AA 274
Principles of Robotic Autonomy

Stereo vision and structure from motion

Stanford University

Autonomous Systems Lab

Logistics

- It's the final (project) stretch!
  - All sections are open office hours for project discussion with TAs

Monday: 5:30 – 7:30pm (virtual) **rabrown1**
Tuesday: 4:30 – 6:30pm (in-person) **lewt**
Wednesday: 10am – 12pm (in-person) **somrita**
Wednesday: 12pm – 2pm (in-person) **schneids**
Wednesday: 5 – 7pm (in-person) **rabrown1**

Thursday: 11:45am – 1:45pm (in-person) **somrita**
Thursday: 4:30 – 6:30pm (virtual) **lewt**
Friday: 9:45am – 11:45am (in-person) **rdyro**
Friday: 12 – 2pm (in-person) **schneids**

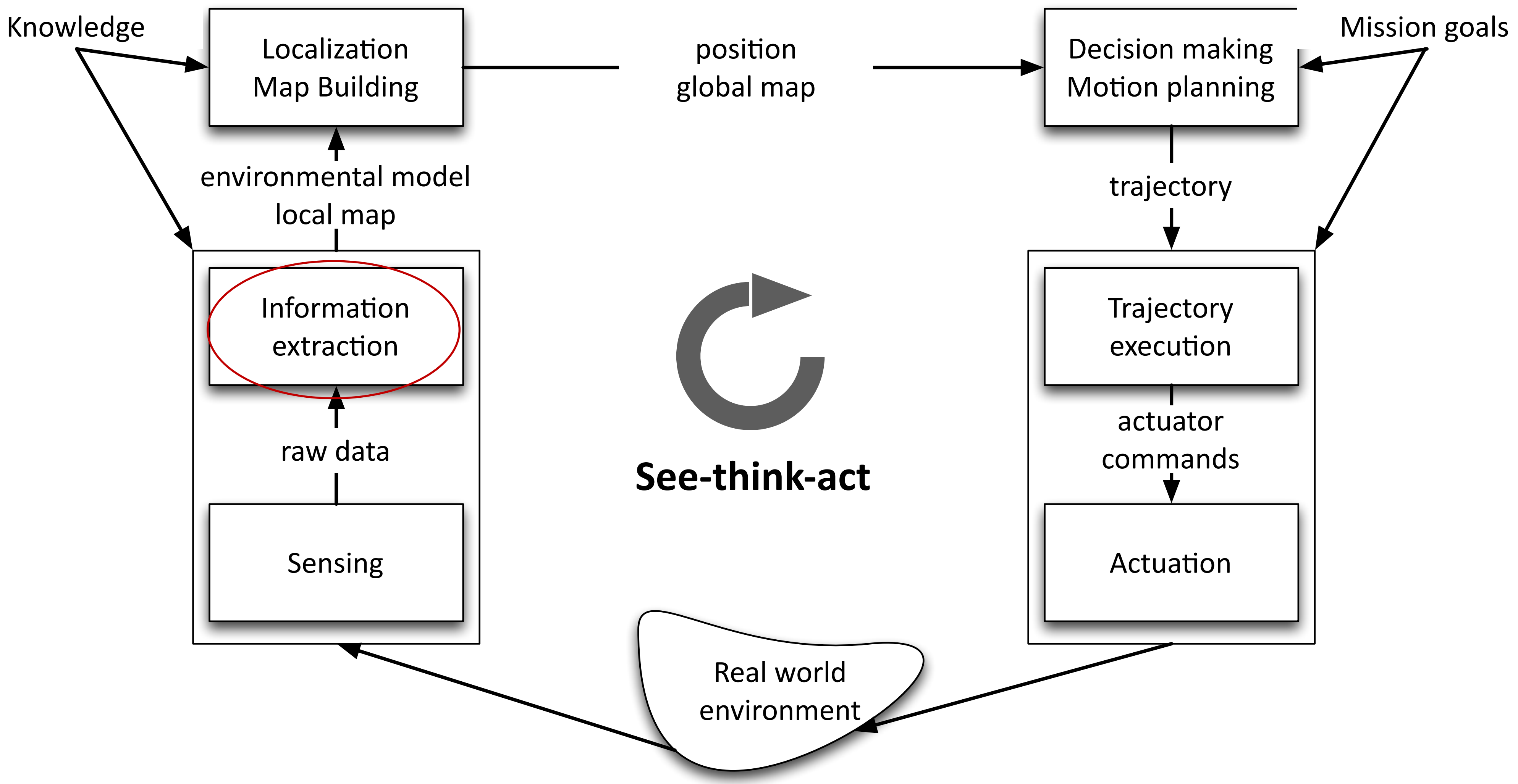
  - Final project check-in (see Section 8) due today
  - Final project demos: Wednesday, December 8th, 8:30 – 11:30am

Today's lecture

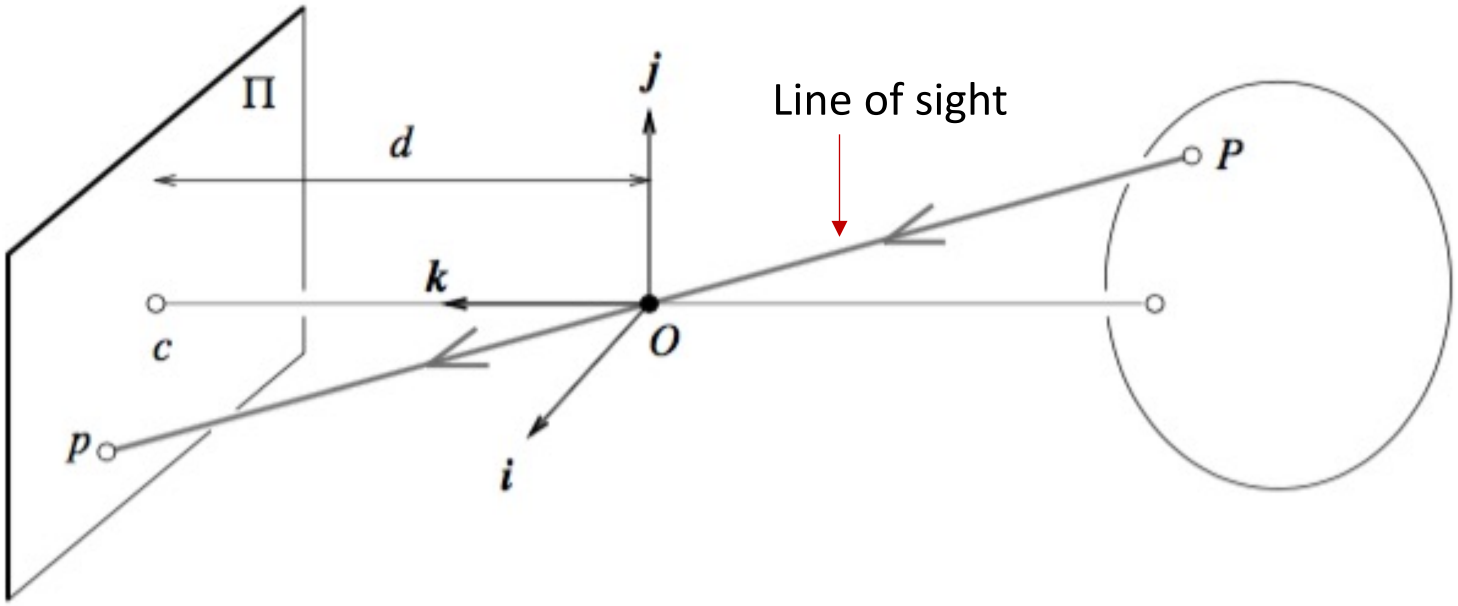
- Aim
  - Learn fundamental geometric concepts needed for 3D reconstruction
  - Learn basic techniques to recover scene structure, chiefly stereo and structure from motion

- Readings
  - SNS: 4.2.5 – 4.2.7
  - D. A. Forsyth and J. Ponce [FP]. Computer Vision: A Modern Approach (2nd Edition). Prentice Hall, 2011. Sections 7.1 and 7.2.

The see-think-act cycle



Measuring depth



$$p^h = K[R \quad t]P_W^h$$

Homogeneous coordinates

Once the camera is calibrated, can we measure the location of a point *P* in 3D given its known observation *p*?

