# Principles of Robot Autonomy I

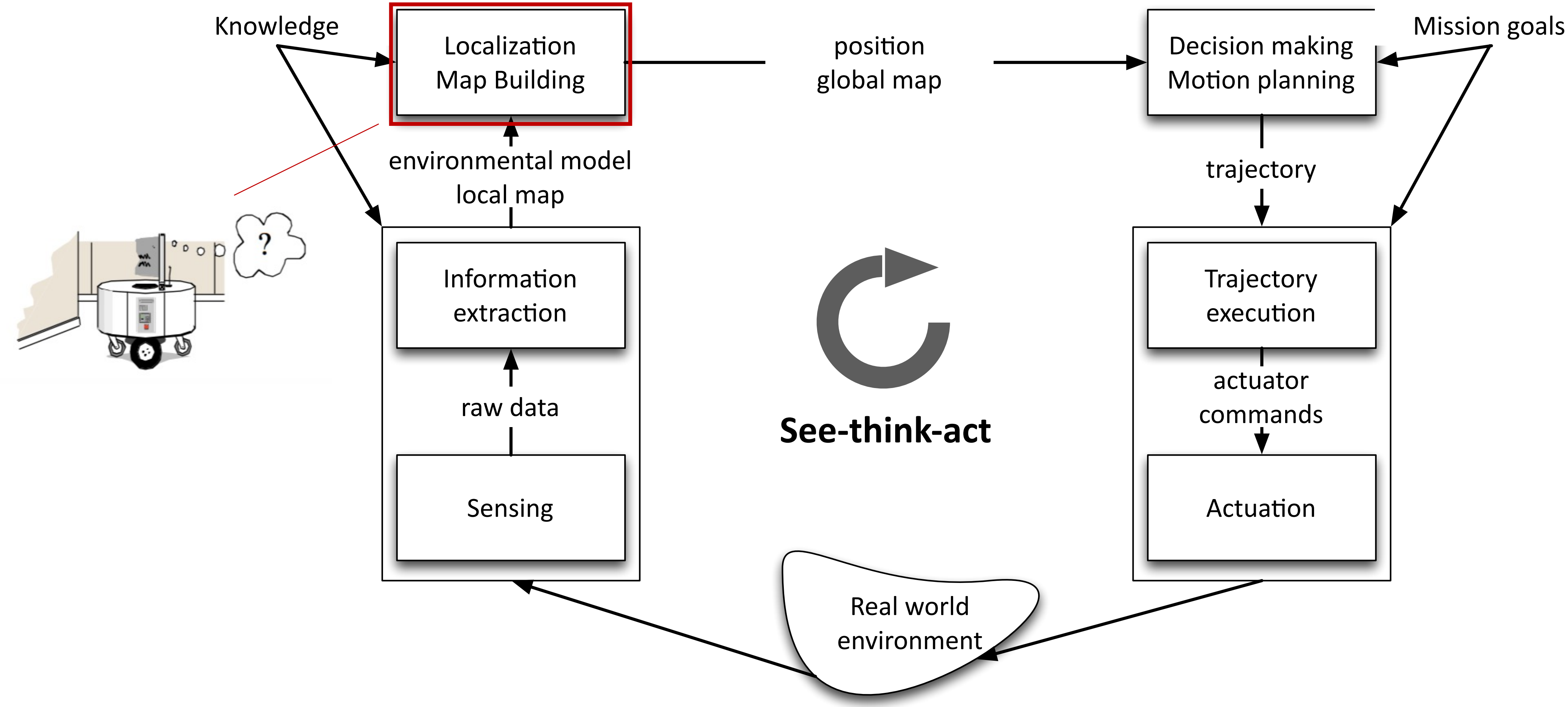
## Non-parametric filtering

Stanford University

IPRL

Autonomous Systems Lab

# Module 3


Knowledge
Localization
Map Building
position
global map
Decision making
Motion planning
Mission goals
environmental model
local map
trajectory
Information
extraction
raw data
Sensing
See-think-act
Trajectory
execution
actuator
commands
Actuation
Real world
environment
?

