Principles of Robot Autonomy I

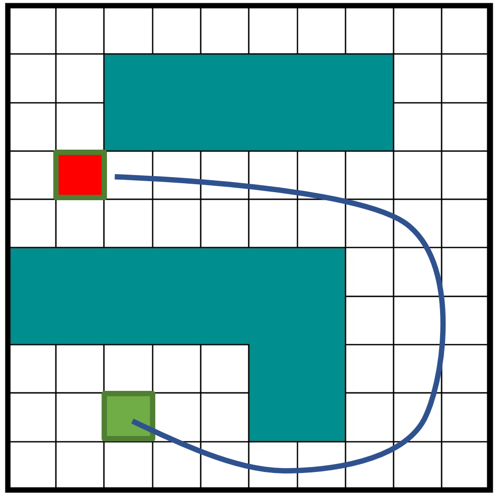
Motion planning I: graph search methods

Stanford University

Autonomous Systems Lab

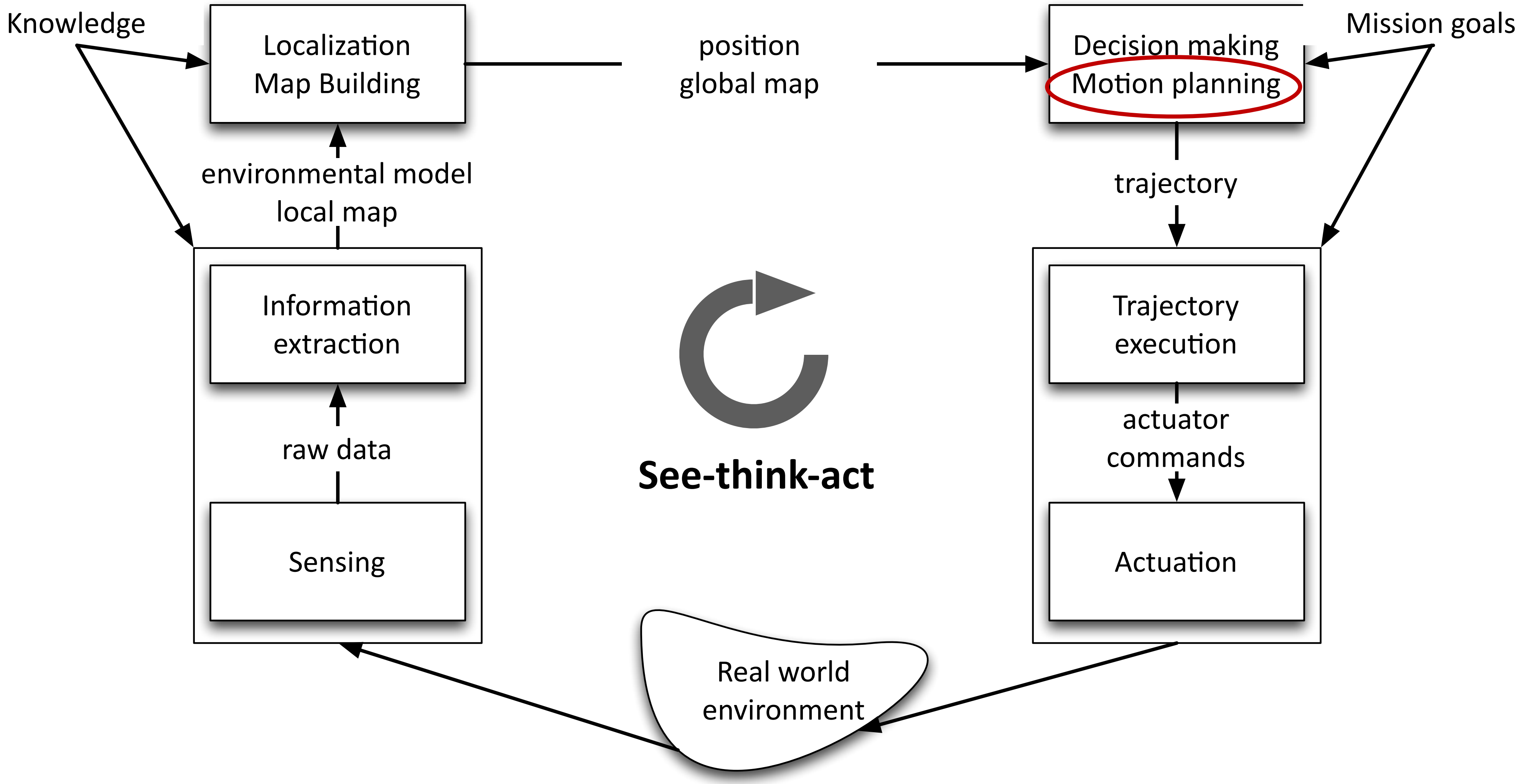
Motion planning

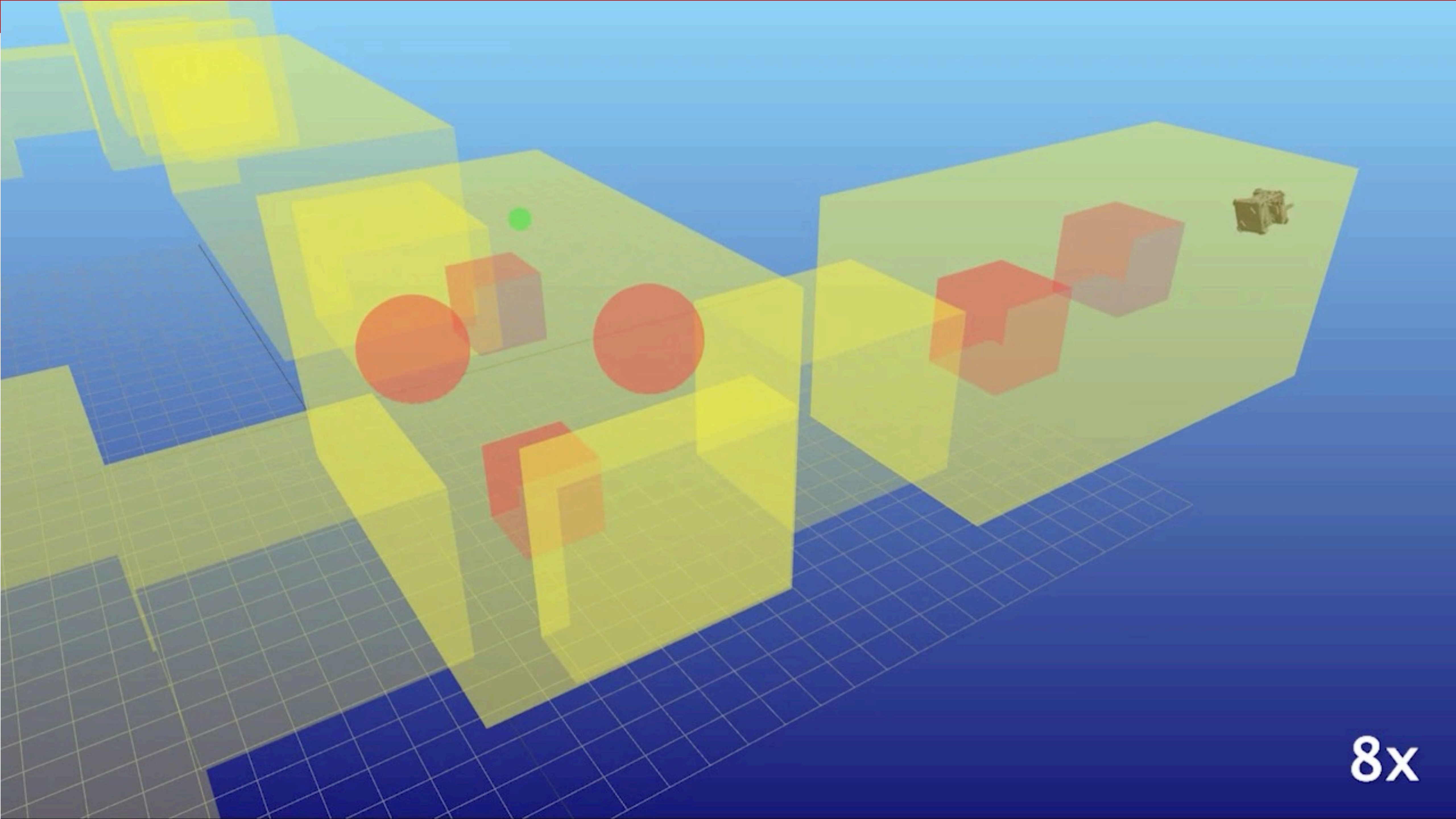
Compute sequence of actions that drives a robot from an initial condition to a terminal condition while avoiding obstacles, respecting motion constraints, and possibly optimizing a cost function



- Aim
  - Introduction to motion planning
  - Learn about search-based methods for motion planning
- Readings:
  - D. Bertsekas. Dynamic Programming and Optimal Control, Vol I. Section 2.3.
  - S. LaValle. Planning Algorithms. Sections 6.1-6.3, 6.5.

