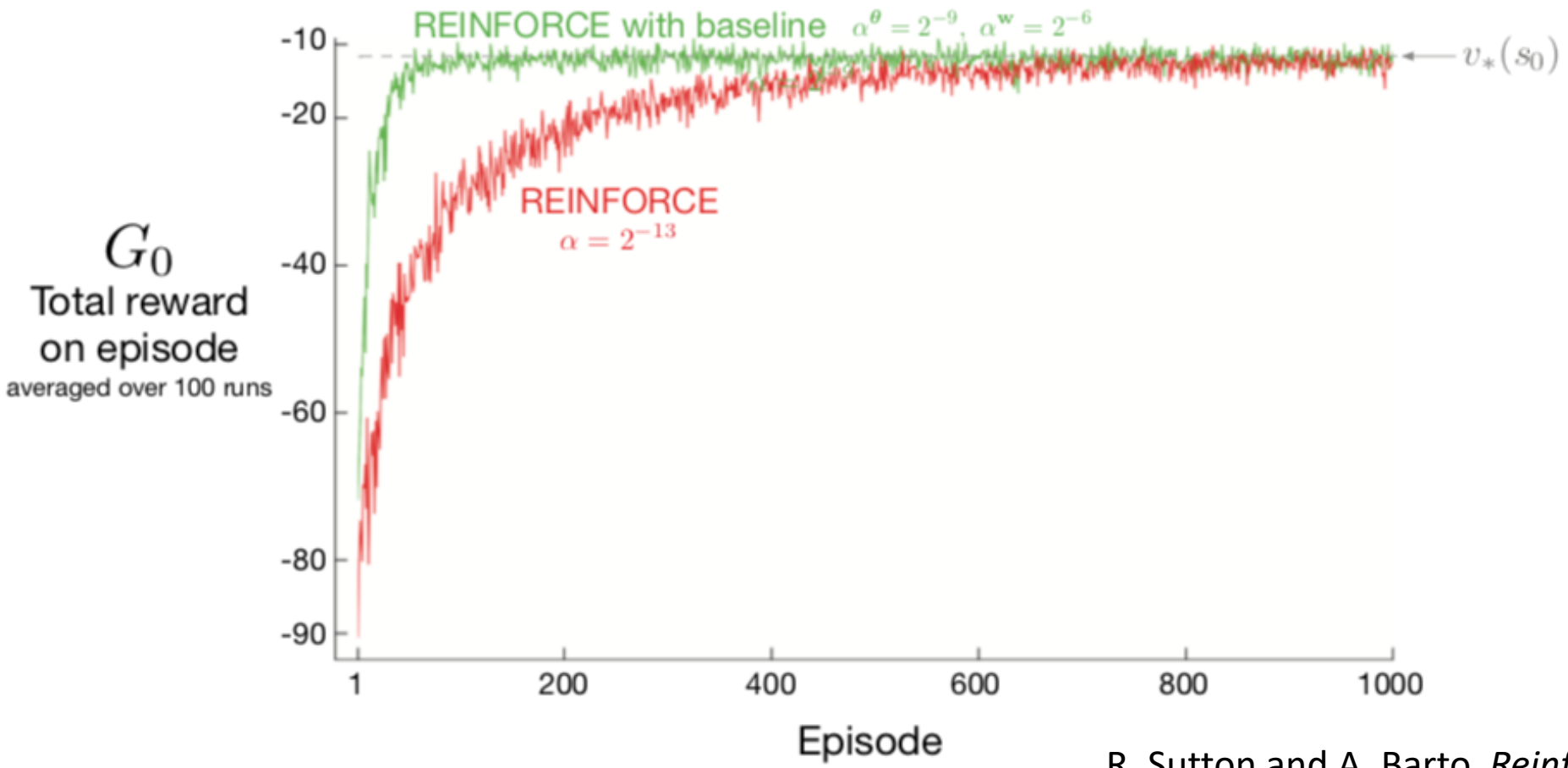
Let's estimate the policy gradient at $\theta = 0$ with two samples of u, [1,1] and [-1, 1]:

$$\nabla_\theta J(\theta) \approx \frac{1}{2}(101[1,1] + 99[-1,1]) = [1, 100]$$

Subtracting a baseline $b = 100$:

$$\nabla_\theta J(\theta) \approx \frac{1}{2}\big((101 - 100)[1,1] + (99 - 100)[-1,1]\big) = [1, 0]$$

