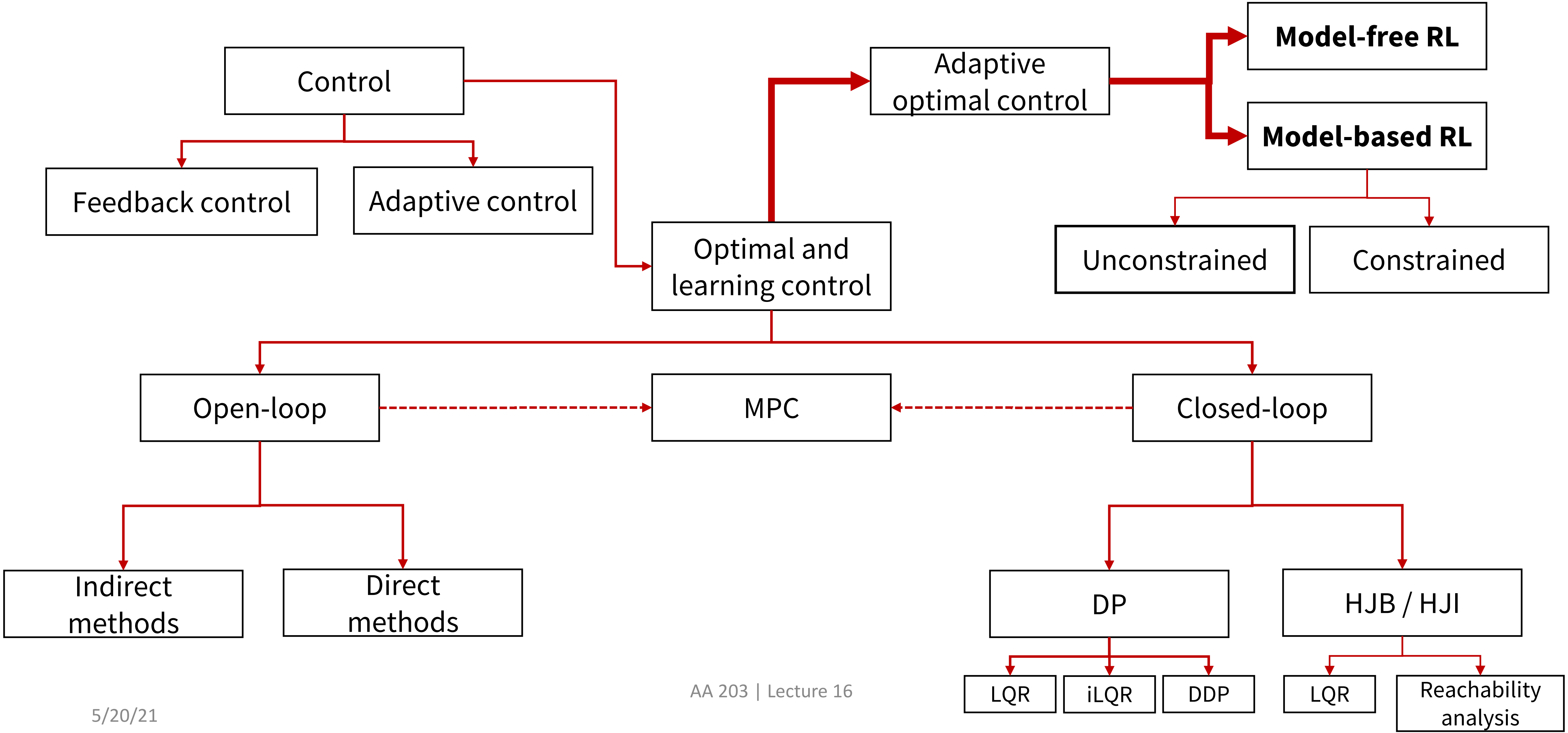
AA203
Optimal and Learning-based Control

Combining model and policy learning

Stanford University

Autonomous Systems Lab

Roadmap


Control
Feedback control
Adaptive control
Optimal and learning control
Adaptive optimal control
Model-free RL
Model-based RL
Unconstrained
Constrained
Open-loop
MPC
Closed-loop
Indirect methods
Direct methods
DP
HJB / HJI
LQR
iLQR
DDP
LQR
Reachability analysis

Combining MB and MF RL ideas

- Review model-based RL
- Combining model and policy learning in the tabular setting
- Combinations in the nonlinear setting

- Readings:
  - R. Sutton and A. Barto. *Reinforcement Learning: An Introduction,* 2018.
  - Several papers, referenced throughout.