The see-think-act cycle

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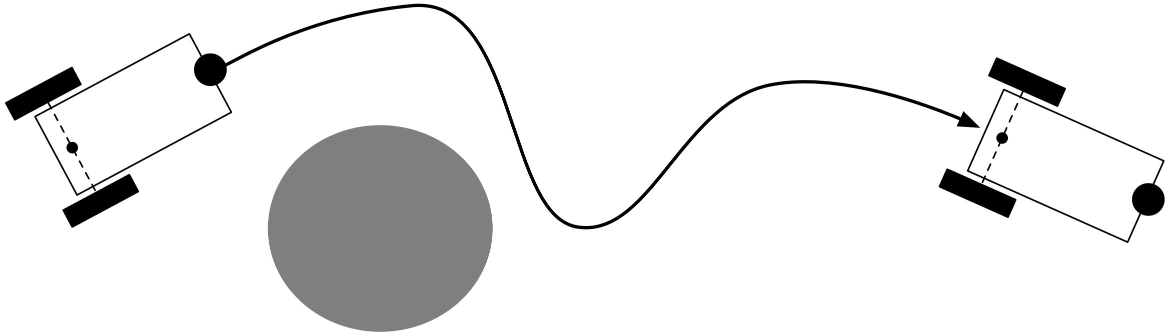
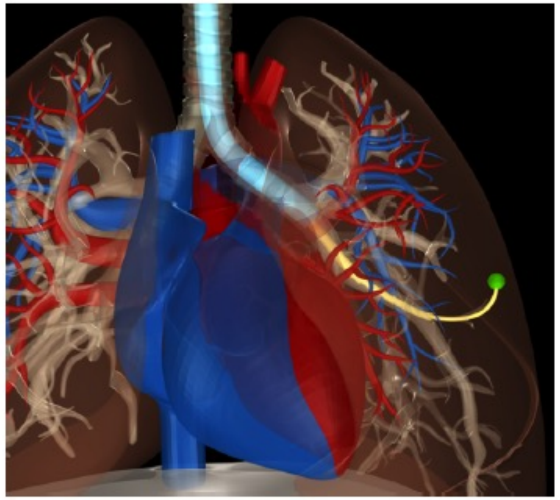
Examples from:
https://ompl.kavrakilab.org/gallery.html

More examples of motion planning

- Steering autonomous vehicles
- Controlling humanoid robot
- Surgery planning
- Protein folding
- …



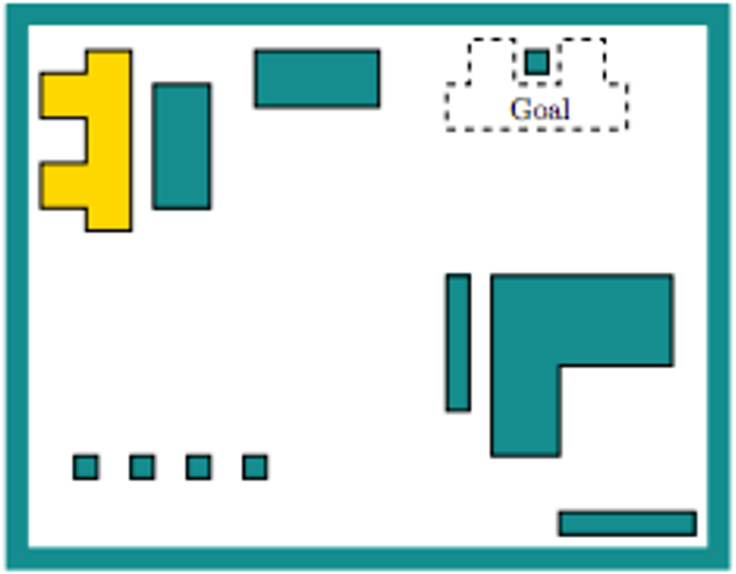


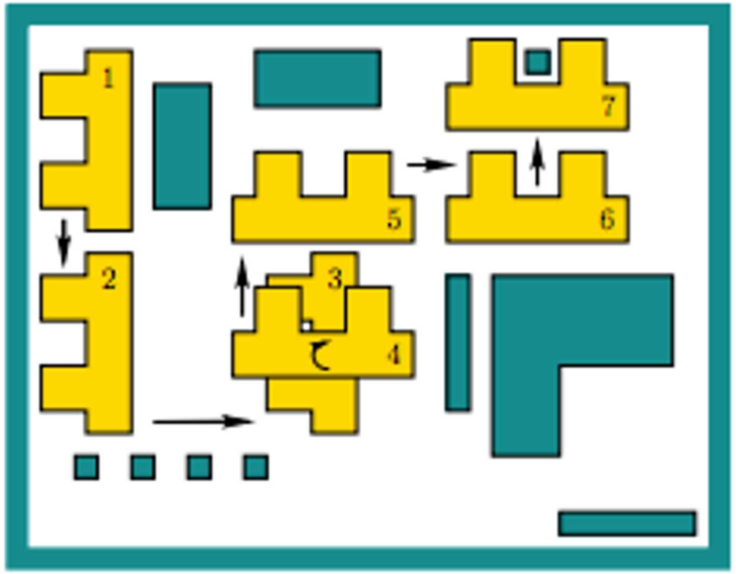
Some history

- Formally defined in the 1970s
- Development of exact, combinatorial solutions in the 1980s
- Development of sampling-based methods in the 1990s
- Deployment on real-time systems in the 2000s
- Current research: inclusion of differential and logical constraints, planning under uncertainty, parallel implementation, feedback plans and more

Simplest setup

- Assume 2D workspace: $\mathcal{W} \subseteq \mathbb{R}^2$
- $\mathcal{O} \subset \mathcal{W}$ is the obstacle region with polygonal boundary
- Robot is a rigid polygon
- Problem: given initial placement of robot, compute how to gradually move it into a desired goal placement so that it never touches the obstacle region





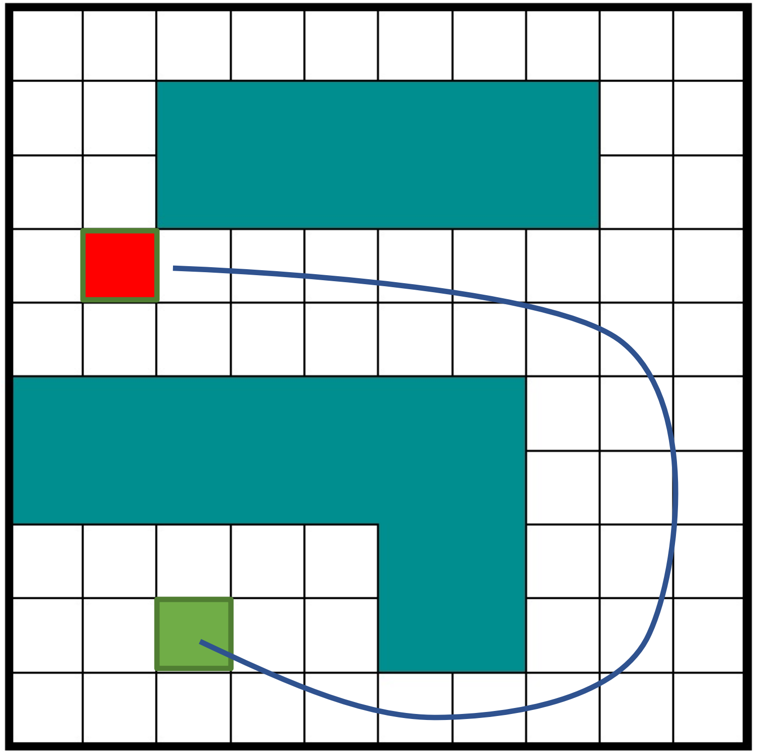
Popular approaches

- *Potential fields* [Rimon, Koditschek, '92]: create forces on the robot that pull it toward the goal and push it away from obstacles
- *Grid-based planning* [Stentz, '94]: discretizes problem into grid and runs a graph-search algorithm (Dijkstra, A*, …)
- *Combinatorial planning* [LaValle, '06]: constructs structures in the configuration (C-) space that completely capture all information needed for planning
- *Sampling-based planning* [Kavraki et al, '96; LaValle, Kuffner, '06, etc.]: uses collision detection algorithms to probe and incrementally search the C-space for a solution, rather than completely characterizing all of the C_{free} structure