# Today's lecture

- Aim
  - Learn about non-parametric filters

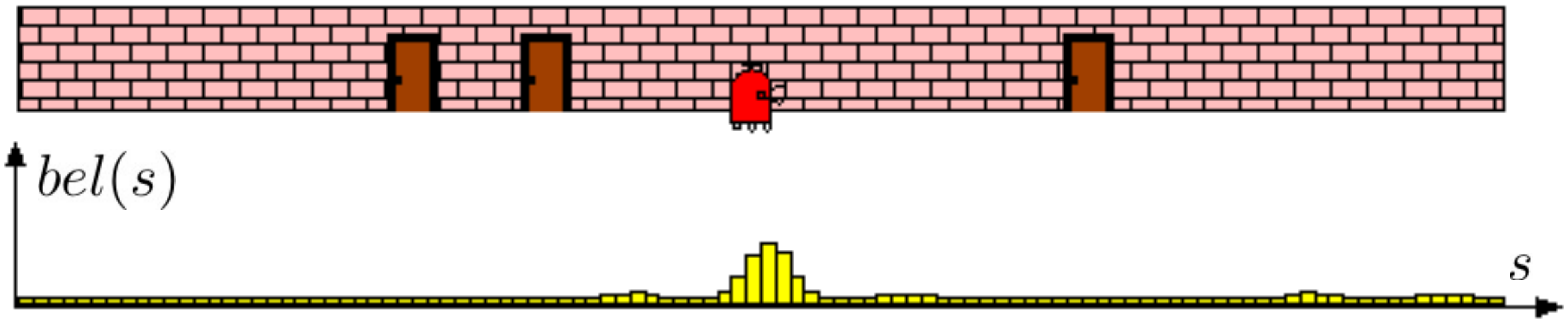
- Readings
  - S. Thrun, W. Burgard, and D. Fox. Probabilistic robotics. MIT press, 2005. Sections 3.1 – 3.4, 4.1, 4.3, 7.1

# Instantiating the Bayes’ filter

- Tractable implementations of Bayes’ filter exploit structure and / or approximations; two main classes
  - Parametric filters: e.g., KF, EKF, UKF, etc.
  - Non parametric filters: e.g., histogram filter, particle filter, etc.

# Histogram filter

- Key idea: use *discrete* Bayes' filter as an approximate inference tool for *continuous* state spaces



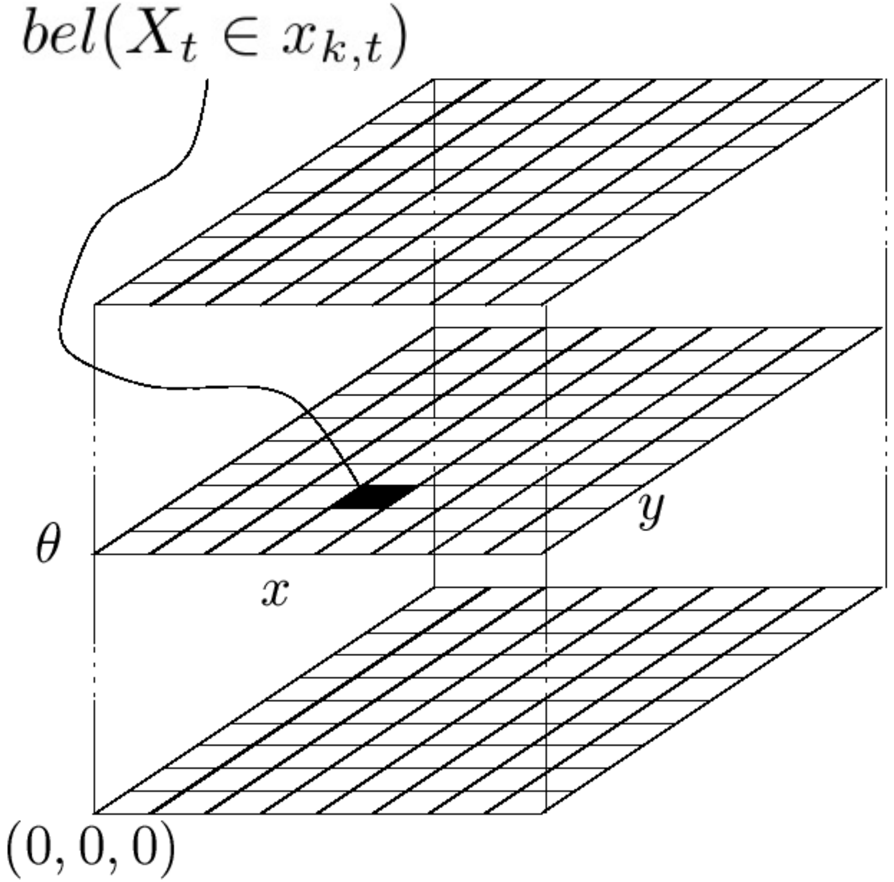
- Step #1: histogram filters decompose a continuous space into finitely many bins