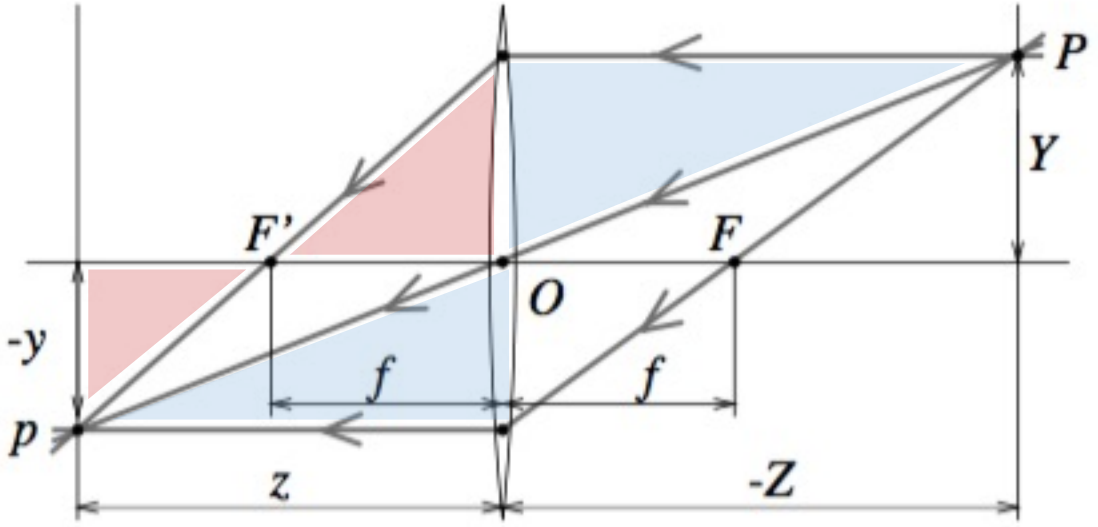
- No: one can only say that *P* is located *somewhere* along the line joining *p* and *O*!

Recovering structure

- Structure: 3D scene to be reconstructed by having access to 2D images

- Common methods
  1. Through recognition of landmarks (e.g., orthogonal walls)
  2. Depth from focus: determines distance to one point by taking multiple images with better and better focus
  3. Stereo vision: processes two distinct images taken at the *same time* and assumes that the relative pose between the two cameras is *known*
  4. Structure from motion: processes two images taken with the same or different cameras at *different times* and from different *unknown* positions

Depth from focus



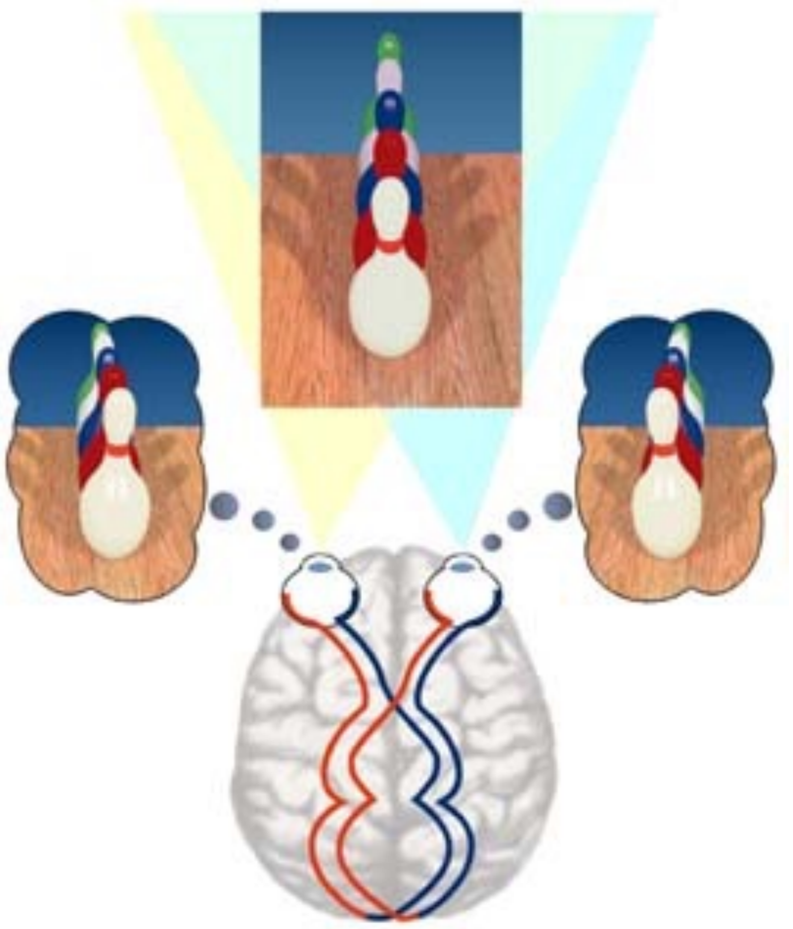
Credit: FP Chapter 1

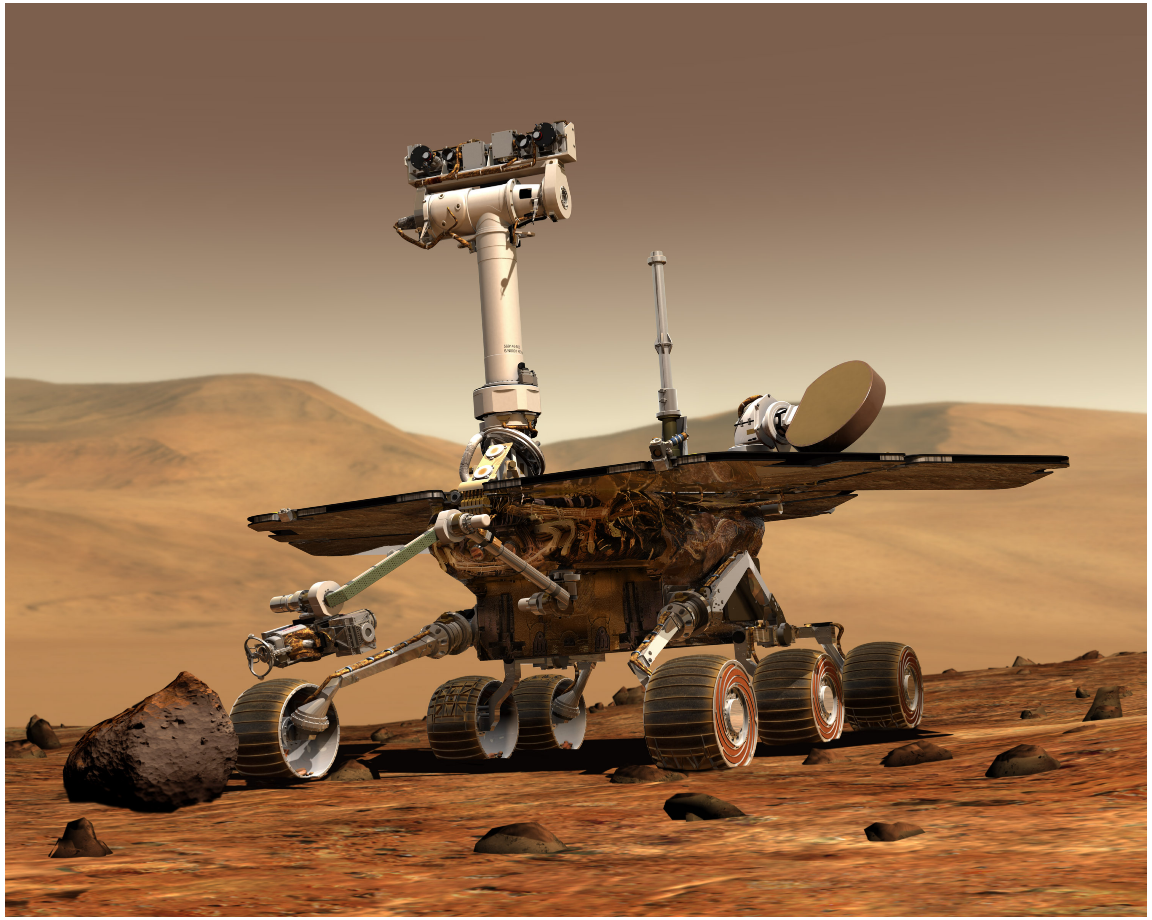
$$\Rightarrow \frac{1}{z} + \frac{1}{Z} = \frac{1}{f}$$