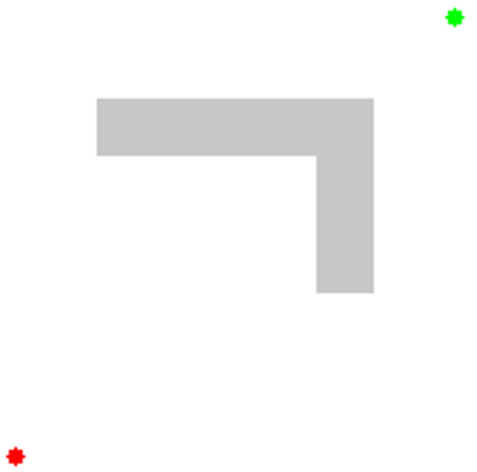
Grid-based approaches

- Discretize the continuous world into a grid
  - Each grid cell is either free or forbidden
  - Robot moves between adjacent free cells
  - **Goal:** find sequence of free cells from start to goal

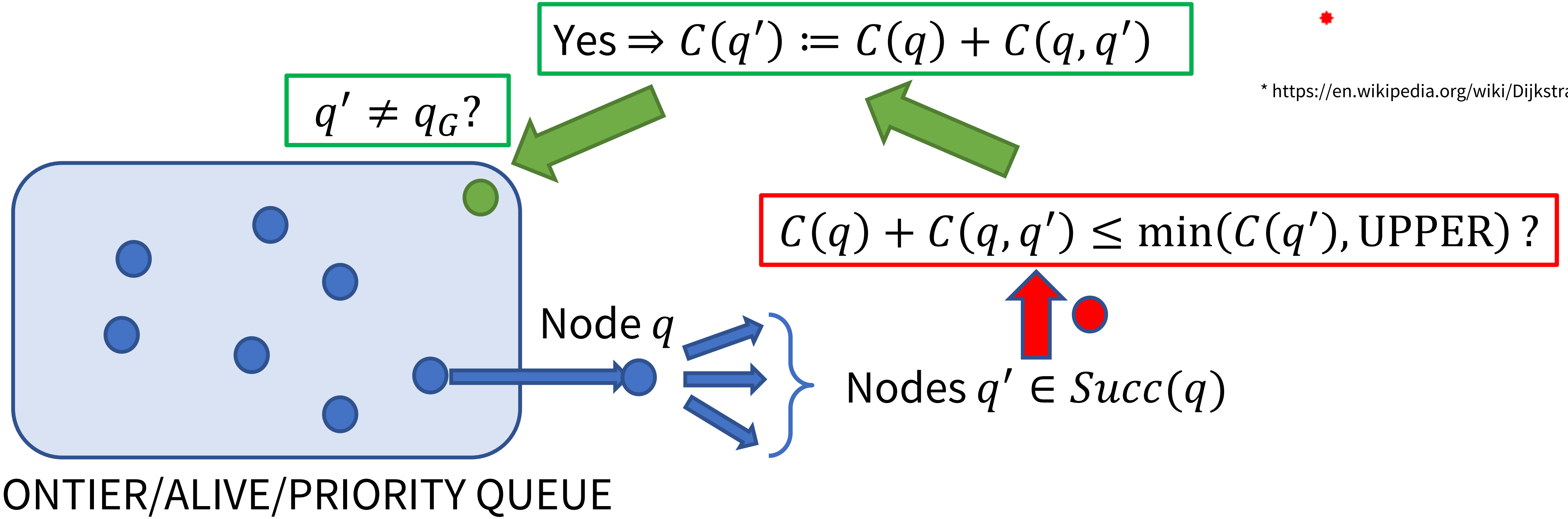
- Mathematically, this corresponds to pathfinding in a discrete graph $G = (V, E)$
  - Each vertex $v \in V$ represents a free cell
  - Edges $(v, u) \in E$ connect adjacent grid cells



Graph search algorithms



- Having determined decomposition, how to find "best" path?
- **Label-Correcting Algorithms**: $C(q)$: *cost-of-arrival* from q_I to q



* https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm

Label correcting algorithm

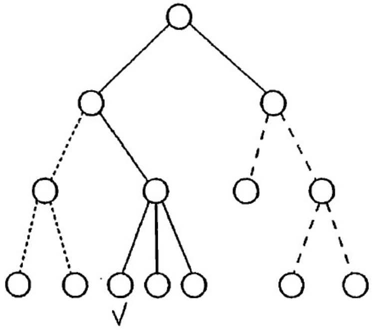
Step 1. Remove a node q from frontier queue and for each child q' of q, execute step 2

Step 2. If $C(q) + C(q, q') \leq \min(C(q'), \mathrm{UPPER})$, set $C(q') \coloneqq C(q) + C(q, q')$ and set q to be the parent of q'. In addition, if $q' \neq q_G$, place q' in the frontier queue if it is not already there, while if $q' = q_G$, set UPPER to the new value $C(q) + C(q, q_G)$

Step 3. If the frontier queue is empty, terminate, else go to step 1

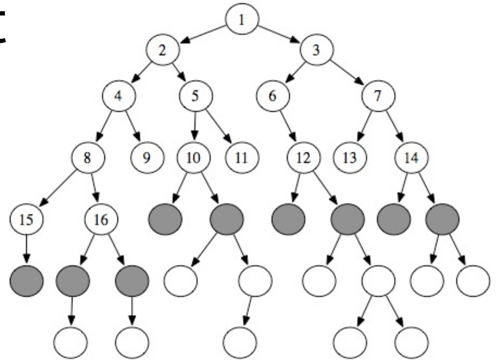
Initialization: set the labels of all nodes to ∞, except for the label of the origin node, which is set to 0

GetNext() ?



Depth-First-Search (DFS): Maintain Q as a **stack** – Last in/first out

- Lower memory requirement (only need to store part of graph)



Breadth-First-Search (BFS, Bellman-Ford): Maintain Q as a **list** – First in/first first out

- Update cost for all edges up to current depth before proceeding to greater depth
- Can deal with negative edge (transition) costs

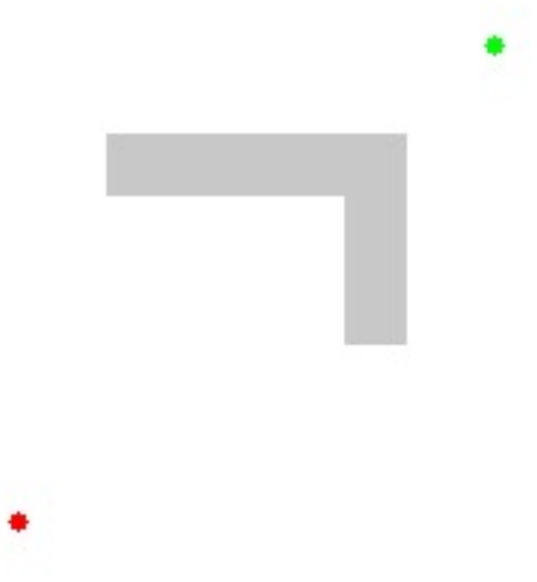
Best-First (BF, Dijkstra): Greedily select next q: $q = \operatorname{argmin}_{q \in Q} C(q)$

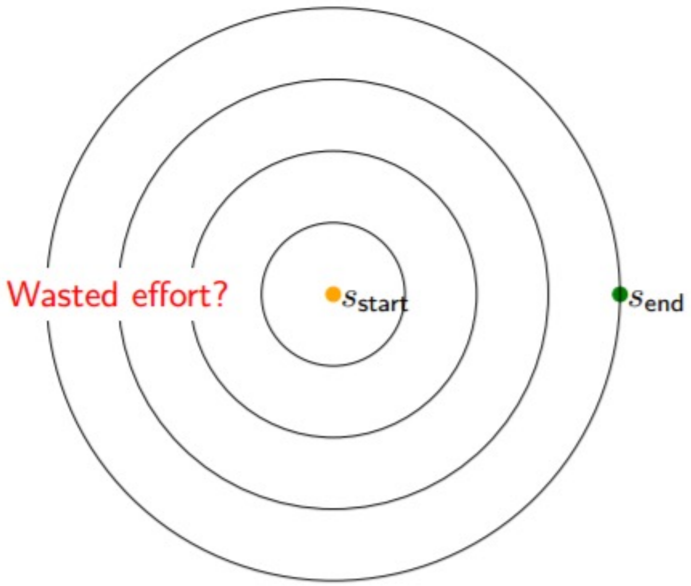
- Node will enter the frontier queue at most *once*
- Requires costs to be non-negative

Correctness and improvements

Theorem

If a feasible path exists from q_I to q_G, then algorithm terminates in finite time with $C(q_G)$ equal to the optimal cost of traversal, $C^*(q_G)$.