$$\mathrm{dom}(X_t) = x_{1,t} \cup x_{2,t} \cup \ldots x_{K,t}$$

State space

$\{x_{k,t}\}$: convex regions forming a partition of state space (e.g., grid cell)

# Example



# Histogram filter

- Step #2: assign to each region $x_{k,t}$ a probability $p_{k,t}$; probabilities are then approximated according to a piecewise scheme

$$p(x_t) \equiv \frac{p_{k,t}}{|x_{k,t}|}, \quad \text{for all } x_t \in x_{k,t} \qquad \Rightarrow \qquad p(X_t \in x_{k,t}) = \int_{x_{k,t}} \frac{p_{k,t}}{|x_{k,t}|}\, dx_t = p_{k,t}$$

# Histogram filter

- Step #3: discretize motion and measurements models, i.e.,

$$p(x_t \mid u_t,\, x_{t-1}) \qquad \text{and} \qquad p(z_t \mid x_t)$$

1. Select mean state as representative state

$$\hat{x}_{k,t} = |x_{k,t}|^{-1} \int_{x_{k,t}} x_t \, dx_t$$

2. Approximate measurement model

$$p(z_t \mid x_{k,t}) \approx p(z_t \mid \hat{x}_{k,t})$$

3. Approximate transition model

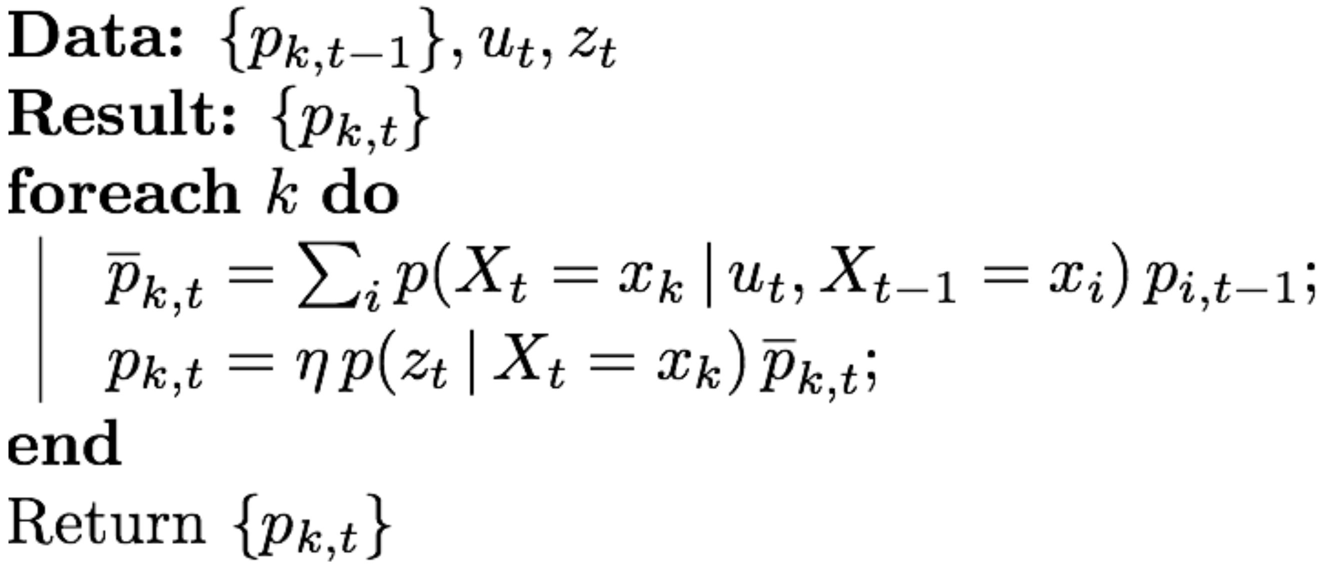
$$p(x_{k,t} \mid u_t,\, x_{i,t-1}) \approx \eta \, |x_{k,t}| \, p(\hat{x}_{k,t} \mid u_t,\, \hat{x}_{i,t-1})$$