Thin lens equation

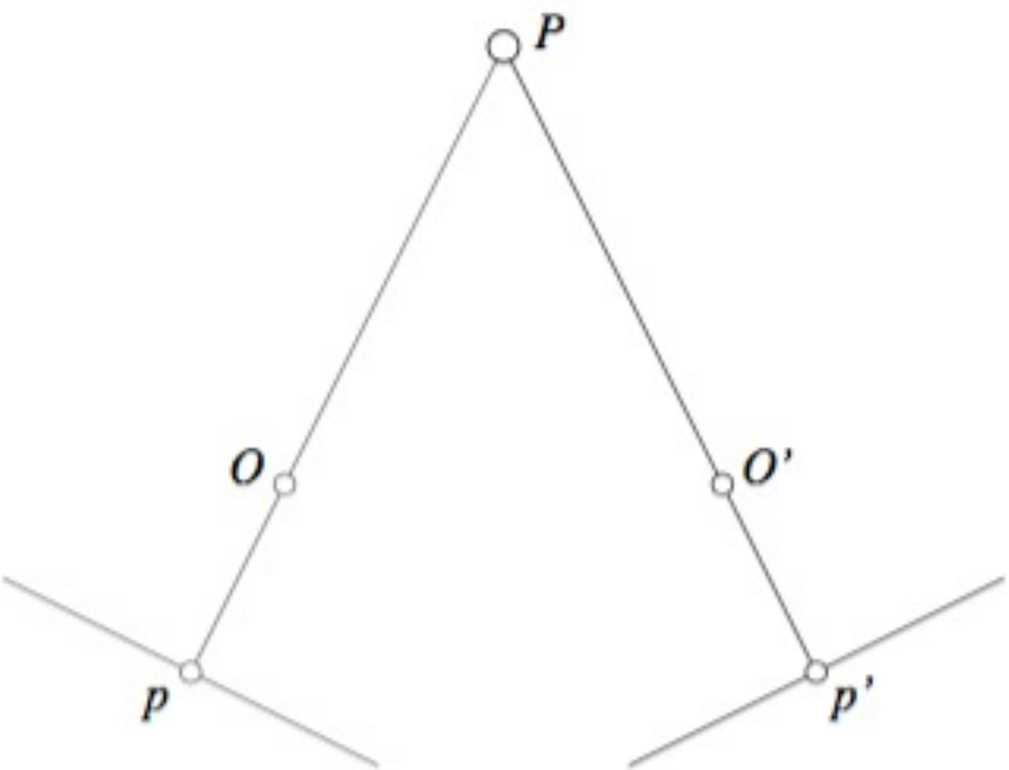
- Take several images until the projection of point *P* is in focus; let *z* denote the distance at which the image is in focus
- Since we know *z* and *f*, through the thin lens equation we obtain *Z*

Stereopsis (why we have two eyes)



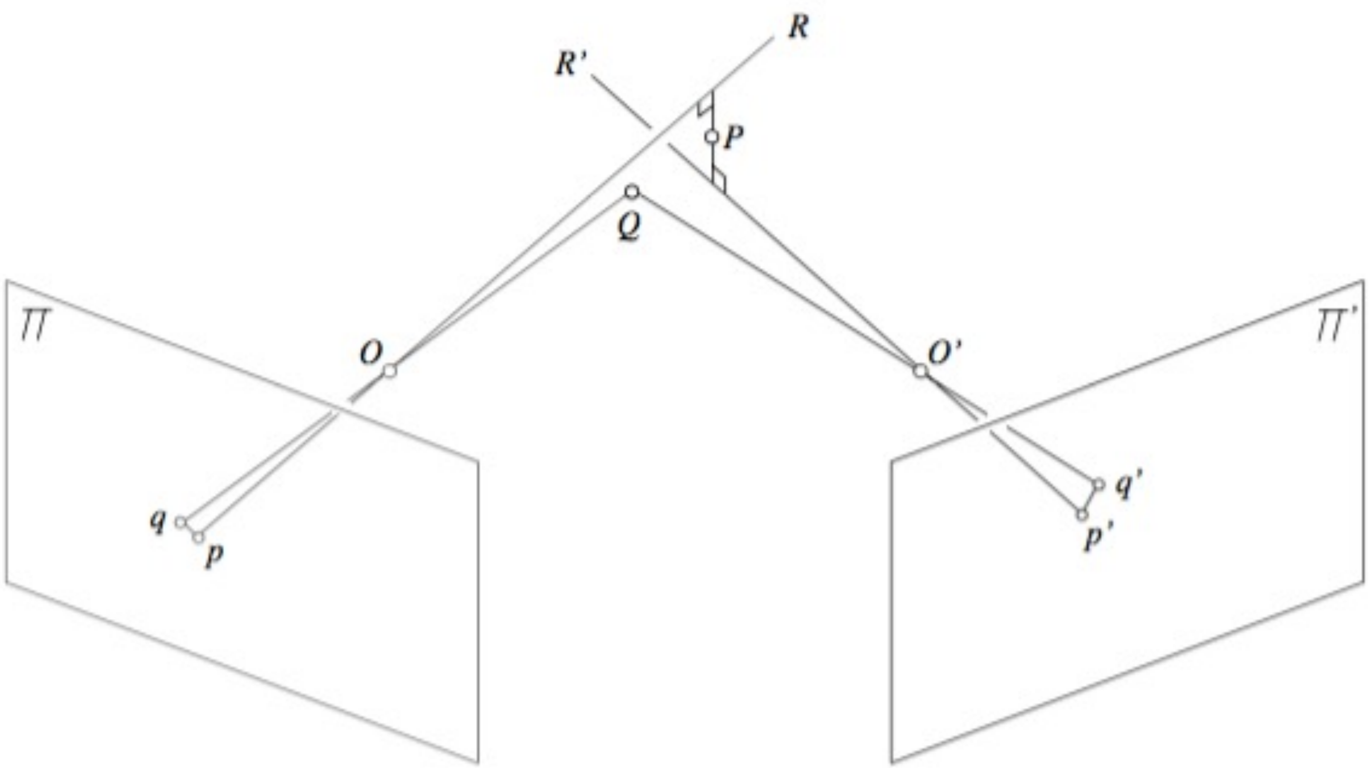


Binocular reconstruction



- Given: *calibrated* stereo rig and two image matching points p and p'
- Find corresponding scene point by intersecting the two rays $\overline{Op}$ and $\overline{O'p'}$ (process known as triangulation)