A*: Improving Dijkstra

- Dijkstra orders by optimal "*cost-to-arrival*"
- Faster results if order by "*cost-to-arrival*"+ *(approximate)* "*cost-to-go*"
- That is, strengthen test

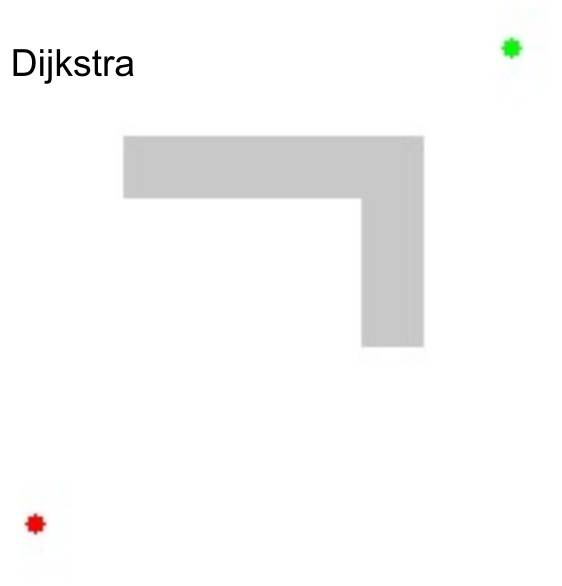
$$C(q) + C(q, q') \leq \text{UPPER}$$

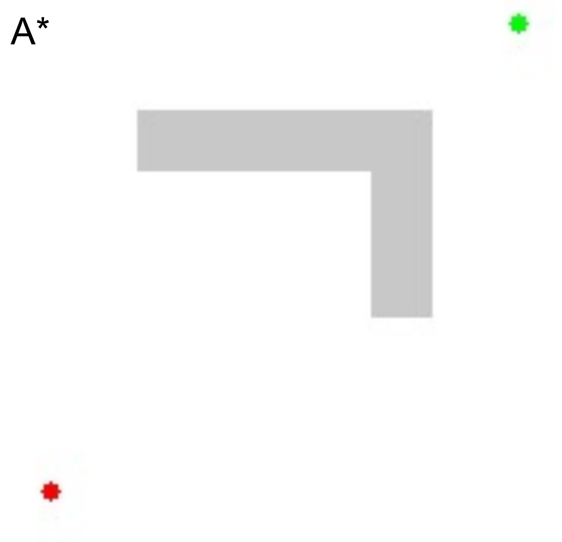
to

$$C(q) + C(q, q') + h(q') \leq \text{UPPER}$$

where $h(q)$ is *a* heuristic for optimal cost-to-go (specifically, a positive *underestimate*)

- In this way, fewer nodes will be placed in the frontier queue
- This modification still guarantees that the algorithm will terminate with a shortest path



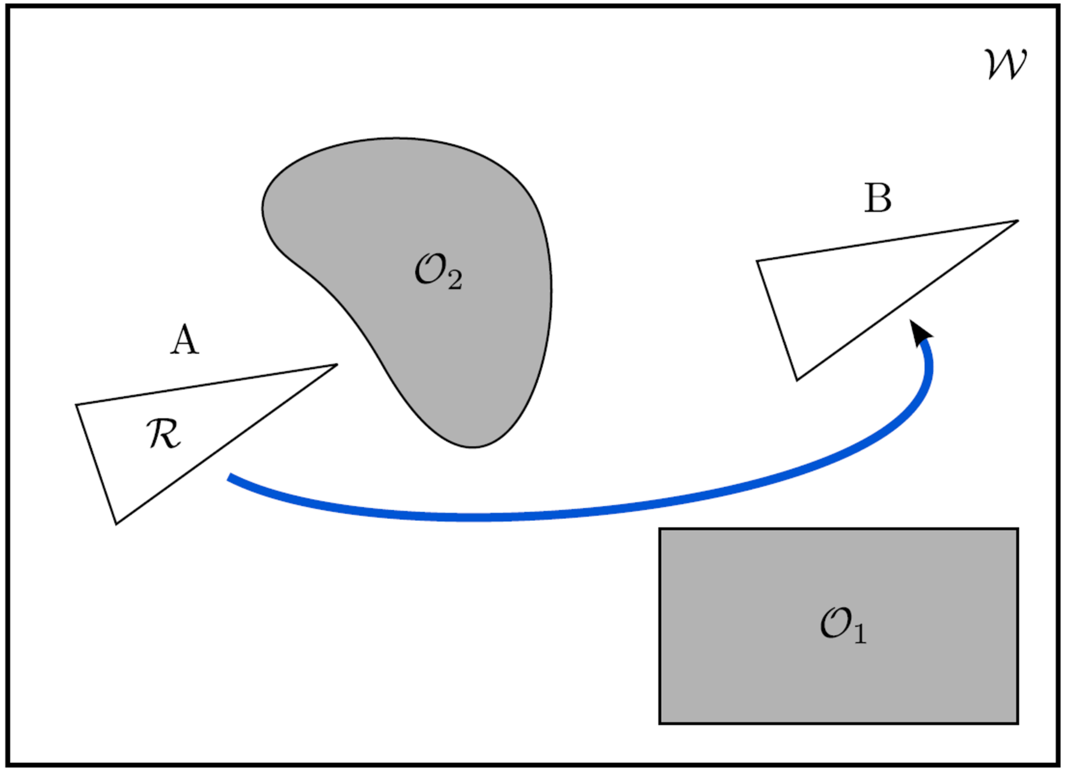


Grid-based approaches: summary

- Pros:
  - Simple and easy to use
  - Fast (for some problems)
- Cons:
  - Resolution dependent
    - Not guaranteed to find solution if grid resolution is not small enough
  - Limited to simple robots
    - Grid size is exponential in the number of DOFs

Back to continuous motion planning

- A robot is a geometric entity operating in continuous space
- *Combinatorial techniques* for motion planning capture the structure of this continuous space
  - Particularly, the regions in which the robot is not in collision with obstacles
- Such approaches are typically complete
  - i.e., guaranteed to find a solution;
  - and sometimes even an optimal one