Review: model-based RL

Choose initial policy π_θ

Loop over episodes:

Get initial state x

Loop until end of episode:

$u \leftarrow \pi_\theta(x)$

Take action u in environment, receive next state x' and reward r

Update model based on x, u, x', r

Update policy π_θ based on updated model

$x \leftarrow x'$

Dyna: combining model-free and model-based RL

Dyna-Q:

Init $Q(x,u), model(x,u)$ for all x, u; initialize state x

Loop forever:

$u \leftarrow argmax_u Q(x,u)$ (possibly with exploration)

Take action u in environment, receive next state x' and reward r

$Q(x,u) \leftarrow Q(x,u) + \alpha[r + \gamma \max_{u'} Q(x',u') - Q(x,u)]$

$model(x,u) \leftarrow x', r$

For $n = 1, \dots, N$:

$x, u \leftarrow$ random previously observed state/action pair

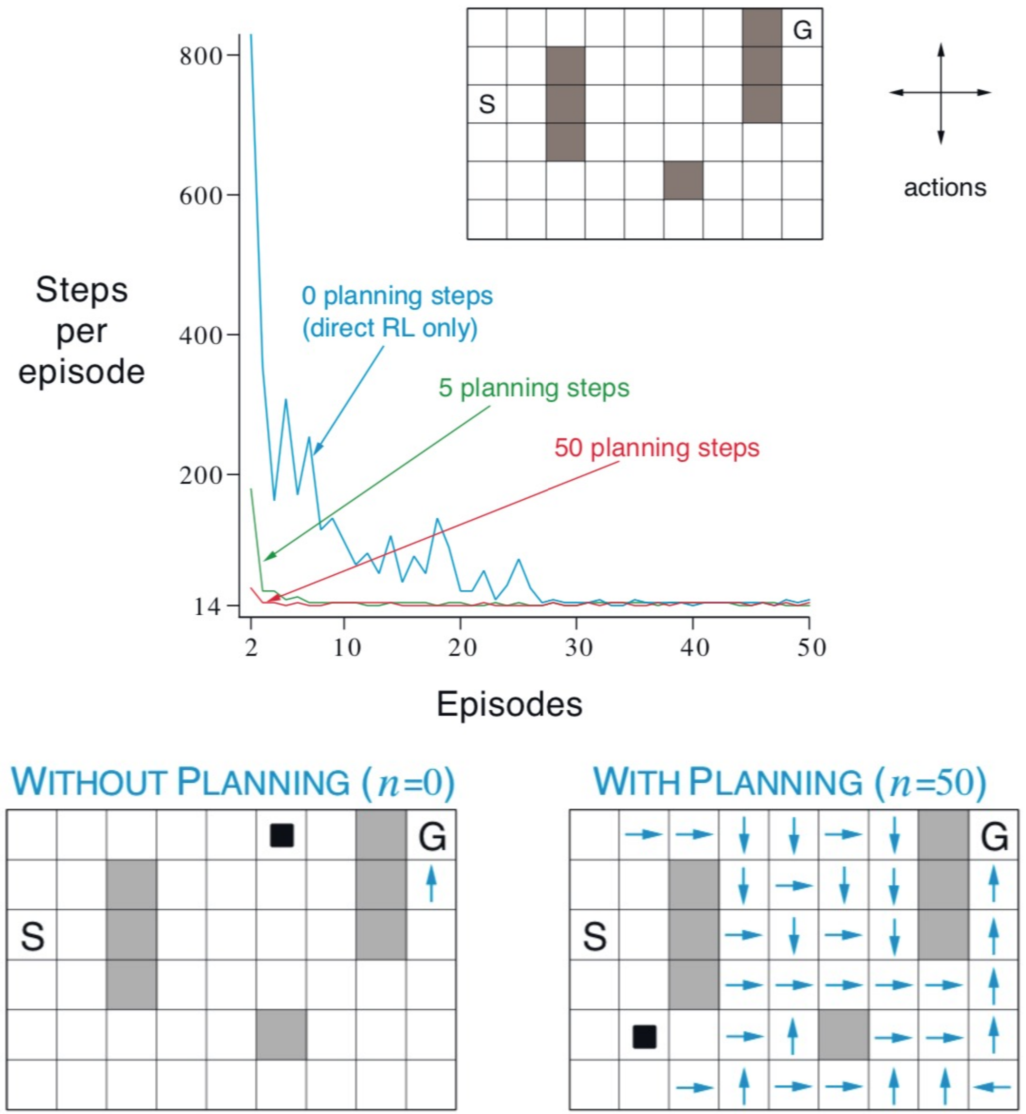
$x', r \leftarrow model(x,u)$

$Q(x,u) \leftarrow Q(x,u) + \alpha[r + \gamma \max_{u'} Q(x',u') - Q(x,u)]$

Dyna performance: deterministic maze

Main idea of Dyna: interleave simulated and real experience in policy optimization.

Allows early model-based training acceleration, without performance limitations of model-based methods.



How to optimize policy?

Question: what should policy be?

	Tabular MDP	Continuous MDP
Limited horizon open loop	Monte Carlo tree search or search of finite horizon action sequence	Model predictive control
Closed-loop policy optimization	Dynamic programming: value iteration or policy iteration	**Main focus of today's lecture**

Why do limited search? Typically, if policy optimization is too expensive.

- Example: game of Go or other very large MDPs

Policy optimization with nonlinear dynamics models

- How can we optimize our policy?
- Simple local approach:
  - iLQR
  - DDP
  - trajectory optimization + time varying LQR

- What about more complex policies than linear feedback?