- Step #4: execute discrete Bayes' filter with discretized probabilities

# Histogram filter

- Then one can run the usual discrete Bayes' filter

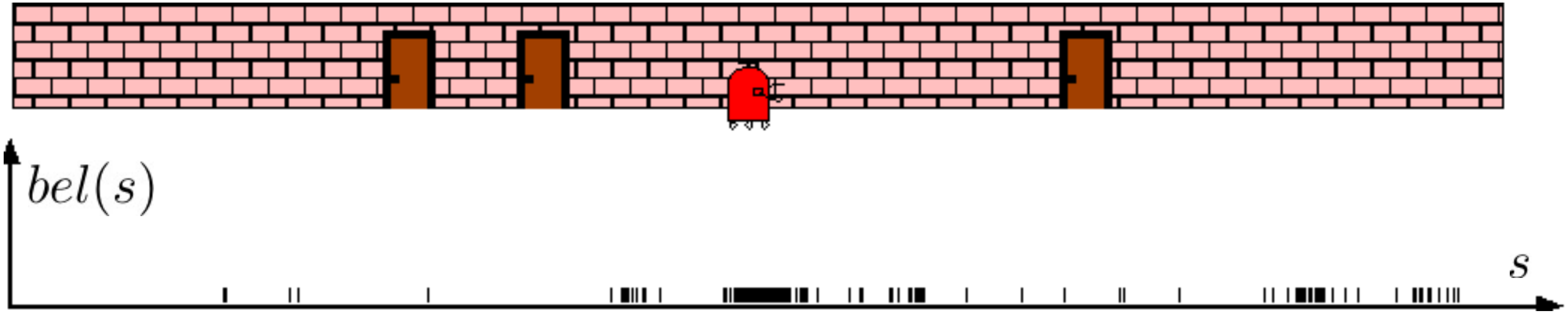
- Belief $bel(x_t)$ represented as pmf $\{p_{k,t}\}$



```
**Data:** $\{p_{k,t-1}\}, u_t, z_t$
**Result:** $\{p_{k,t}\}$
**foreach** $k$ **do**
    $\overline{p}_{k,t} = \sum_i p(X_t = x_k \,|\, u_t, X_{t-1} = x_i)\, p_{i,t-1}$;
    $p_{k,t} = \eta\, p(z_t \,|\, X_t = x_k)\, \overline{p}_{k,t}$;
**end**
Return $\{p_{k,t}\}$
```

# Particle filter

- Key idea: represent posterior $bel(x_t)$ by a set of random samples



- Allows one to represent non-Gaussian distributions and handle nonlinear transformations in a direct way