Approximate triangulation



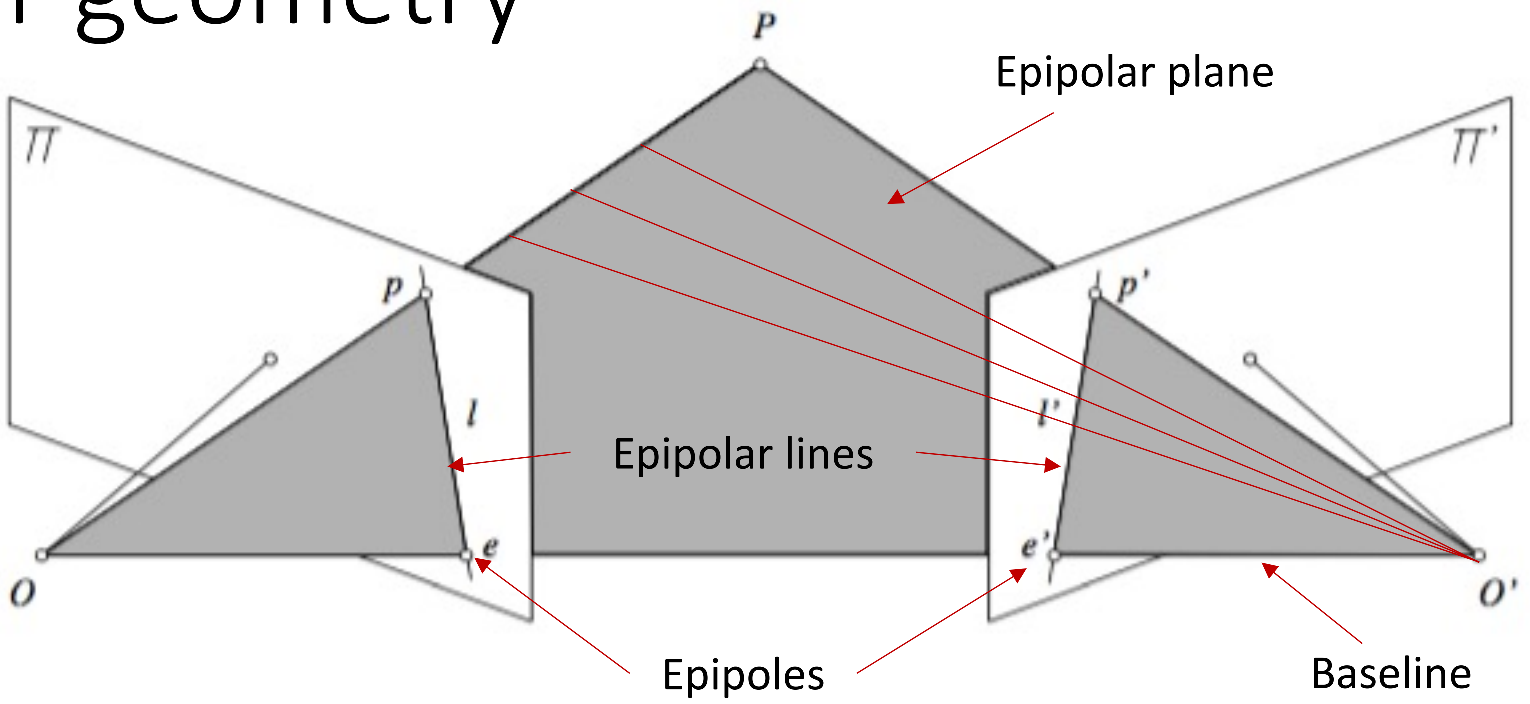
- Due to noise, triangulation problem is often solved as finding the point *Q* with images q and q' that minimizes

$$\underbrace{d^2(p,q) + d^2(p',q')}_{\text{Re-projection error}}$$

Stereo vision process

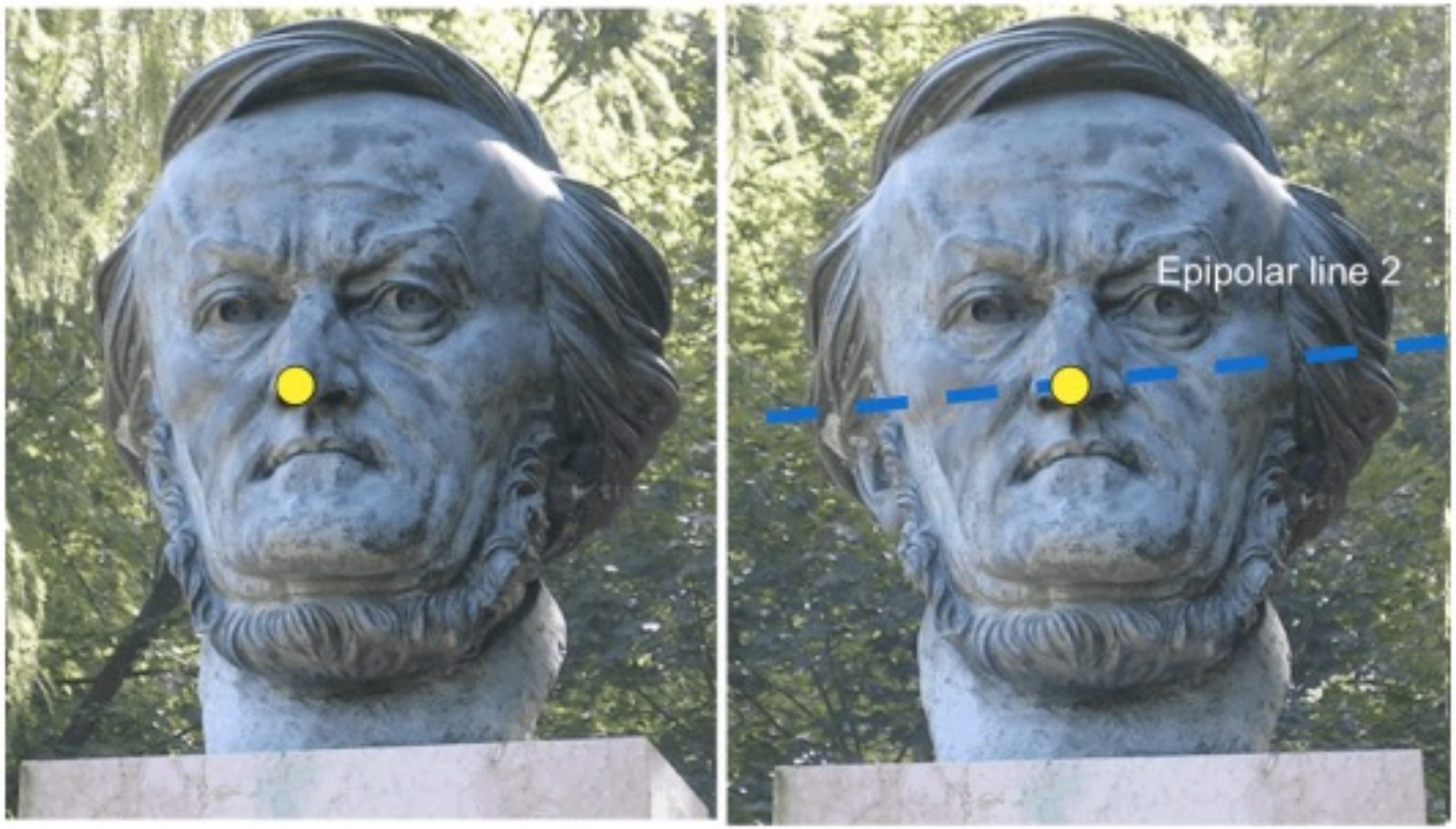
- Stereo vision consists of two steps:
  1. *fusion* of features observed by two (or more) cameras -> correspondence
  2. *reconstruction* of their three-dimensional preimages -> triangulation
- Step 2 is relatively easy; Step 1 requires you to establish correct correspondences and avoid erroneous depth measurements
- Several constraints can be leveraged to simplify Step 1 (e.g., similarity constraint, continuity constraints, etc.); most important: epipolar constraint

Epipolar geometry



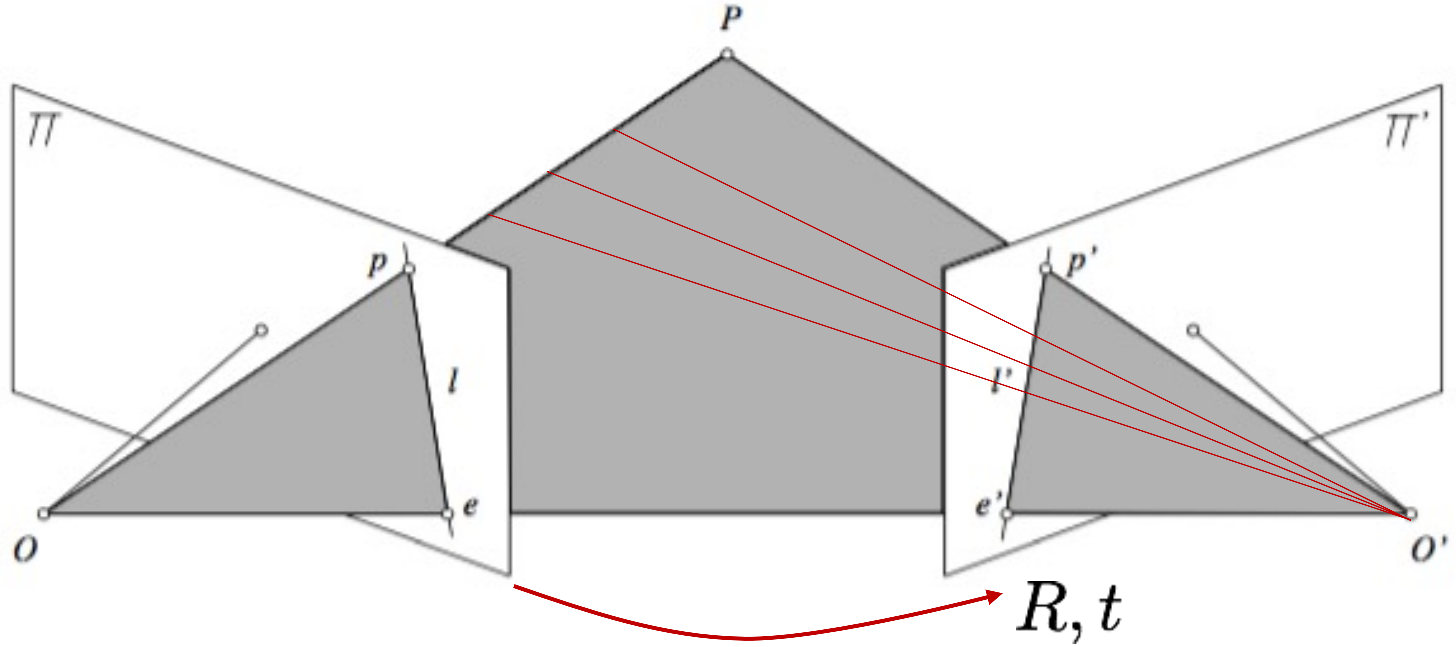
- Consider images p and p' of a point P observed by two cameras
- These five points all belong to the *epipolar plane* defined by p, O, O', or equivalently, p', O, O'
- Epipolar constraint: potential matches for p must lie on epipolar line l' (and vice-versa)