Policy optimization with models

- Want to optimize π_θ via

$$\theta^* = argmax_\theta \mathrm{E}_{x_0}[V^{\pi_\theta}(x_0)]$$

Approach: fit model $f_\phi(x,u)$, define value w.r.t. this model as

$$V^{\pi,f}(x) = \sum_t \mathrm{E}_{x_t \sim f, u_t \sim \pi}[r(x_t, u_t)]$$

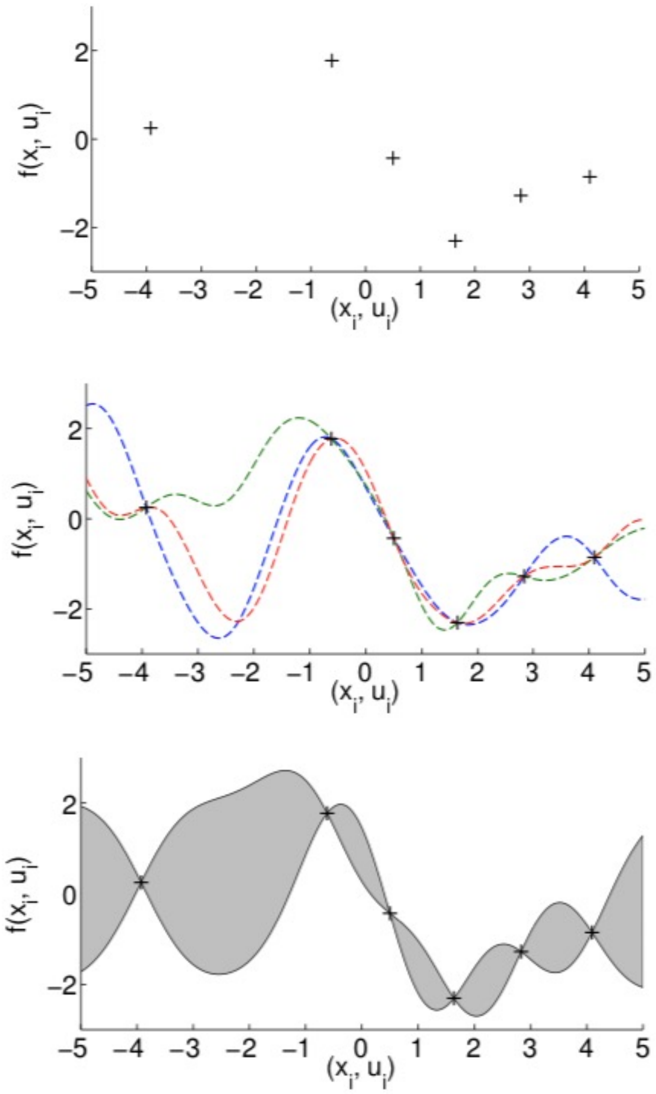
Want to compute gradient of this value w.r.t. policy parameters:

$$\theta \leftarrow \theta + \alpha \nabla_\theta V^{\pi_\theta, f_\phi}(x)$$

Case study: PILCO

Deisenroth and Rasmussen, *Probabilistic inference for learning control,* ICML 2011.

- Approach: use Gaussian process for dynamics model
  - Gives measure of *epistemic* uncertainty
  - Extremely sample efficient
- Pair with arbitrary (possibly nonlinear) policy
- By propagating the uncertainty in the transitions, capture the effect of small amount of data

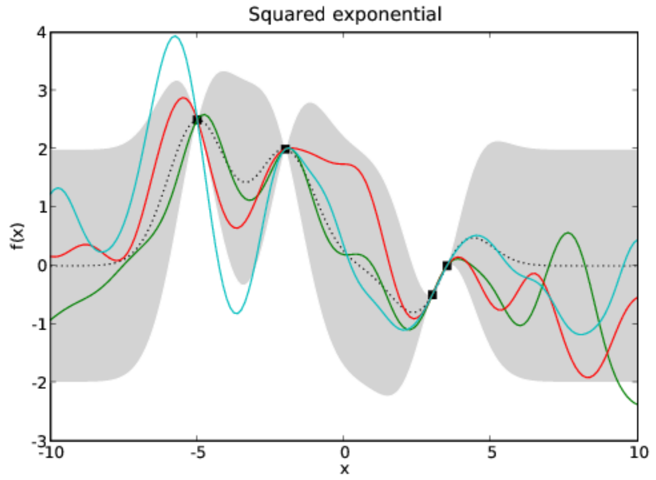


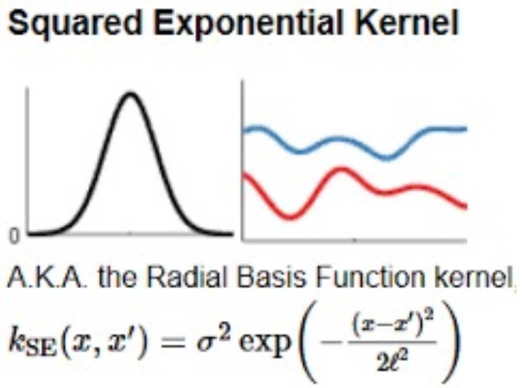
GP reminder

- Gaussian processes: Gaussian distributions over functions
- Typically, initialize with zero mean; behavior determined entirely by **kernel**

$$cov(x, x') = k(x, x')$$

- Standard kernel choice: squared exponential, used in PILCO
  - Has smooth interpolating behavior