# Particle filter

- Samples of posterior distribution are called *particles*, denoted as

$$\mathcal{X}_t := x_t^{[1]}, x_t^{[2]}, \ldots, x_t^{[M]}$$

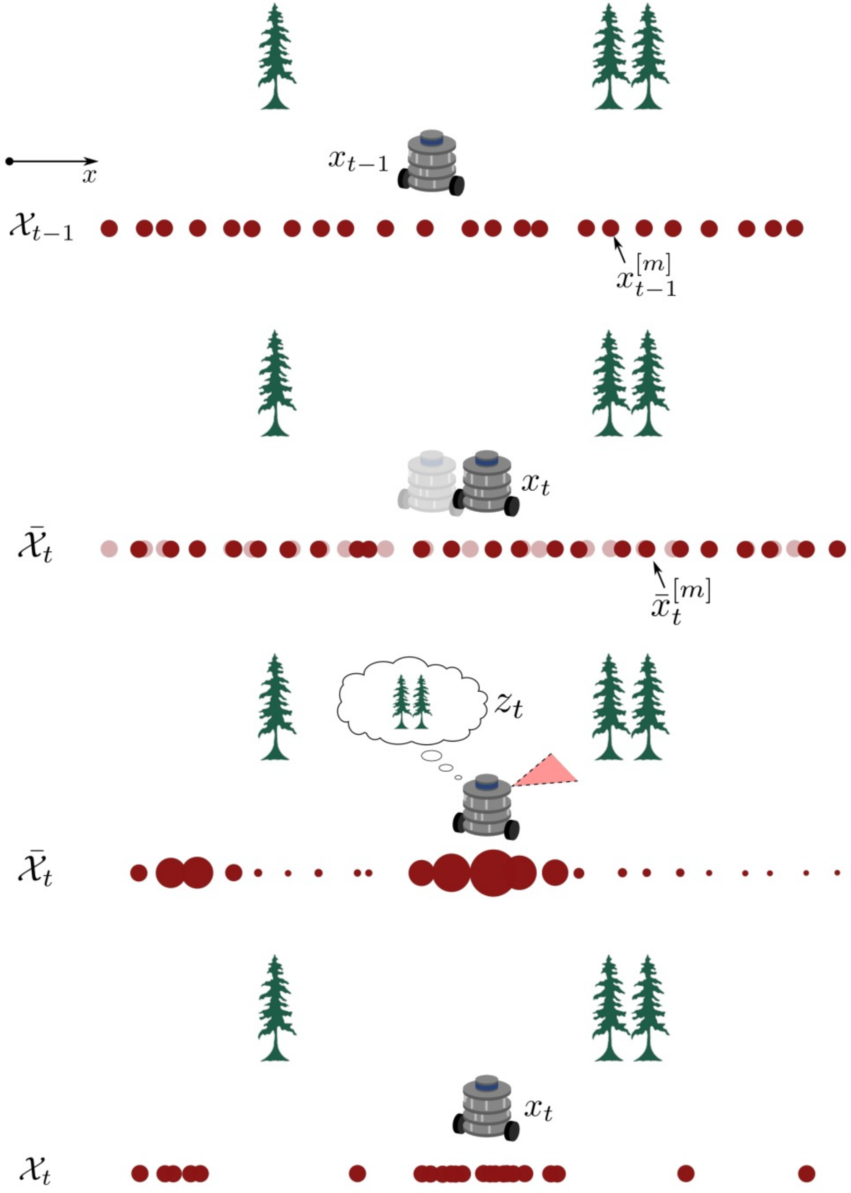
- A particle represents a hypothesis about what the true world state might be at time $t$
- Ideally, particles should be distributed according to

$$x_t^{[m]} \sim p(x_t \mid z_{1:t},\, u_{1:t}) = bel(x_t)$$

- Matching exact only as $M \rightarrow \infty$, but $M \approx 1000$ usually good enough
- A particle filter constructs the particle set $\mathcal{X}_t$ from the particle set $\mathcal{X}_{t-1}$ recursively, with the goal of matching the distribution $bel(x_t)$

# Particle filter: example

- Resampling can be a high variance process (e.g., “weight collapse” can be a problem) motivating the development of lower variance schemes and/or recovery processes
- Many extensions/variants (e.g., Gaussian Sum Particle Filtering in which belief is represented as a Gaussian Mixture Model)