Epipolar constraint



- Search for matches can be restricted to the epipolar line instead of the whole image! → one dimensional search

Epipolar constraint: derivation



- Epipolar constraint: $\overline{Op}$, $\overline{O'p'}$, and $\overline{OO'}$ must be coplanar, or

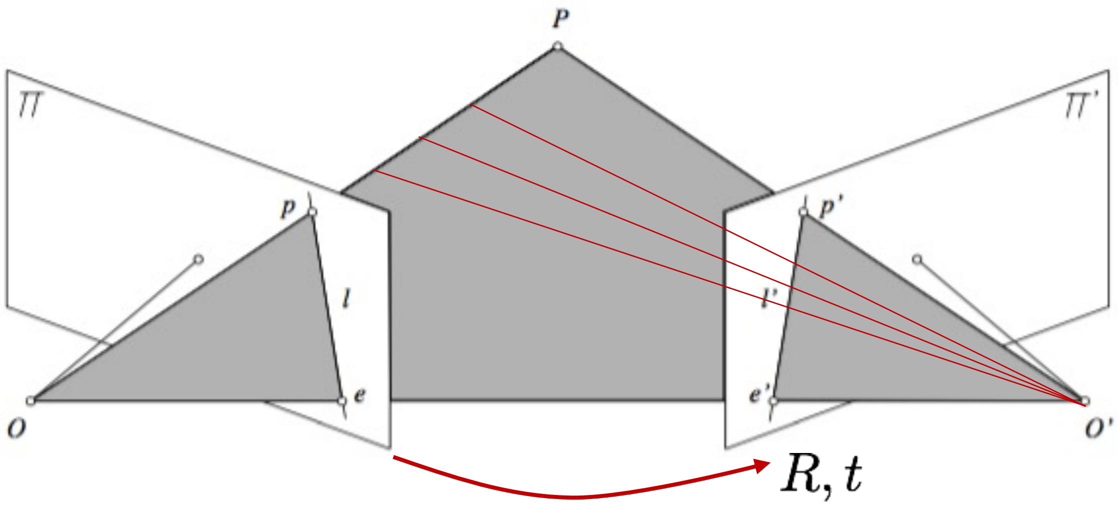
$$\overline{Op} \cdot [\overline{OO'} \times \overline{O'p'}] = 0$$

Aside: matrix notation for cross product

- Cross product can be expressed as the product of a skew-symmetric matrix and a vector

$$a \times b = \underbrace{\begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}}_{:= [a]_\times} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = [a]_\times b$$

Epipolar constraint: derivation



- Assume that the world reference system is co-located with camera 1
- After some algebra, epipolar constraint becomes [FP, Section 7.1]

$$p^T F p' = 0$$

where: $F = K^{-T}\,[t]_{\times} R\, K'^{-1}$

Key facts

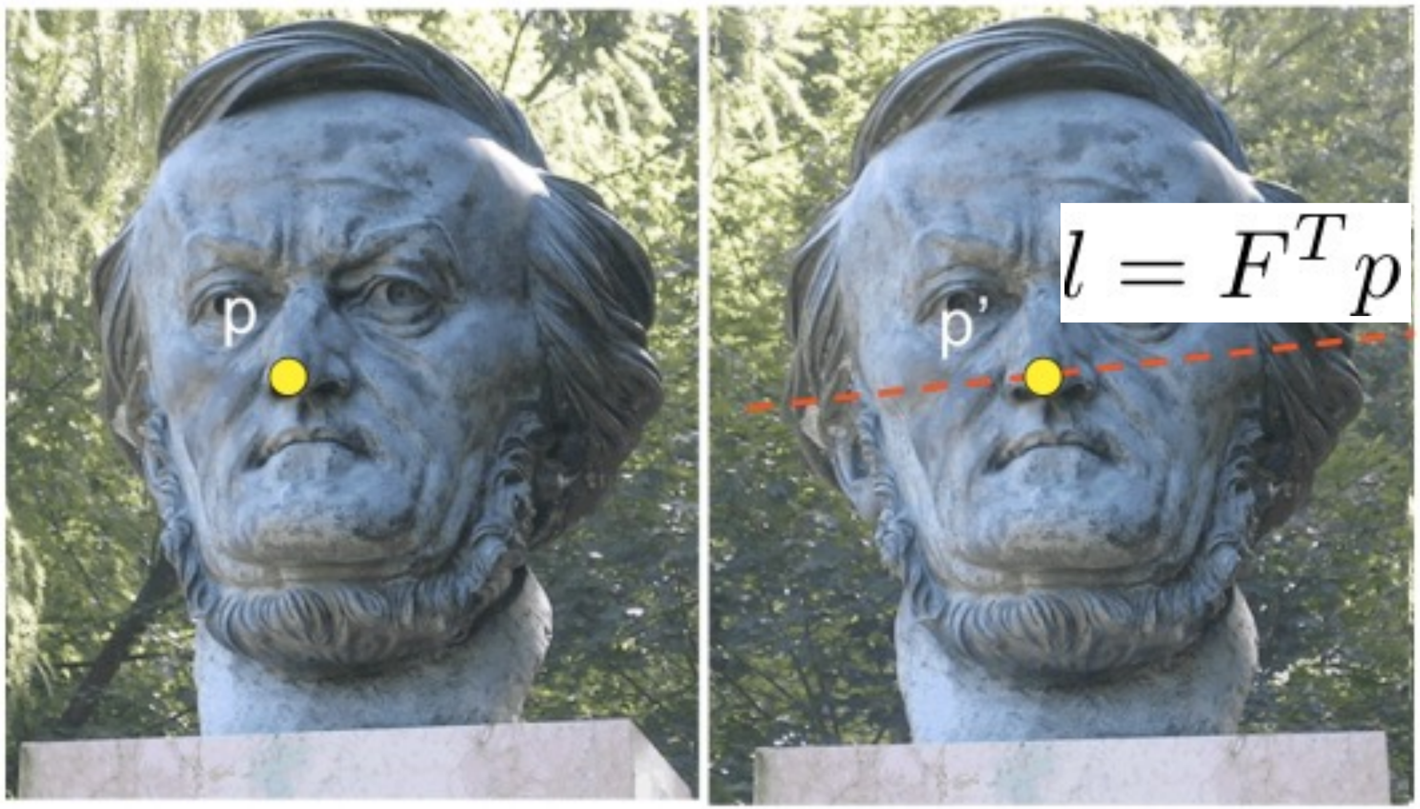
- F is referred to as the fundamental matrix

- $l = Fp'$ (resp. $l' = F^T p$) represents the epipolar line corresponding to the point p' (resp. p) in the first (resp. second) image. This exploits the homogenous notation for lines.