PILCO mechanics

For GP conditioned on data, one step prediction is Gaussian

$$p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) = \mathcal{N}\left(\mathbf{x}_t \mid \mu_t, \mathbf{\Sigma}_t\right),$$
$$\mu_t = \mathbf{x}_{t-1} + \mathbb{E}_f[\Delta_t],$$
$$\mathbf{\Sigma}_t = \mathrm{var}_f[\Delta_t].$$

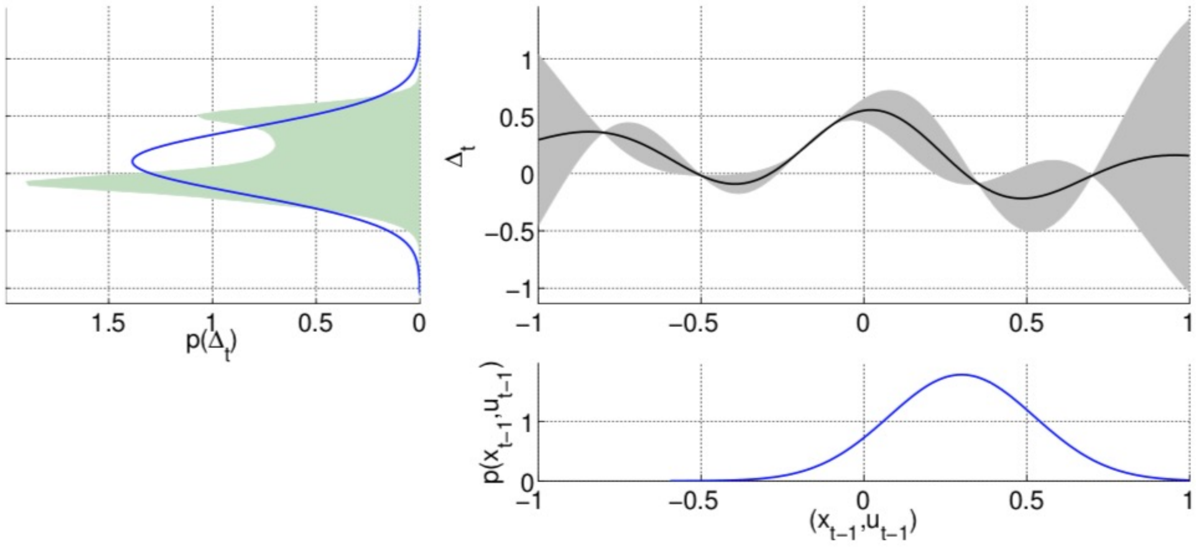
with $\Delta_{\mathrm{t}} = x_t - x_{t-1} + \epsilon$, $\epsilon \sim N(0, \Sigma_\epsilon)$, and

$$m_f(\tilde{\mathbf{x}}_*) = \mathbb{E}_f[\Delta_*] = \mathbf{k}_*^\top(\mathbf{K} + \sigma_\varepsilon^2\mathbf{I})^{-1}\mathbf{y} = \mathbf{k}_*^\top\beta,$$
$$\sigma_f^2(\Delta_*) = \mathrm{var}_f[\Delta_*] = k_{**} - \mathbf{k}_*^\top(\mathbf{K} + \sigma_\varepsilon^2\mathbf{I})^{-1}\mathbf{k}_*$$

For $k_* = k\left(\widetilde{X}, \tilde{x}_*\right), k_{**} = k(\tilde{x}_*, \tilde{x}_*), K_{ij} = k\left(\tilde{x}_i, \tilde{x}_j\right)$, with $\tilde{x} = [x^T, u^T]^T$.

Uncertainty propagation

- We have the one step posterior predictive
- But, need to make multistep predictions: so, need to derive multi-step predictive distribution
- Turn to approximating distribution at each time with a Gaussian via *moment matching*



Uncertainty propagation

- Because of the squared exponential kernel, mean and variance can be computed in closed form
- Choose cost

$$c(\mathbf{x}) = 1 - \exp(-\|\mathbf{x} - \mathbf{x}_{\text{target}}\|^2/\sigma_c^2)$$

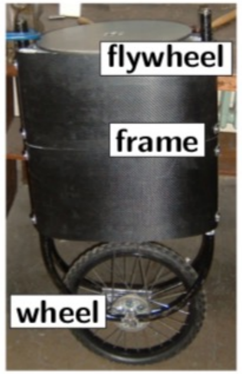
which is similarly squared exponential; thus expected cost can be computed, factoring in uncertainty.