Performance improvement on gridworld



R. Sutton and A. Barto, *Reinforcement Learning: An Introduction,* 2018.

Actor-critic

Particularly good baseline choice: value function

Actor-critic: use both **actor** (policy π_θ) and **critic** (value function V_w).

Loop forever:

Generate episode $x_0, u_0, r_0, x_1, u_1, r_1$ … with π_θ

Loop for all $t = 0, \dots, N-1$:

$G \leftarrow \sum_{k=t+1}^{N} r_k$

$\delta_w \leftarrow G - V_w(x_t)$ (fits V_w through MC policy evaluation)

$w \leftarrow w + \alpha_w \delta_w \nabla_w V_w(x_t)$

$\theta \leftarrow \theta + \alpha_\theta \delta_w \nabla_\theta \log \pi_\theta(u_t | x_t)$ (baselined policy gradient)

Policy gradient theorem with Q function

- Previously, have used $J(\theta) = \mathrm{E}_{x_0,u_0,\ldots}[\sum_{t\geq 0} r(x_t, u_t)]$
- Note that, by definition (of Q),

$$J(\theta) = \mathrm{E}_{u_t \sim \pi(\cdot|x_t)}[Q^\pi(x_t, u_t)]$$

Yields policy gradient

$$\nabla_\theta J(\theta) = \mathrm{E}_{u_t \sim \pi_\theta(\cdot|x_t)}[Q^\pi(x_t, u_t) \nabla_\theta \log \pi_\theta(u_t|x_t)]$$

Note that $Q^\pi(x_t, u_t) = \mathrm{E}_{u_t \sim \pi_\theta(\cdot|x_t), x_{t+1}}[r(x_t, u_t) + V^\pi(x_{t+1})]$

Advantage policy gradient

- Combining the Q function policy gradient and the value baselines, we have

$$\nabla_\theta J(\theta) = \mathrm{E}[\, \delta^\pi \nabla_\theta \log \pi_\theta(u_t|x_t)]$$

For $\delta^\pi = \underbrace{\left(r_t + V^\pi(x_{t+1}) - V^\pi(x_t)\right)}_{\text{“TD advantage estimate”}}$. This is the TD error for policy evaluation!

- Note that $\mathrm{E}_\pi[\delta^\pi|x,u] = Q^\pi(x,u) - V^\pi(x) = A^\pi(x,u)$.
  - This is called the **advantage**.
    (the value of taking a specific action vs. following your policy)

Advantage actor-critic (A2C)

Loop forever:

Generate episode $x_0, u_0, r_0, x_1, u_1, r_1$ … with π_θ

Loop for all $t = 0, \ldots, N-1$:

$$\delta_w \leftarrow r_t + V_w(x_{t+1}) - V_w(x_t)$$

(a "one point" estimate of the advantage)

$$w \leftarrow w + \alpha_w \delta_w \nabla_w V_w(x_t)$$

(now fits V_w by minimizing TD error)

$$\theta \leftarrow \theta + \alpha_\theta \delta_w \nabla_\theta \log \pi_\theta(u_t | x_t)$$

Alternative estimators

- Many possible estimators for the advantage

- A problem with $r_t + V_w(x_{t+1}) - V_w(x_t)$ is that during learning, $V_w(x_t)$ may be quite inaccurate, and thus this will be a biased estimator of $A^\pi(x, u)$

- Using a multistep TD error, we get the "τ-step advantage estimate":

$$\delta \leftarrow \underbrace{r_t + r_{t+1} + \ldots + r_{t+\tau} + V_w(x_{t+\tau+1})}_{\text{less biased estimate of expected return (or even unbiased, if you sum all the way to end of episode), but higher variance}} - V_w(x_t)$$

As τ gets larger, this gets closer to Monte Carlo with value baseline.