- $F^T e = Fe' = 0$ → F is also singular (as t is parallel to the coordinate vectors of the epipoles)

- F has 7 DoF (9 elements – common scaling – det(F)=0)

Usefulness of fundamental matrix



- Assume *F* is given
- Given a point in image 1, one can compute the corresponding epipolar line in image 2 without any additional information needed!

Estimating the fundamental matrix

- 8-point algorithm

$$p = [u, v, 1]^T, \quad p' = [u', v', 1]^T \quad \Rightarrow \quad [u, v, 1] \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [uu', uv', u, vu', vv', v, u', v', 1] \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0 \quad \Rightarrow \quad Wf = 0$$

f

nx9 matrix of known coefficients

- Given $n \geq 8$ correspondences, one then solves

$$\min_{f \in R^9} \quad \|Wf\|^2 \quad \Rightarrow \tilde{F}$$
$$\text{subject to} \quad \|f\|^2 = 1$$

Enforcing the rank constraint

- $\tilde{F}$ satisfies the epipolar constraints, but is not necessarily singular (hence, is not necessarily a proper fundamental matrix)
- Enforce rank constraint (again, via SVD decomposition)

$$\text{Find } F \text{ that minimizes } \quad \|F - \tilde{F}\|^2 \qquad \leftarrow \text{Frobenius norm}$$
$$\text{subject to } \quad \det(F) = 0$$

- 8-point algorithm
  1. Use linear least squares to compute $\tilde{F}$
  2. Enforce rank-2 constraint via SVD

Parallel image planes

- Assume image planes are parallel
- Epipolar lines are horizontal
- *v* coordinates are equal
  - Easier triangulation
  - Easier correspondence problem
- Is it possible to warp images to simulate a parallel image plane?

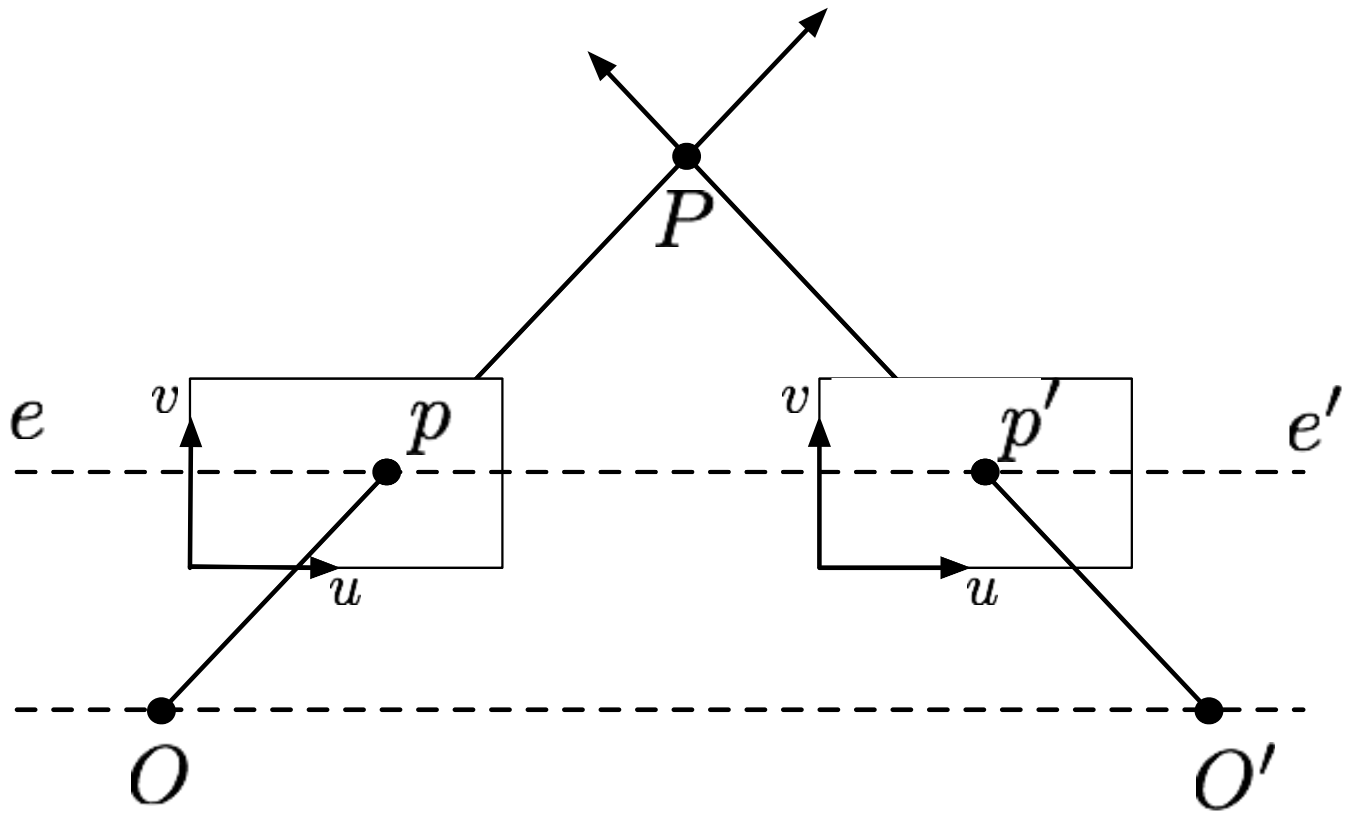
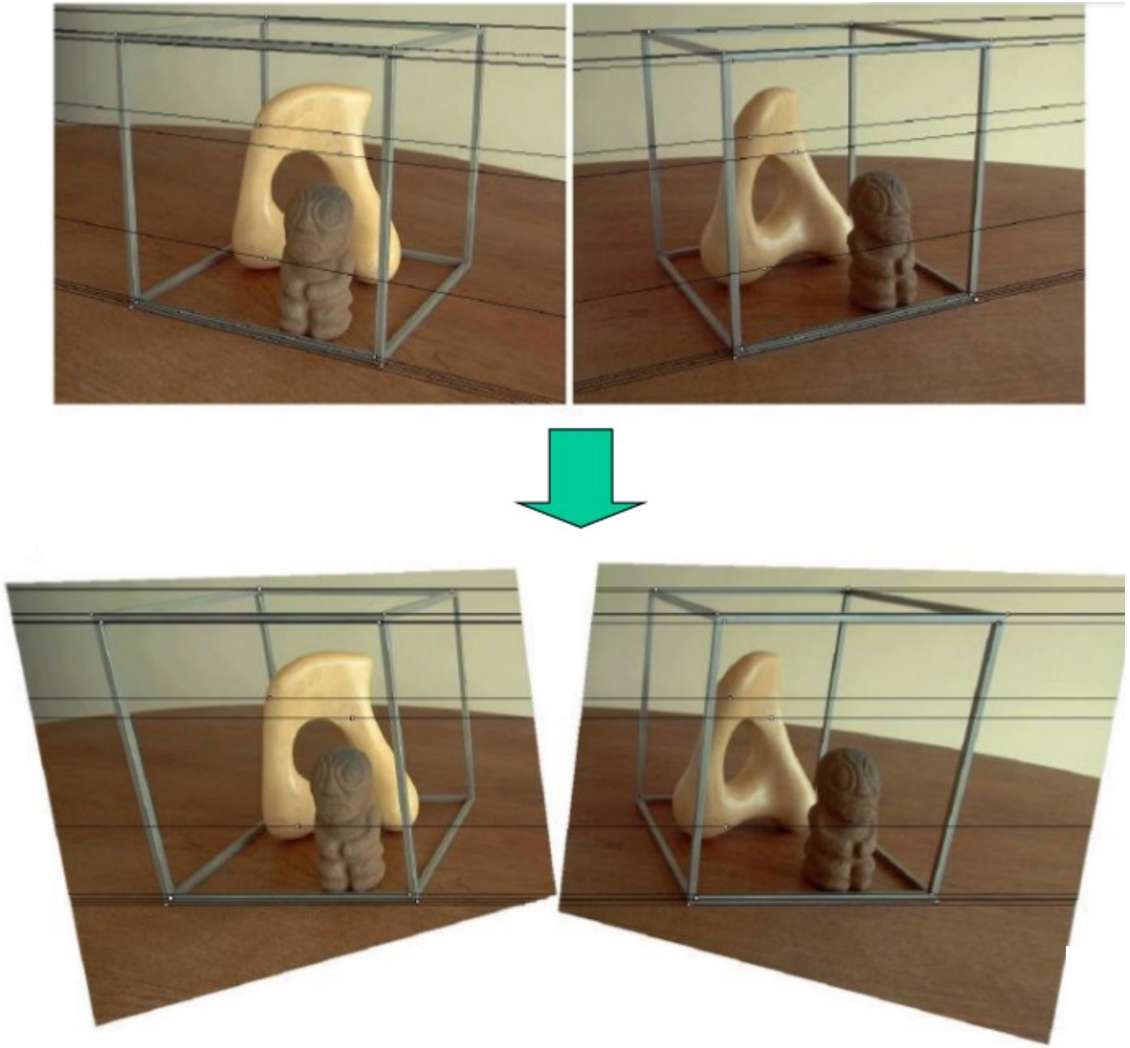