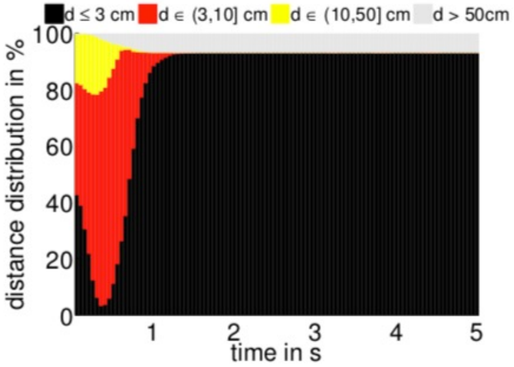
- Choose also radial basis function or linear policy, to enable analytical uncertainty propagation

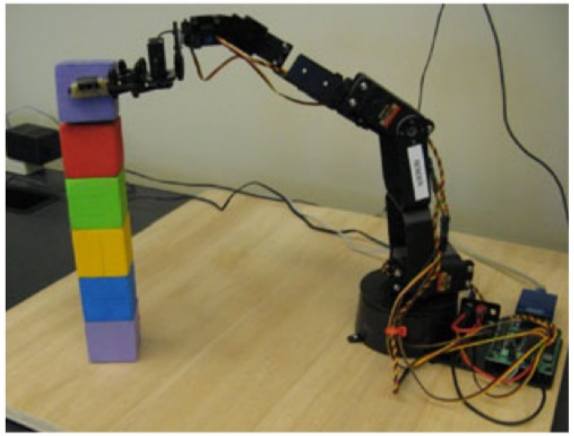
PILCO Summary

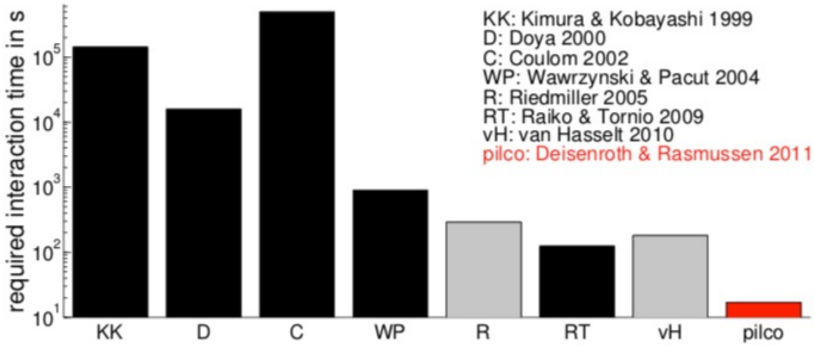
- Uncertainty prop: leverage specific form to derive analytical expressions for mean and variance of trajectory under policy.
- Can use chain rule (aka backprop through time) to compute the gradient of expected total cost w.r.t. policy parameters
- Algorithm:
  - Roll out policy to get new measurements; update model
  - Compute (locally) optimal policy via gradient descent
  - Repeat

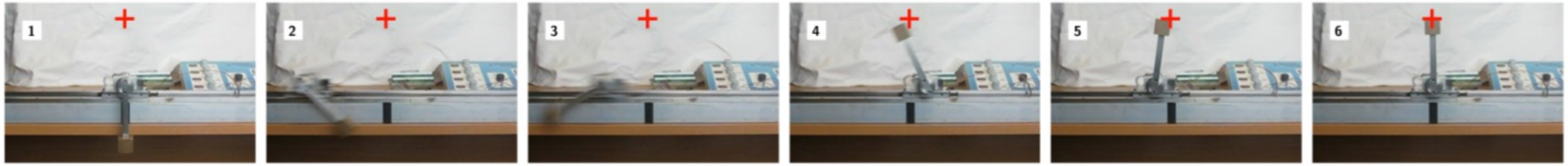
PILCO results







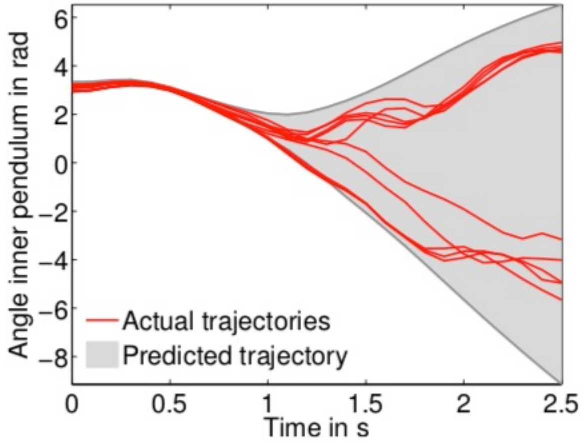


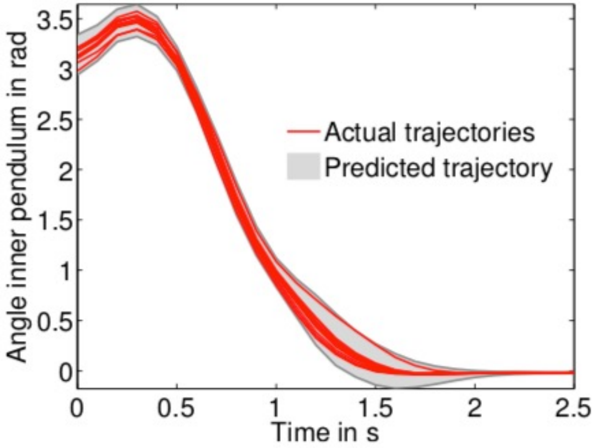


For more results and algorithm info: Deisenroth, Fox, and Rasmussen, *Gaussian Processes for Data-Efficient Learning in Robotics and Control*, TPAMI 2015.