Deterministic policy gradient (DPG) [Silver et al., ICML 2014]

- Instead of using stochastic policy with value estimation baseline:
  - Maintain estimate of Q function via minimizing TD error
  - Optimize deterministic policy via

$$\max_{\theta} \mathrm{E}_x[Q(x, \pi_\theta(x))]$$

- Policy simply *amortizes* optimization of the Q function (the "argmax" of policy improvement).
- Can be used off policy, relatively unstable in practice.

Double Q-learning

Addressing maximization bias:

- Several possible solutions; in general, want to avoid using *max of estimates* as *estimate of max*.
- Double Q-learning [van Hasselt, NeurIPS 2010]: use two independent estimates Q_1, Q_2
  - $u^* = \text{argmax}_u Q_1(x, u)$
  - Use value estimate $Q_2(x, u^*)$
- Alternative approach: maintain two independent critics, always use min [Fujimoto et al, ICML 2018]

Trust region policy optimization (TRPO) [Schulman et al., ICML 2015]

- Main idea : instead of choosing step size, use trust region

$$\max \mathrm{E}_{\pi_{\theta_{old}}}\left[\frac{\pi_\theta(u_t|x_t)}{\pi_{\theta_{old}}(u_t|x_t)}\hat{A}_t\right]$$
$$s.t.\,\mathrm{E}_{x\sim\rho_{old}}\left[D_{KL}\left(\pi_{\theta_{old}}(\cdot\,|x)||\pi_\theta(\cdot\,|x)\right)\right] \leq \delta$$

- Can show that this leads to monotonic improvement in the ideal case.
- Simpler, more popular version: proximal policy optimization (PPO).
  - Replaces TRPO CG solve with simple adaptive KL penalty.

Criticism of model-free methods

- Despite recent progress (+ much not discussed here), questions about whether model-free methods are doing more than random search in parameter space.

Why did TD-Gammon Work?

Jordan B. Pollack & Alan D. Blair
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Abstract

Although TD-Gammon is one of the major successes in machine learning, it has not led to similar impressive breakthroughs in temporal difference learning for other applications or even other games. We were able to replicate some of the success of TD-Gammon, developing a competitive evaluation function on a 4000 parameter feed-forward neural network, without using back-propagation, reinforcement or temporal difference learning methods. Instead we apply simple hill-climbing in a relative fitness environment. These results and further analysis suggest that the surprising success of Tesauro's program had more to do with the co-evolutionary structure of the learning task and the dynamics of the backgammon game itself.

Simple random search of static linear policies is competitive for reinforcement learning

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Department of Electrical Engineering and Computer Science
University of California, Berkeley

Abstract

Model-free reinforcement learning aims to offer off-the-shelf solutions for controlling dynamical systems without requiring models of the system dynamics. We introduce a model-free random search algorithm for training static, linear policies for continuous control problems. Common evaluation methodology shows that our method matches state-of-the-art sample efficiency on the benchmark MuJoCo locomotion tasks. Nonetheless, more rigorous evaluation reveals that the assessment of performance on these benchmarks is optimistic. We evaluate the performance of our method over hundreds of random seeds and many different hyperparameter configurations for each benchmark task. This extensive evaluation is possible because of the small computational footprint of our method. Our simulations