Simplest setup revisited

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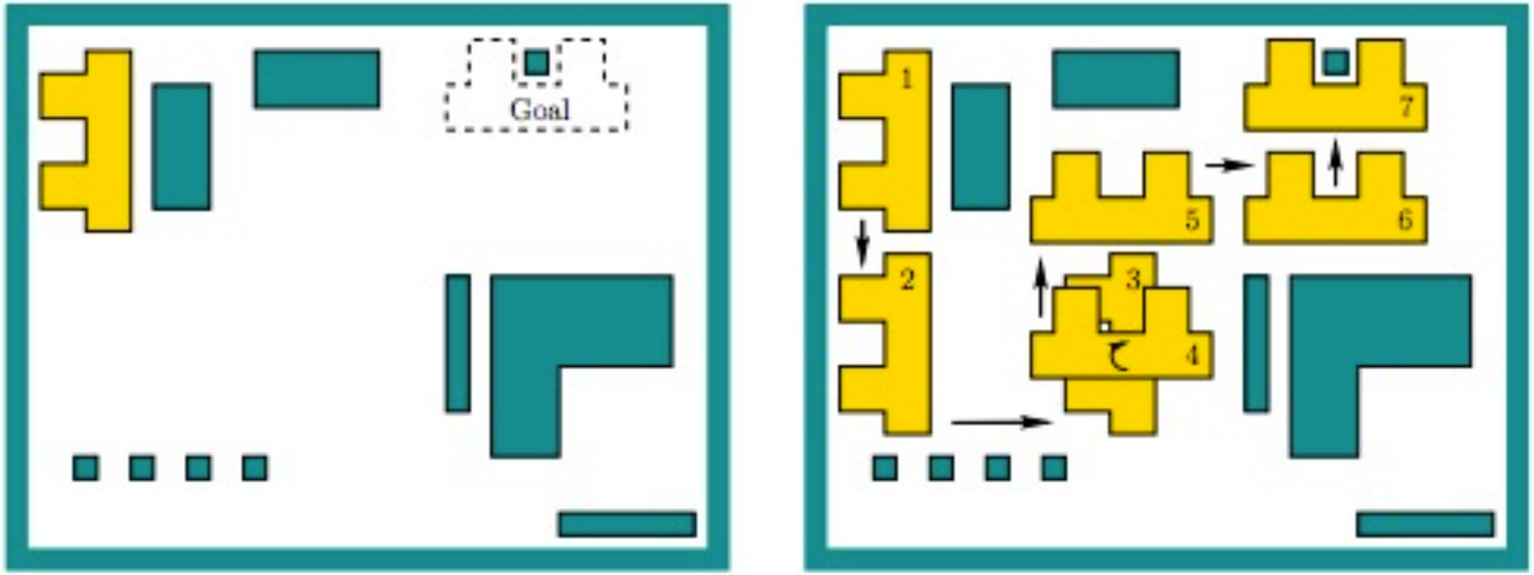
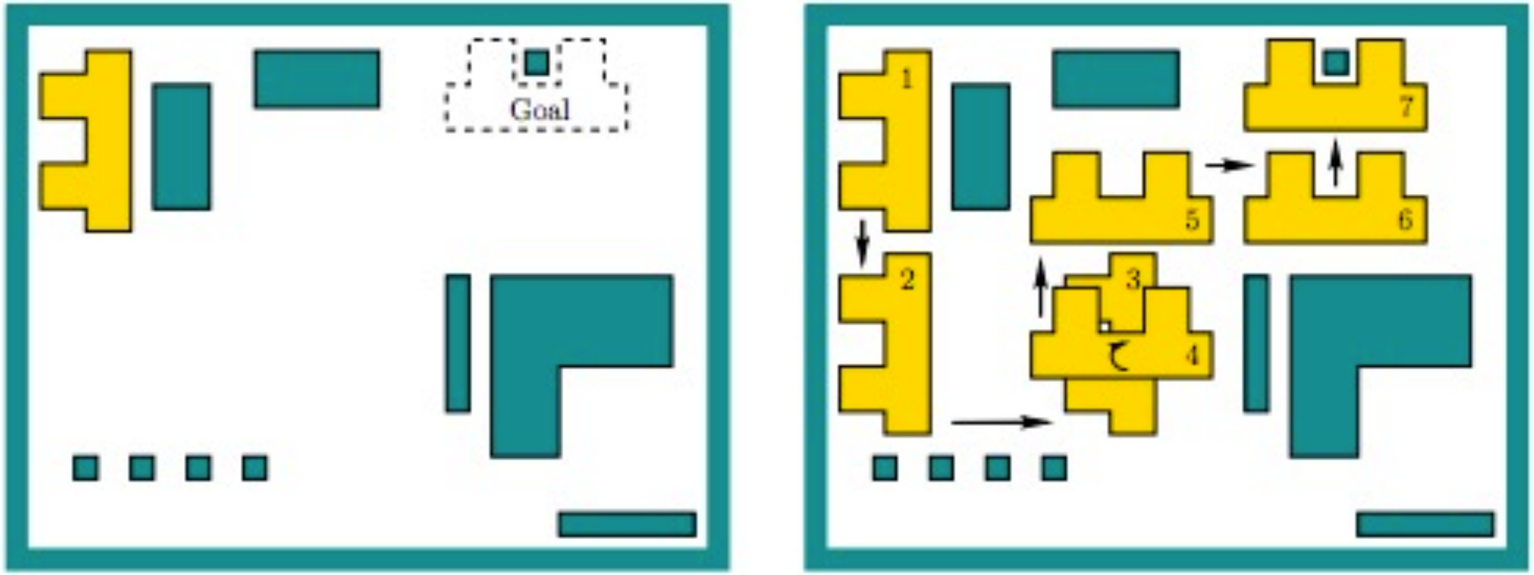


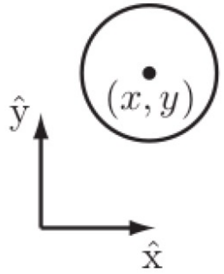
Simplest setup

Key point: motion planning problem described in the real-world, but it really lives in another space -- the configuration (*C*-) space!

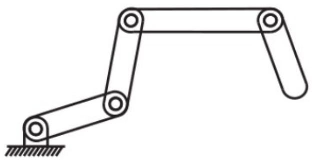


Configuration space

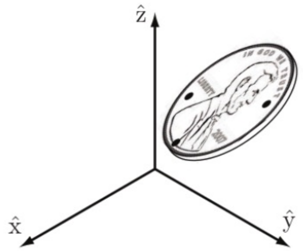
- *C*- space: captures all degrees of freedom (all rigid body transformations)
- More in detail, let $\mathcal{R} \subset \mathbb{R}^2$ be a polygonal robot (e.g., a triangle)
- The robot can rotate by angle θ or translate $(x_t, y_t) \subset \mathbb{R}^2$
- Every combination $q = (x_t, y_t, \theta)$ yields a *unique* robot placement: configuration
- So *C*- space is a subset of $\mathbb{R}^3$
- Note: $\theta \pm 2\pi$ yields equivalent rotations $\Rightarrow$ *C*- space is: $\mathbb{R}^2 \times \mathcal{S}^1$
- Concept of *C*- space extends naturally to higher dimensions (e.g., robot linkages)