PILCO limitations

- Treatment of uncertainty
  - Propagates uncertainty via moment matching, so can't handle multi-modal outcomes
  - Limited in choice of kernel function
  - Doesn't capture temporal correlation
- Efficiency
  - GPs are extremely data efficient; however, *very* slow
  - Policy optimization (done after every rollout) can take on the order of ~1h

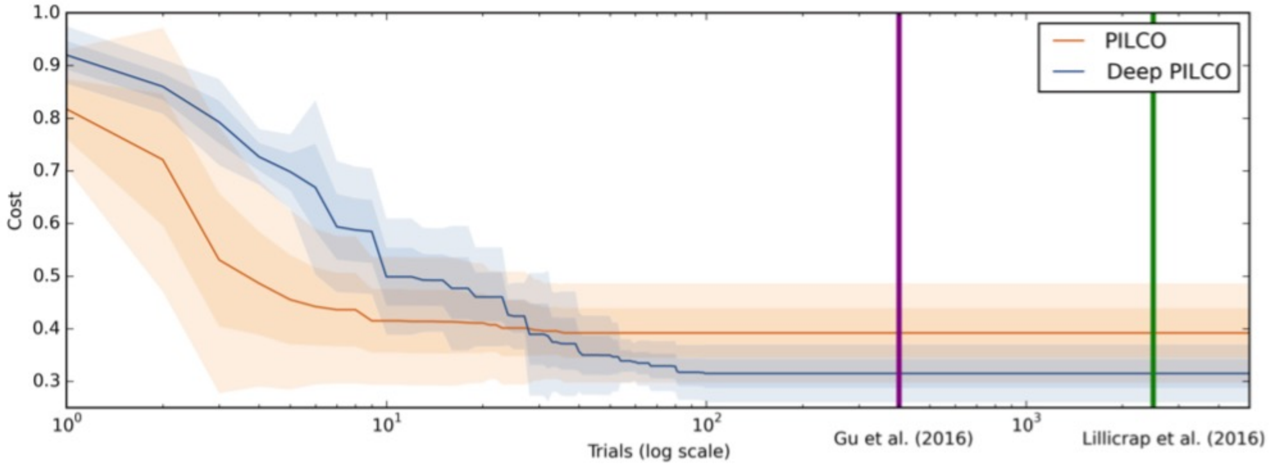




	Bayesian NP model	Deterministic NP model
Learning success	**94.52%**	0%

What about the same principles with neural network models?

- McHutchon, *Modelling nonlinear dynamical systems with Gaussian processes,* PhD thesis, 2014: particle propagation performs poorly.
- Gal, McAllister, Rasmussen, *Improving PILCO with Bayesian neural network dynamics models*, 2017.
  - Use a Bayesian network that provides samples from posterior
  - Again use moment matching; this time not necessary for analytical variance computation, but for performance



For much deeper discussion of gradient estimation with particles, see:
Parmas, Rasmussen, Peters, Doya, *PIPPS: Flexible model-based policy search robust to the curse of chaos,* ICML 2018.

Policy optimization via backpropagation through neural network dynamics

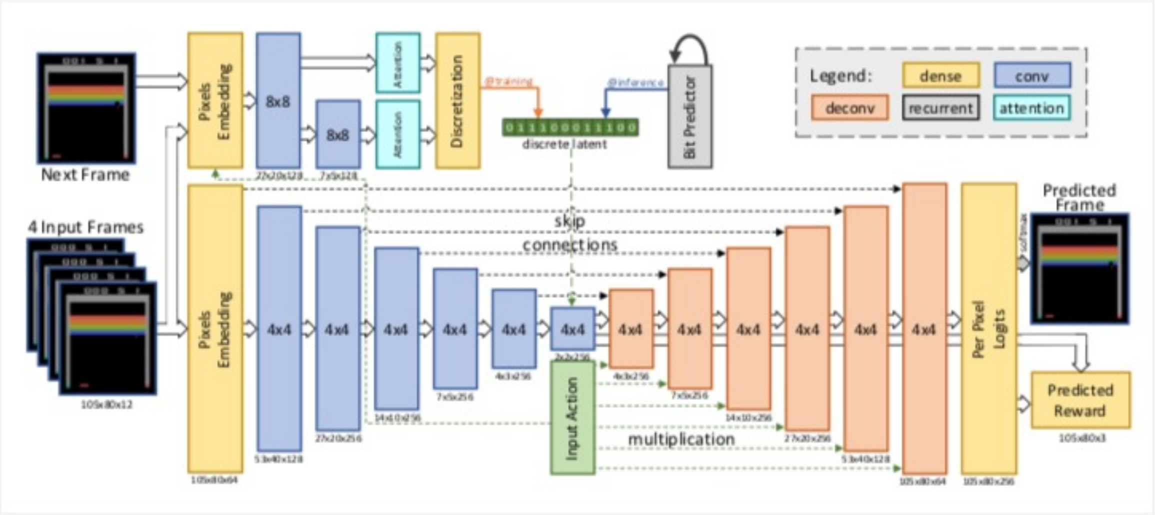
- Backpropagate through computation graph of dynamics and policy
- Same instability as shooting methods in trajectory optimization
  - However, in shooting methods, each time step is an independent action
- Here, the policy is the same at each time step: so very small changes in policy **dramatically** change trajectory
  - Accumulated gradients become very large as you backprop further
  - Similar to exploding/vanishing gradient problems in recurrent NNs