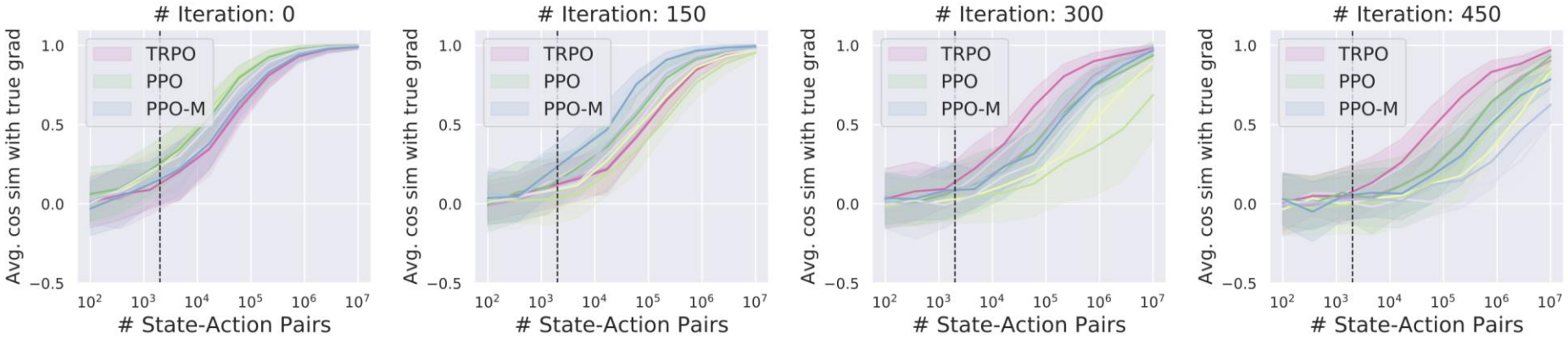
Jacob Andreas ✈️ #acl2022nlp
@jacobandreas

Deep RL is popular because it's the only area in ML where it's socially acceptable to train on the test set.

12:27 PM · Oct 28, 2017 · Twitter Web Client

Are Deep Policy Gradient Algorithms Truly Policy Gradient Algorithms?

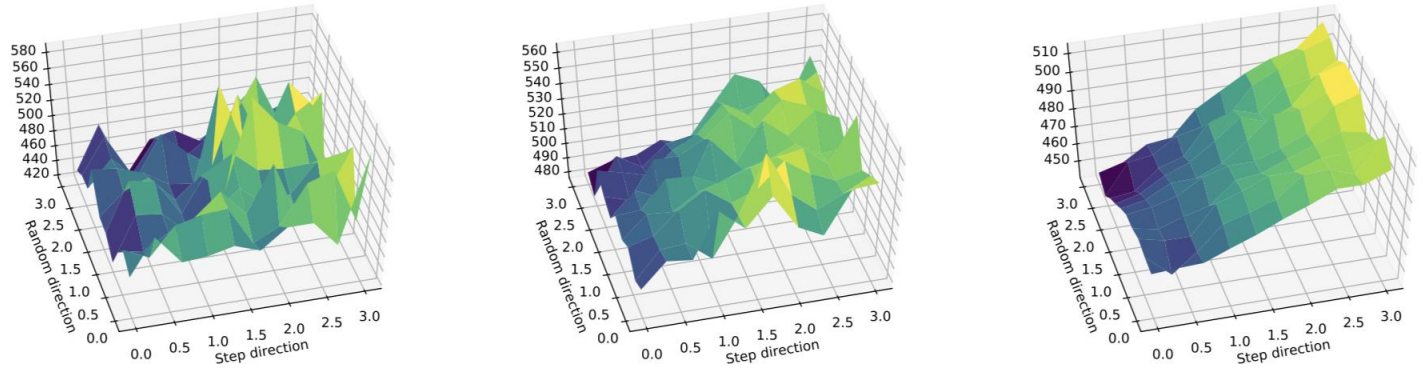
Andrew Ilyas*[1], Logan Engstrom*[1], Shibani Santurkar[1], Dimitris Tsipras[1],
Firdaus Janoos[2], Larry Rudolph[1,2], and Aleksander Mądry[1]



2,000 state-action pairs (19 trajectories)

20,000 state-action pairs (198 trajectories)

100,000 state-action pairs (1068 trajectories)



Why model-free?

- Advantages
  - Very few assumptions
  - Many state of the art methods reach better performance than model-based methods

- Weaknesses
  - Extremely high sample complexity

Next time

- Combining policy optimization with model learning
- AA 203 recap!