$d = 2$



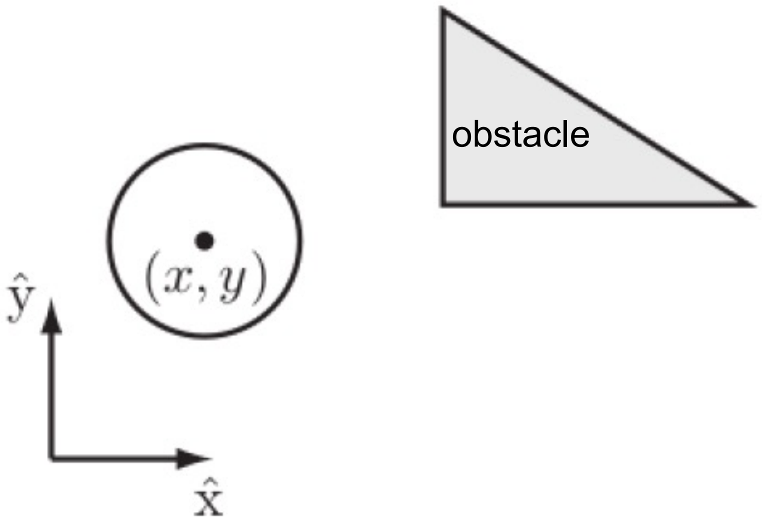
$d = 4$

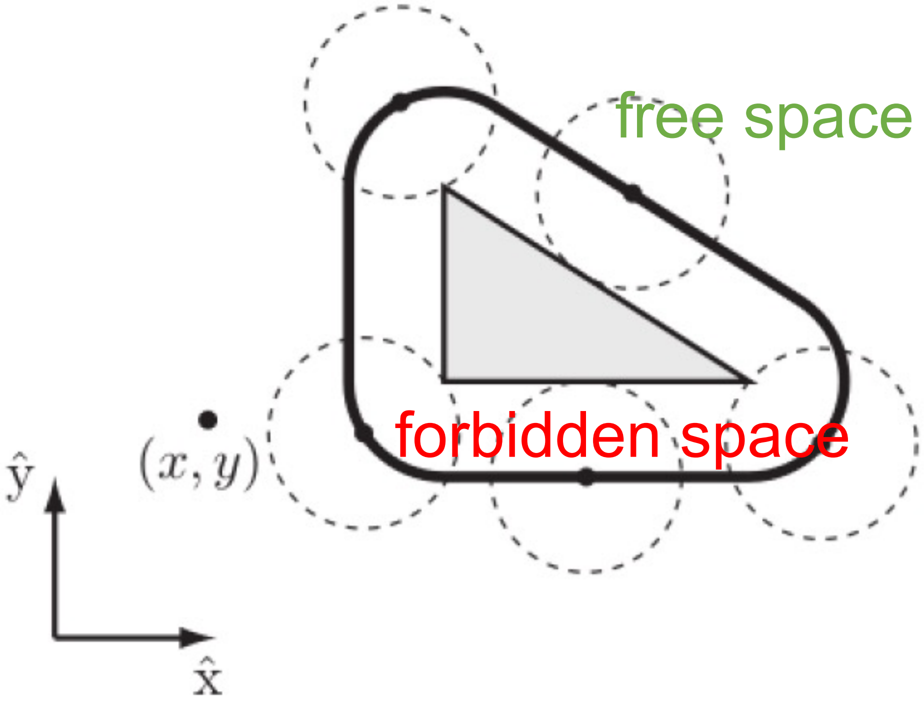


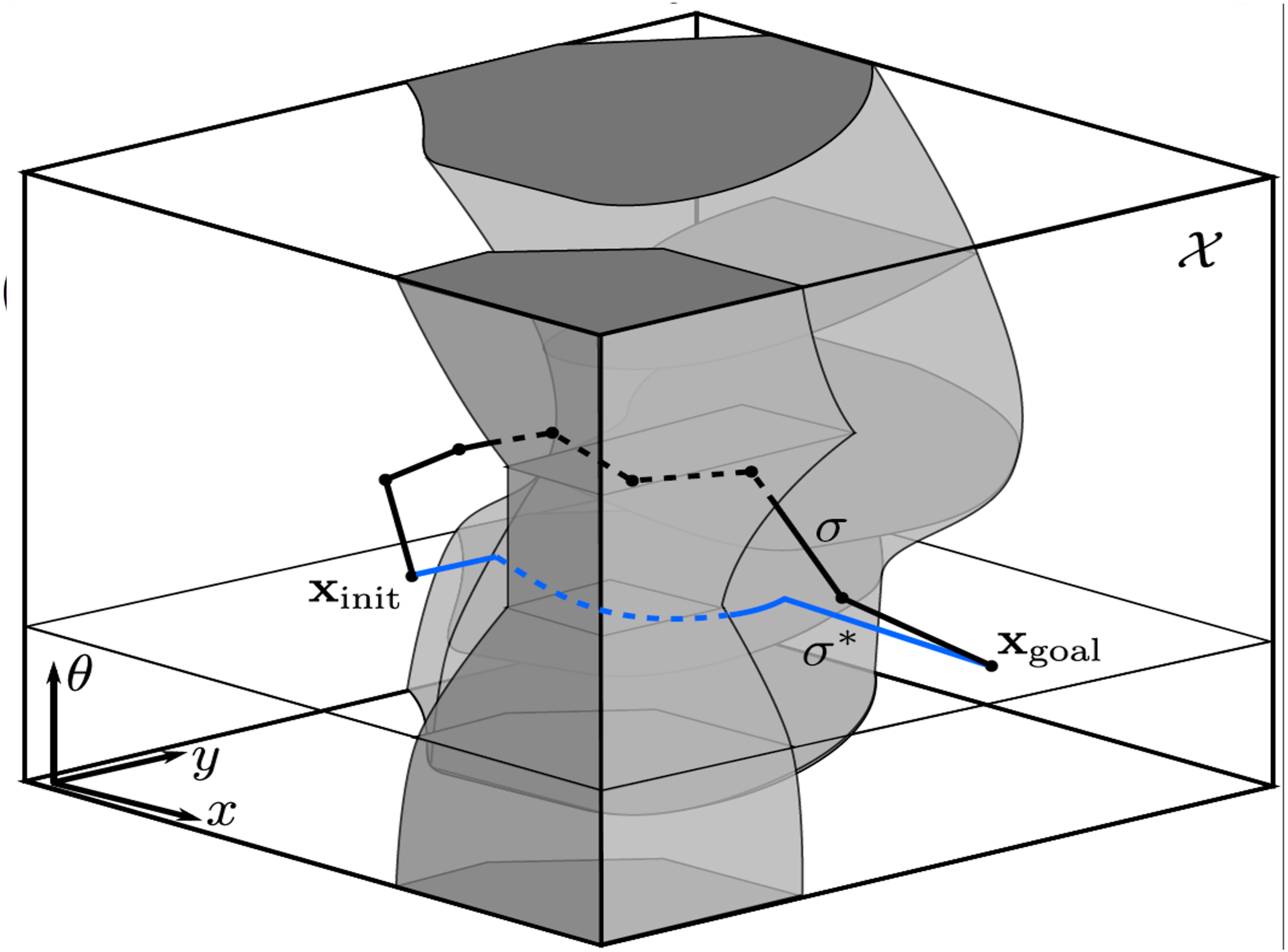
$d = 6$

Configuration free space

- The subset $\mathcal{F} \subseteq \mathcal{C}$ of all collision free configurations is the **free space**



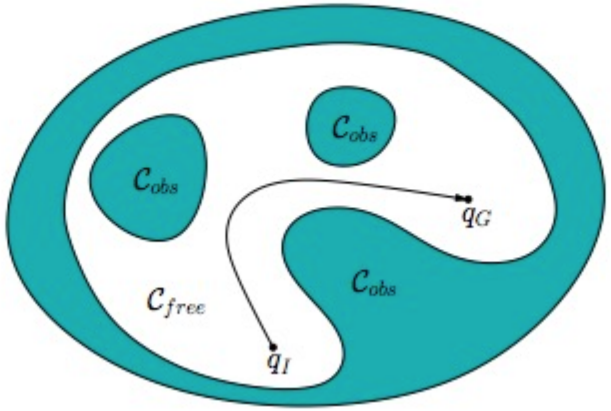




$\mathcal{X}$
σ
$\mathbf{x}_{\text{init}}$
σ^*
$\mathbf{x}_{\text{goal}}$
θ
y
x

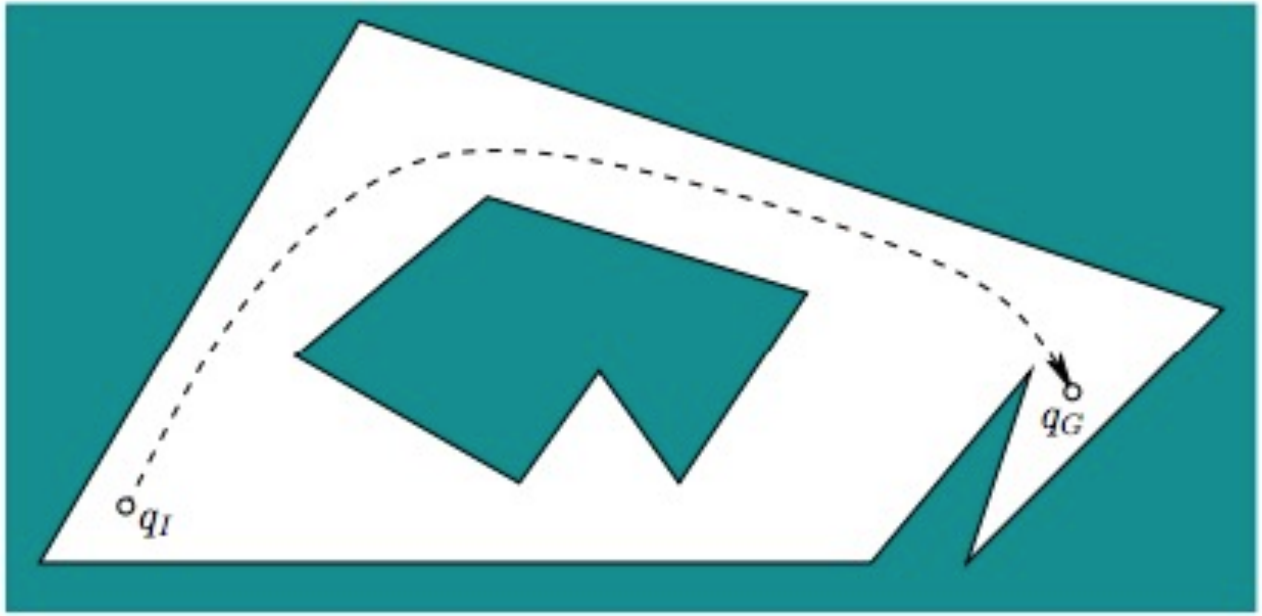
Planning in C-space

- Let $R(q) \subset W$ denote set of points in the world occupied by robot when in configuration q
- Robot in collision $\Leftrightarrow R(q) \cap O \neq \emptyset$
- Accordingly, *free* space is defined as: $C_{free} = \{q \in C | R(q) \cap O = \emptyset\}$
- Path planning problem in C-space: compute a **continuous** path: $\tau: [0,1] \rightarrow C_{free}$, with $\tau(0) = q_I$ and $\tau(1) = q_G$



Combinatorial planning

Key idea: compute a roadmap, which is a graph in which each vertex is a configuration in C_{free} and each edge is a path through C_{free} that connects a pair of vertices