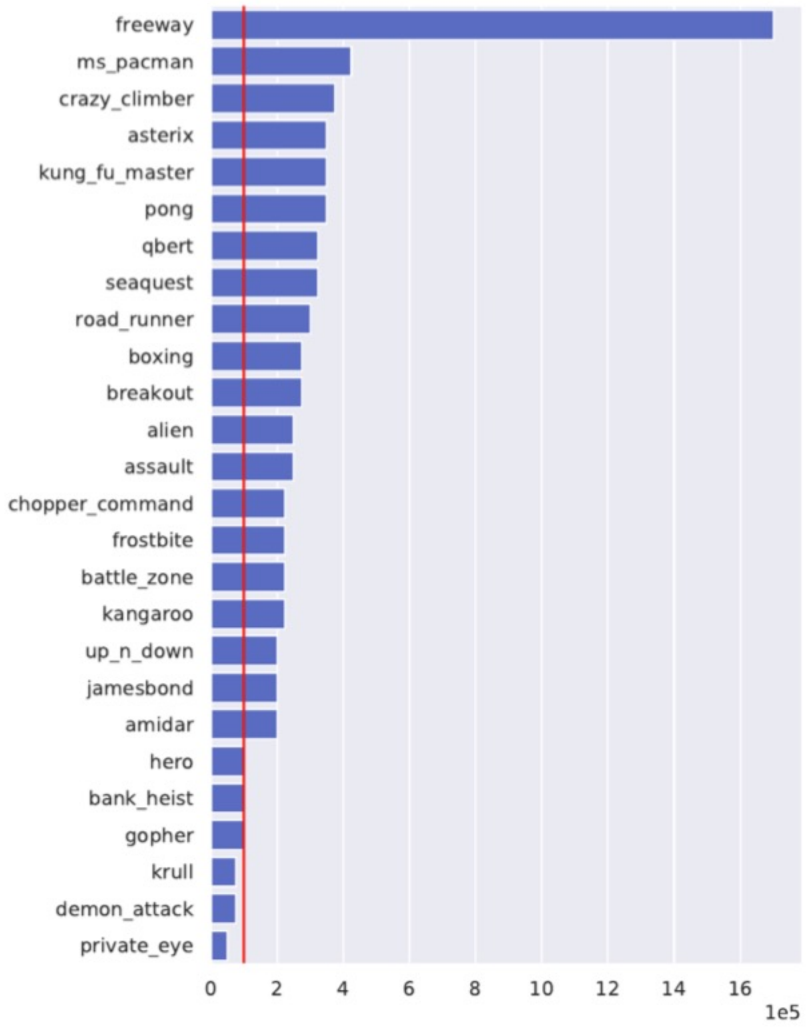
Solution 1: use policy gradient from model-free RL

- E.g., policy gradient algorithm such as TRPO, PPO, Advantage actor critic, etc.
- Doesn't require multiplying many Jacobians, which leads to large gradient

Example: MBRL for Atari

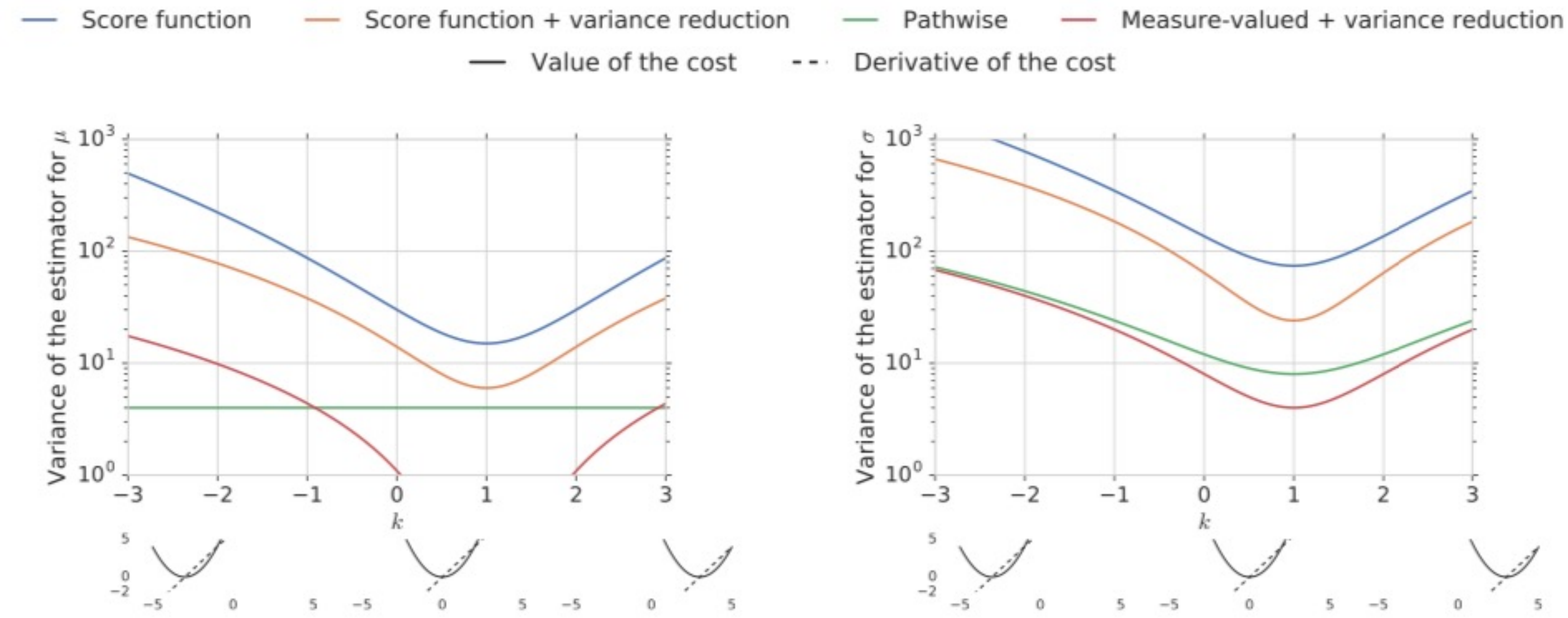
- Atari playing from pixels one of the first major successes of deep RL
- Seems like quintessential domain in which model-free makes sense
- Use video prediction model (shown below) + PPO