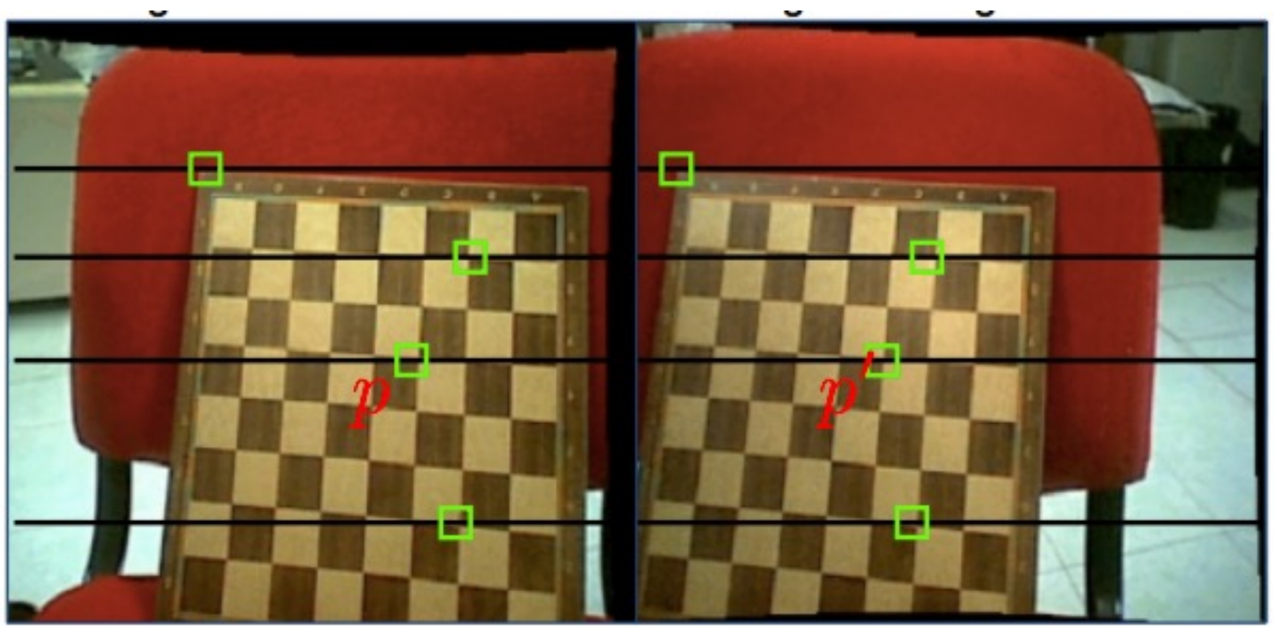
Image rectification



- Achieved by applying an appropriate projective transformation
- Several algorithms exist
- From now on, we assume rectified image pairs

Back to stereo vision process

- Recall that stereo vision consists of two steps:
  1. *fusion* of features observed by two (or more) cameras (correspondence)
  2. *reconstruction* of their three-dimensional preimages (triangulation)
- Correspondence problem



Goal: find corresponding observations p and p'

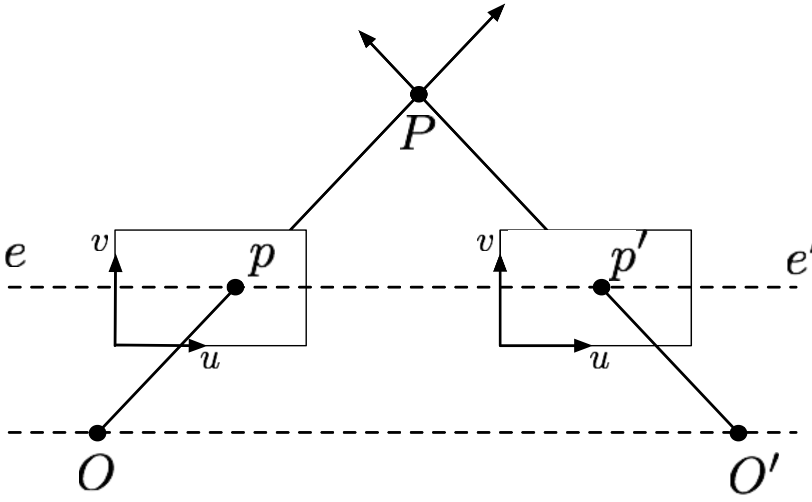
Exploits epipolar constraints

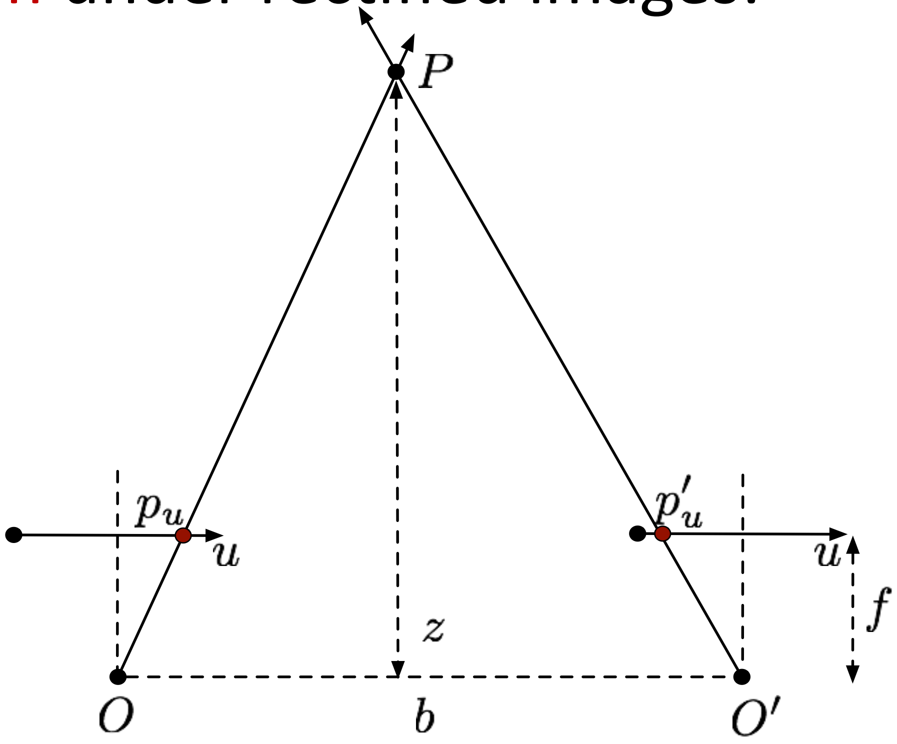
Two classes of algos: *area-based* and *feature-based*

Hard problem: occlusions, repetitive patterns, etc.; more on this later

Triangulation under rectified images

- We already saw how to triangulate correspondences in the general case
- Triangulation problem under rectified images:





From similar triangles:

$$z = \frac{b\,f}{p_u - p'_u}$$

disparity

Large baseline: Object might be visible from one camera, but not the other

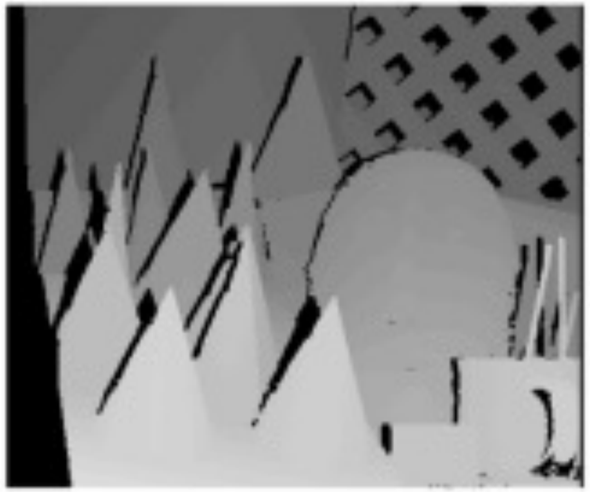
Small baseline: large depth error

Disparity map

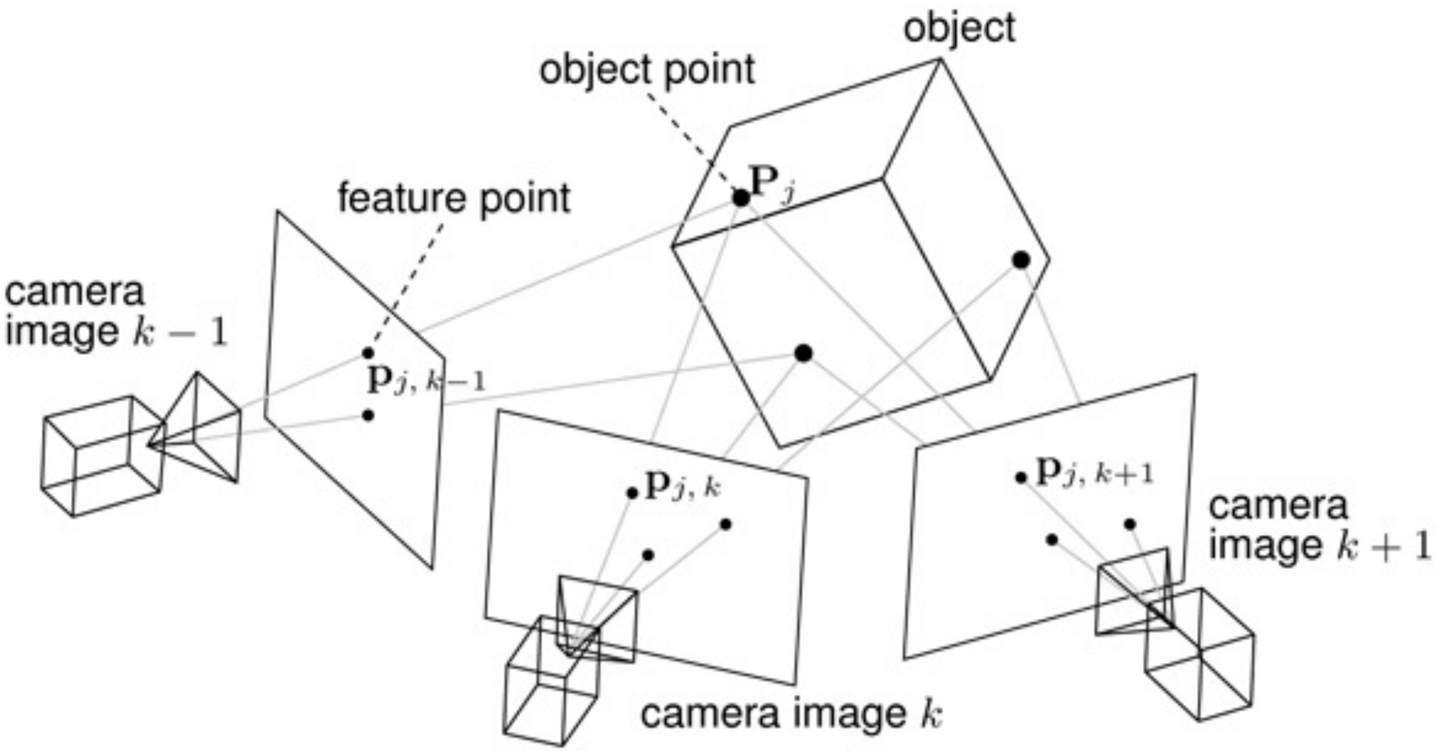
- Disparity: pixel displacement between corresponding points
- Disparity map: holds the disparity values for every pixel
- Nearby objects experience largest disparity

Stereo pair

Disparity map



Method #3: structure from motion (SFM)



Given *m* images of *n* fixed 3D points

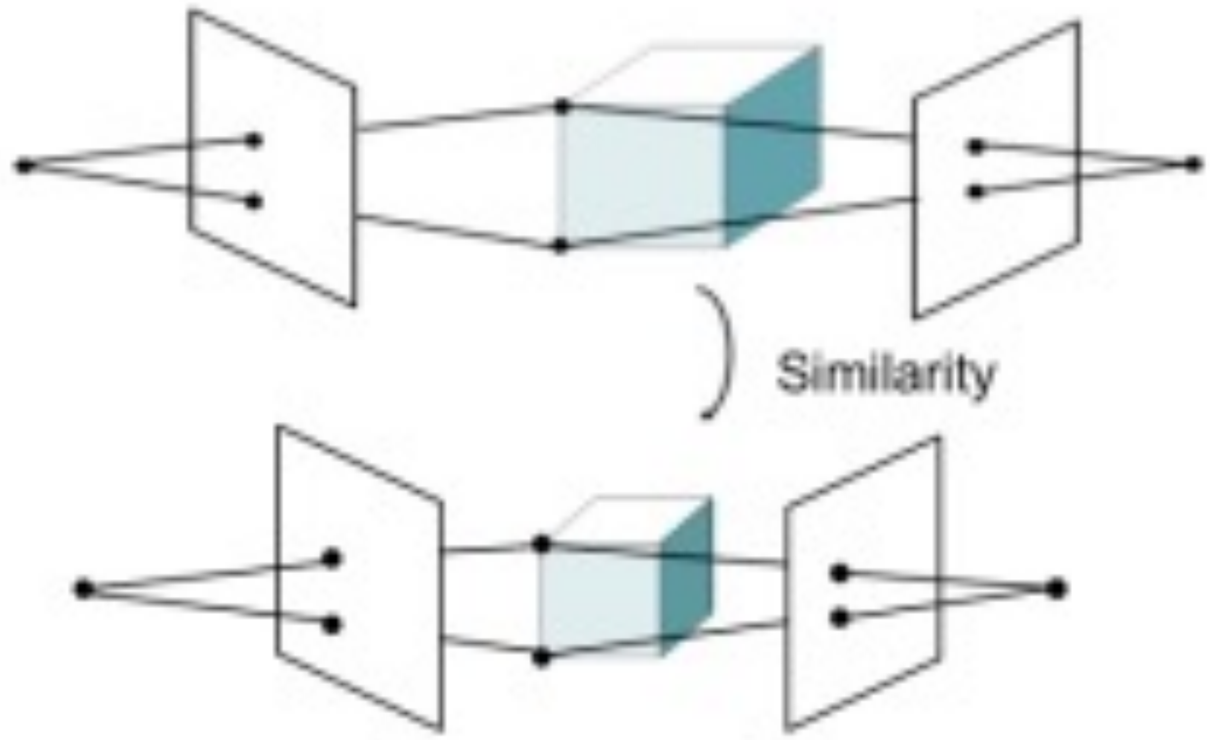
$$p_{j,k}^h = M_k P_j^h$$

Find:

- *m* projection matrices M_k (motion)
- *n* 3D points P_j (structure)

SFM ambiguity

- It is not possible to recover the absolute scale of the observed scene



Solution to SFM problem (high-level)

- Several approaches available:
  - Algebraic approach (by fundamental matrix)
  - Bundle adjustment

- Algebraic approach (2-views)
  1. Compute fundamental matrix F (e.g., via 8-point algorithm)
  2. Use F to estimate projection camera matrices
  3. Use projection camera matrices for triangulation

Application of SFM: visual odometry

- Visual odometry: estimate the motion of the robot by using visual input (and possibly additional information)
  - Single camera: absolute scale must be estimated in other ways
  - Stereo camera: measurements are directly provided in absolute scale