Free-space roadmaps

Given a complete representation of the free space, we compute a roadmap that captures its connectivity

A roadmap should preserve:

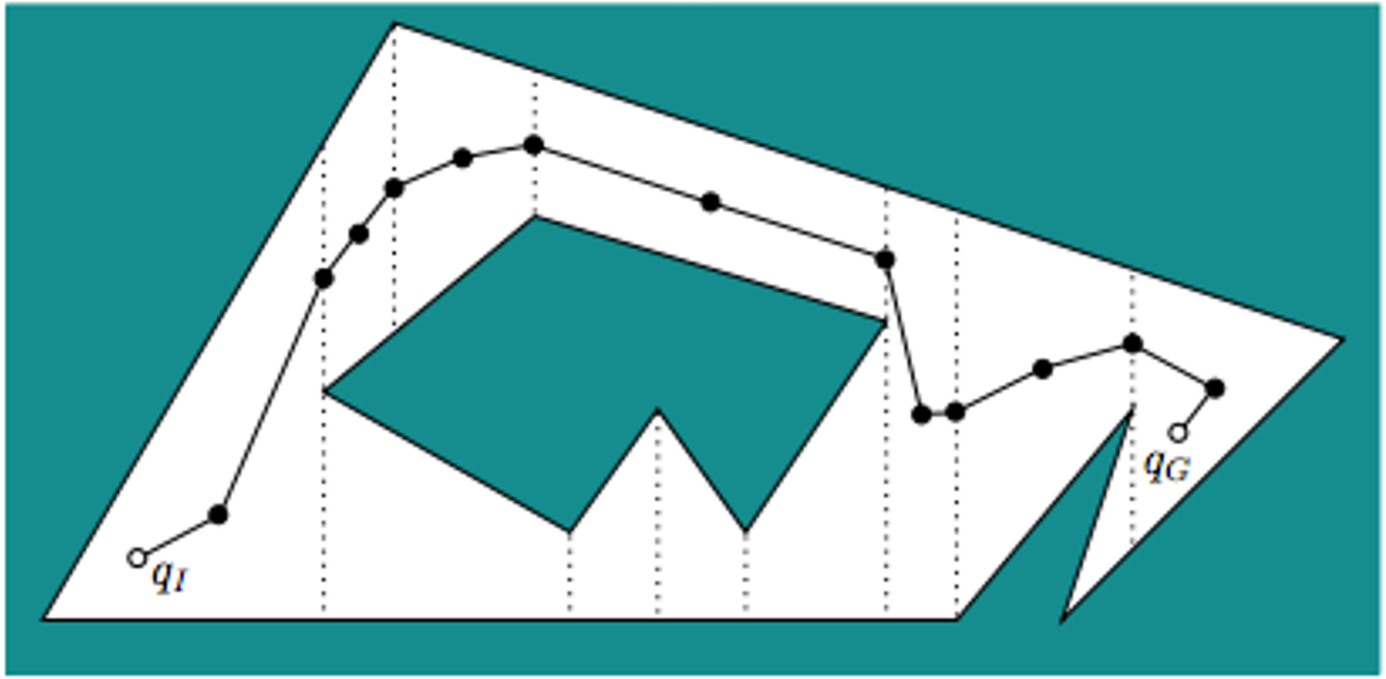
1. Accessibility: it is always possible to connect some q to the roadmap (e.g., $q_I \rightarrow s_1, q_G \rightarrow s_2$)
2. Connectivity: if there exists a path from q_I to q_G, there exists a path on the roadmap from s_1 to s_2

Main point: a roadmap provides a discrete representation of the continuous motion planning problem *without losing* any of the original connectivity information needed to solve it

Cell decomposition

Typical approach: cell decomposition. General requirements:

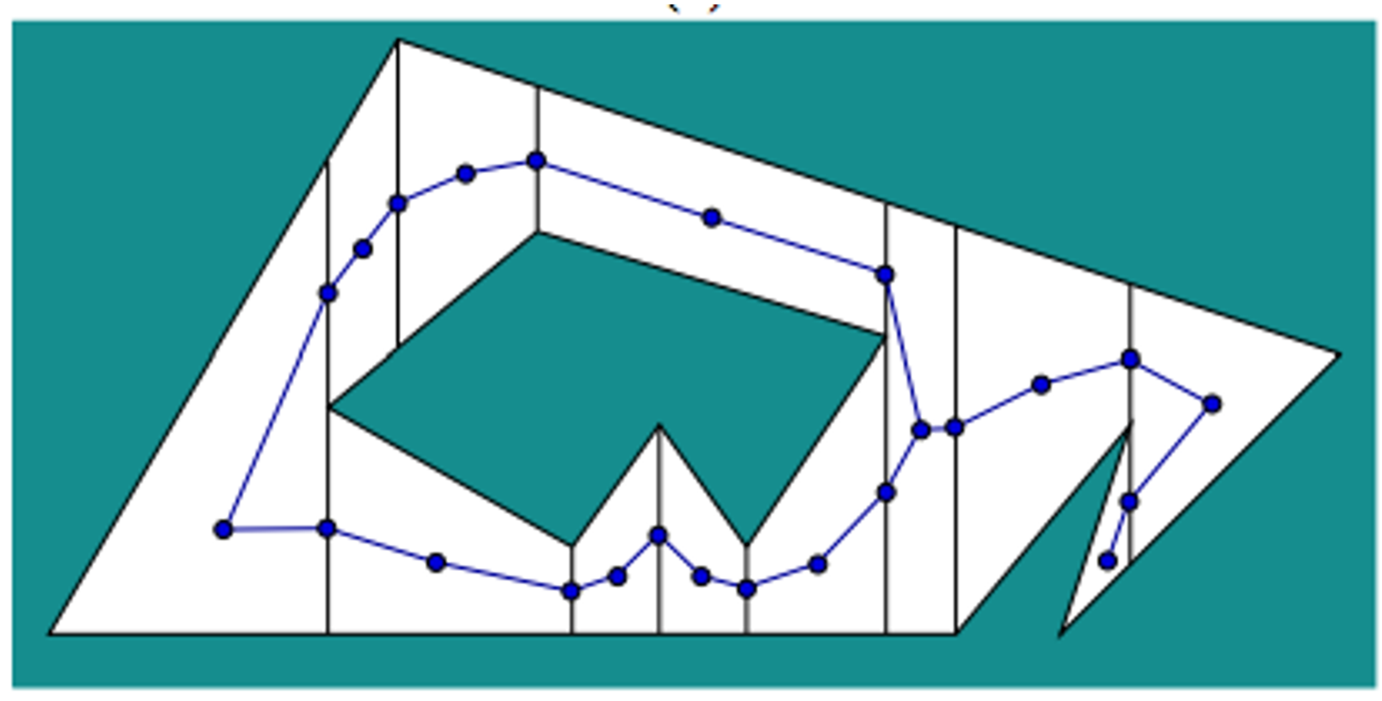
- Decomposition should be easy to compute
- Each cell should be easy to traverse (ideally convex)
- Adjacencies between cells should be straightforward to determine



Computing a trapezoidal cell decomposition

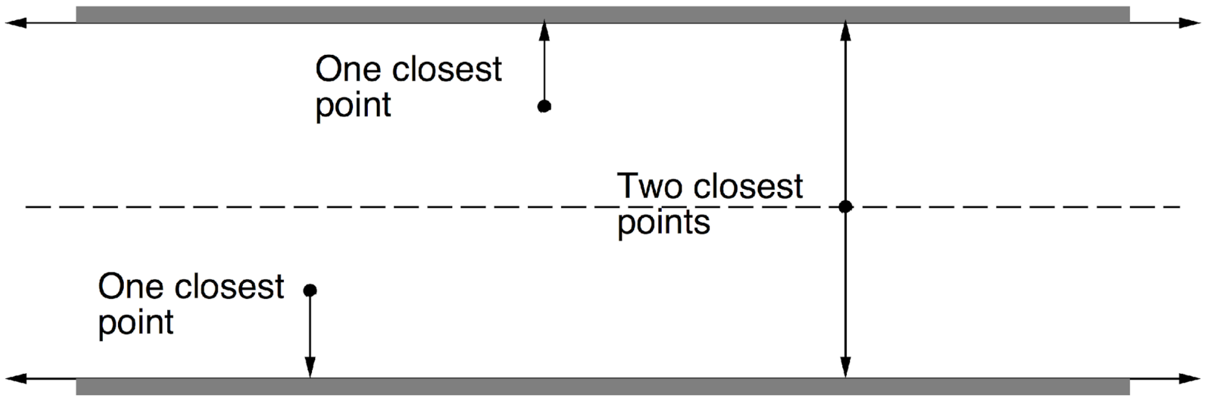
For every vertex (corner) of the forbidden space:

- Extend a vertical ray until it hits the first edge from top and bottom
  - Compute intersection points with all edges, and take the closest ones
  - More efficient approaches exists

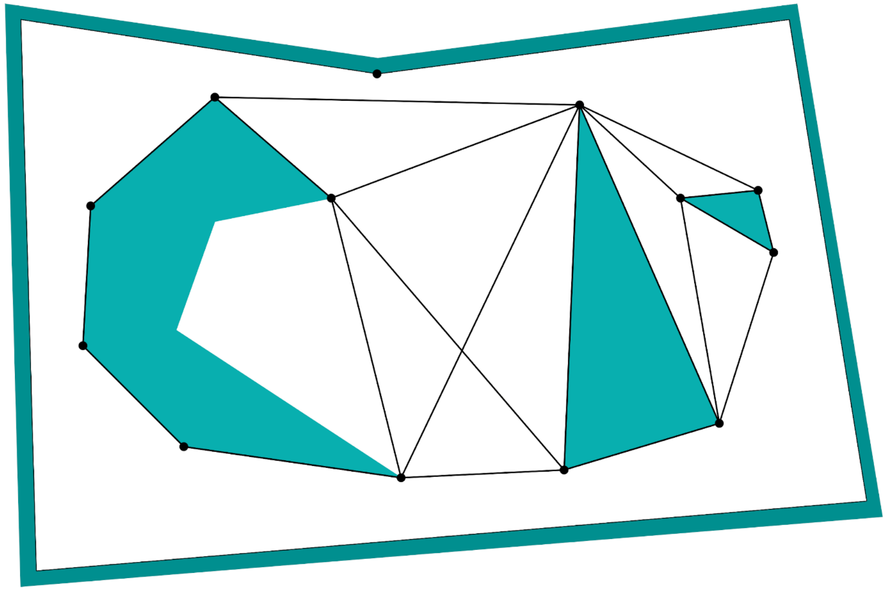


Other roadmaps

Maximum clearance (medial axis)



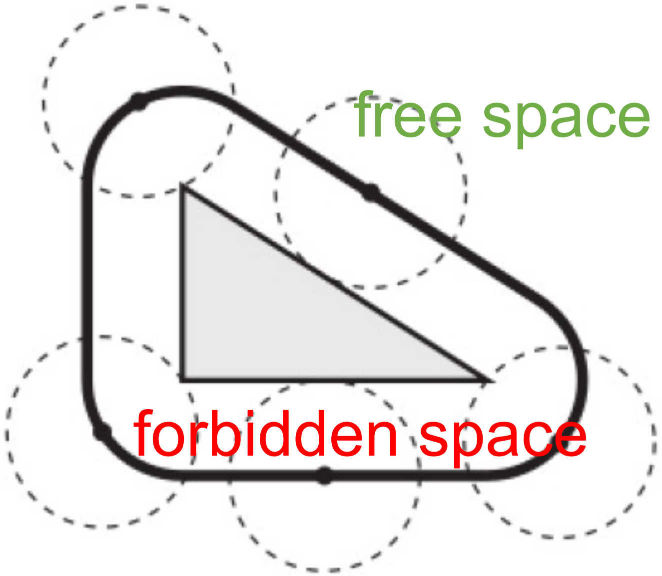
Minimum distance (visibility graph)



Note: No loss in optimality for a proper choice of discretization

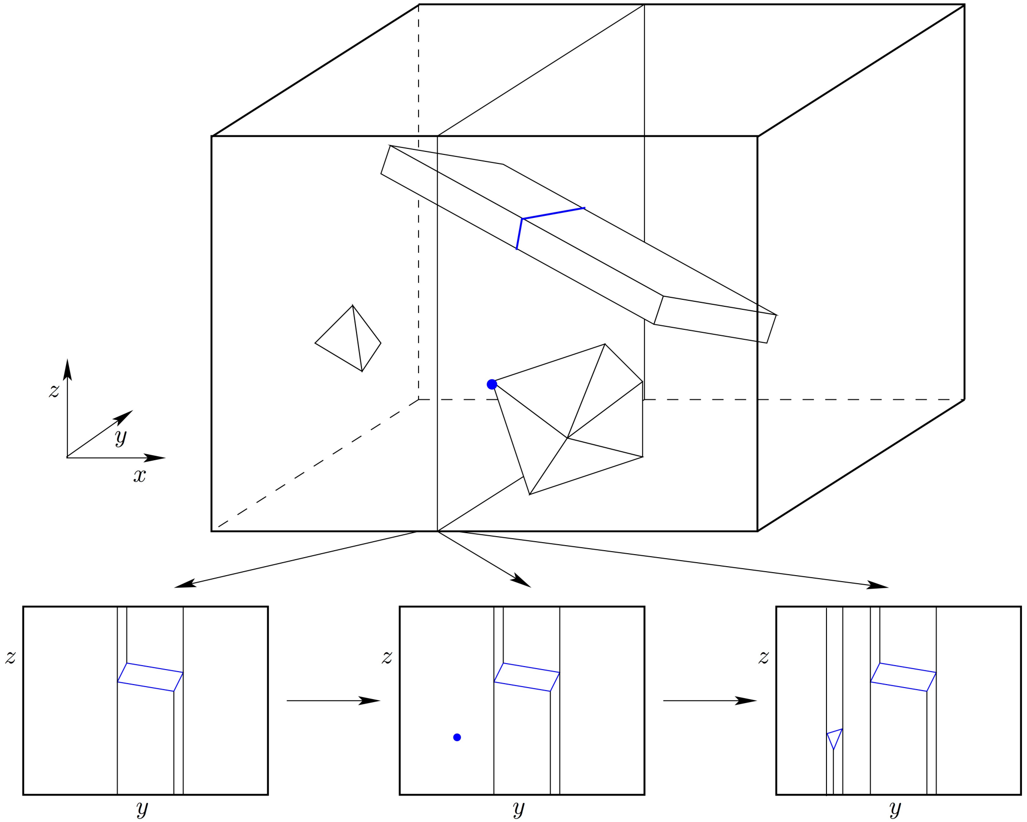
Caveat: free-space computation

- The free space is not known in advance
- We need to compute this space given the ingredients
  - Robot representation, i.e., its shape (polygon, polyhedron, …)
  - Representation of obstacles
- To achieve this we do the following:
  - Contract the robot into a point
  - In return, inflate (or stretch) obstacles by the shape of the robots



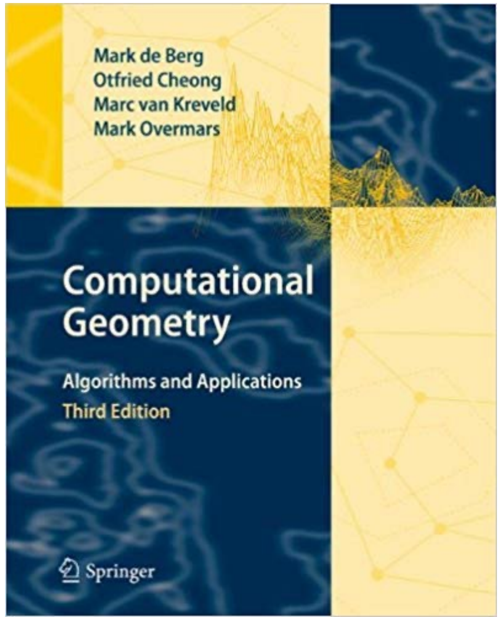
Higher dimensions

- Extensions to higher dimensions is challenging ⇒ algebraic decomposition methods



Additional resources on combinatorial planning

- Visualization of C-space for polygonal robot: https://www.youtube.com/watch?v=SBFwgR4K1Gk
- Algorithmic details for Minkowski sums and trapezoidal decomposition: de Berg et al., “Computational geometry: algorithms and applications”, 2008
- Implementation in C++: Computational Geometry Algorithms Library



Combinatorial planning: summary

- These approaches are complete and even optimal in some cases
  - Do not discretize or approximate the problem
- Have theoretical guarantees on the running time
  - I.e., computational complexity is known
- Usually limited to small number of DOFs
  - Computationally intractable for many problems
- Problem specific: each algorithm applies to a specific type of robot/problem
- Difficult to implement: require special software to reason about geometric data structures (CGAL)

Next time: sampling-based planning