# Particle filter: algorithm

- The temporary particle set $\overline{\mathcal{X}}_t$ represents the belief $\overline{bel}(x_t)$
- The particle set $\mathcal{X}_t$ represents the belief $bel(x_t)$
- Importance factors are used to incorporate measurement $z_t$ in the particle set
- After resampling, particles are (as $M \rightarrow \infty$) distributed as

$$bel(x_t) = \eta \, p(z_t \mid x_t^{[m]}) \, \overline{bel}(x_t)$$

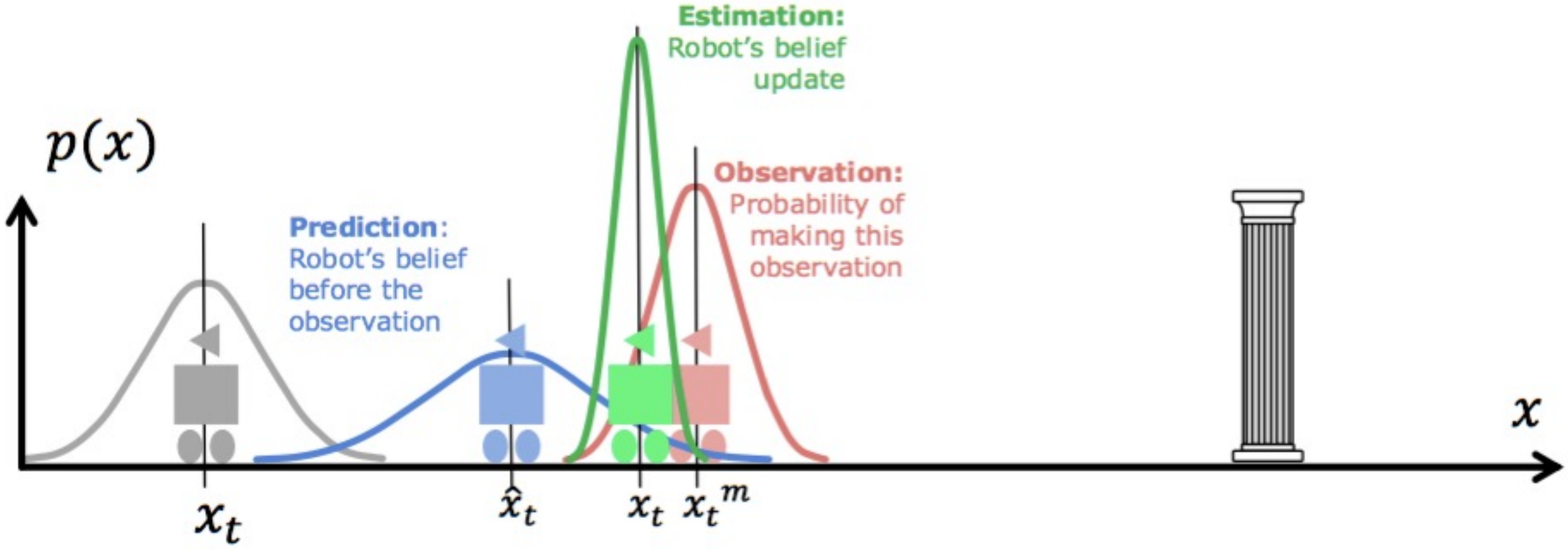
```
Data: $\overbrace{\mathcal{X}_{t-1}}^{bel(x_{t-1})}, u_t, z_t$
Result: $\mathcal{X}_t$
$\overline{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$;
for $i = 1$ to $M$ do
    Sample $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$;
    $w_t^{[m]} = p(z_t \mid x_t^{[m]})$;
    $\overline{\mathcal{X}}_t = \overline{\mathcal{X}}_t \cup \left(x_t^{[m]}, w_t^{[m]}\right)$;
end
for $m = 1$ to $M$ do
    Draw $i$ with probability $\propto w_t^{[i]}$;
    Add $x_t^{[i]}$ to $\mathcal{X}_t$;
end
Return $\underbrace{\mathcal{X}_t}_{bel(x_t)}$
```

Prediction: $\overline{bel}(x_t)$

Importance factor

Correction: $bel(x_t)$

# Next time


$p(x)$
$x$
**Prediction**: Robot's belief before the observation
**Estimation:** Robot's belief update
**Observation:** Probability of making this observation
$x_t$
$\hat{x}_t$
$x_t$
$x_t{}^m$