Aside: Pathwise derivative

Comparing gradient estimators



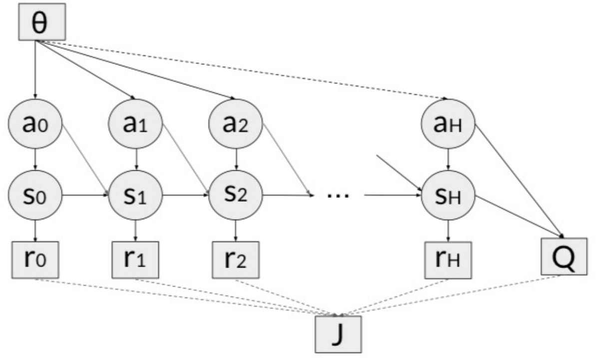
Monte Carlo Gradient Estimation in Machine Learning, Mohamed et al., JMLR 2020.

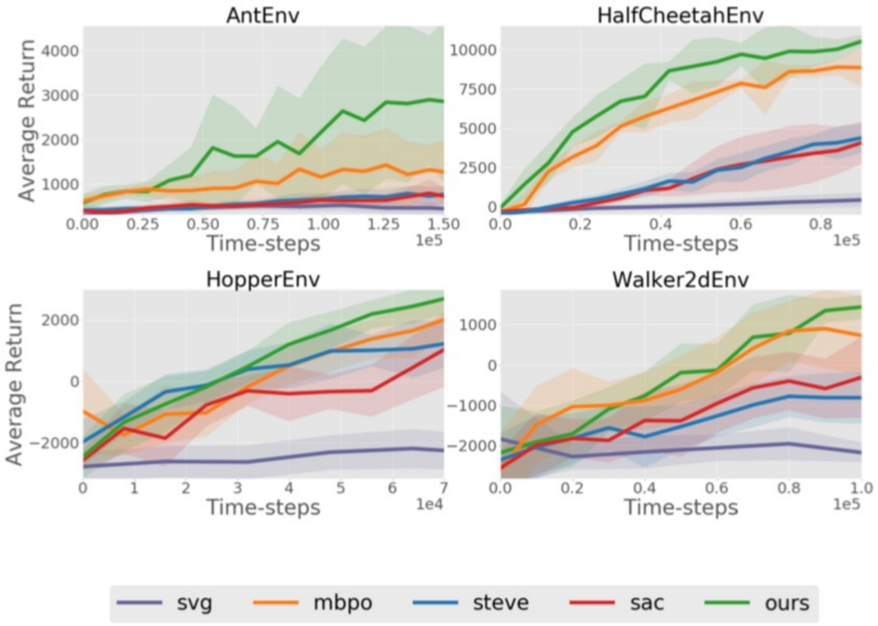
Solution 2: Use value function for tail return

- Clavera, Fu, Abbeel, *Model-augmented actor critic: Backpropagating through paths,* ICLR 2020.
- Stochastic policy and dynamics: compute gradient via pathwise derivative

$$J_\pi(\boldsymbol{\theta}) = \mathbb{E}\left[\sum_{t=0}^{H-1} \gamma^t r(s_t) + \gamma^H \hat{Q}(s_H, a_H)\right]$$

- Use ensemble of dynamics models, two Q functions, Dyna-style training





Summary and Conclusion

- Discussed two possible solutions; infinitely many more
- Very busy research direction! Many topics not covered here
  - Many possible combinations of planning/control, policies, values, and models
- Quite practical: model learning is data efficient and parameterized policy is cheap to evaluate at run time

Next time

- Back to optimal control! Indirect methods