Principles of Robot Autonomy I

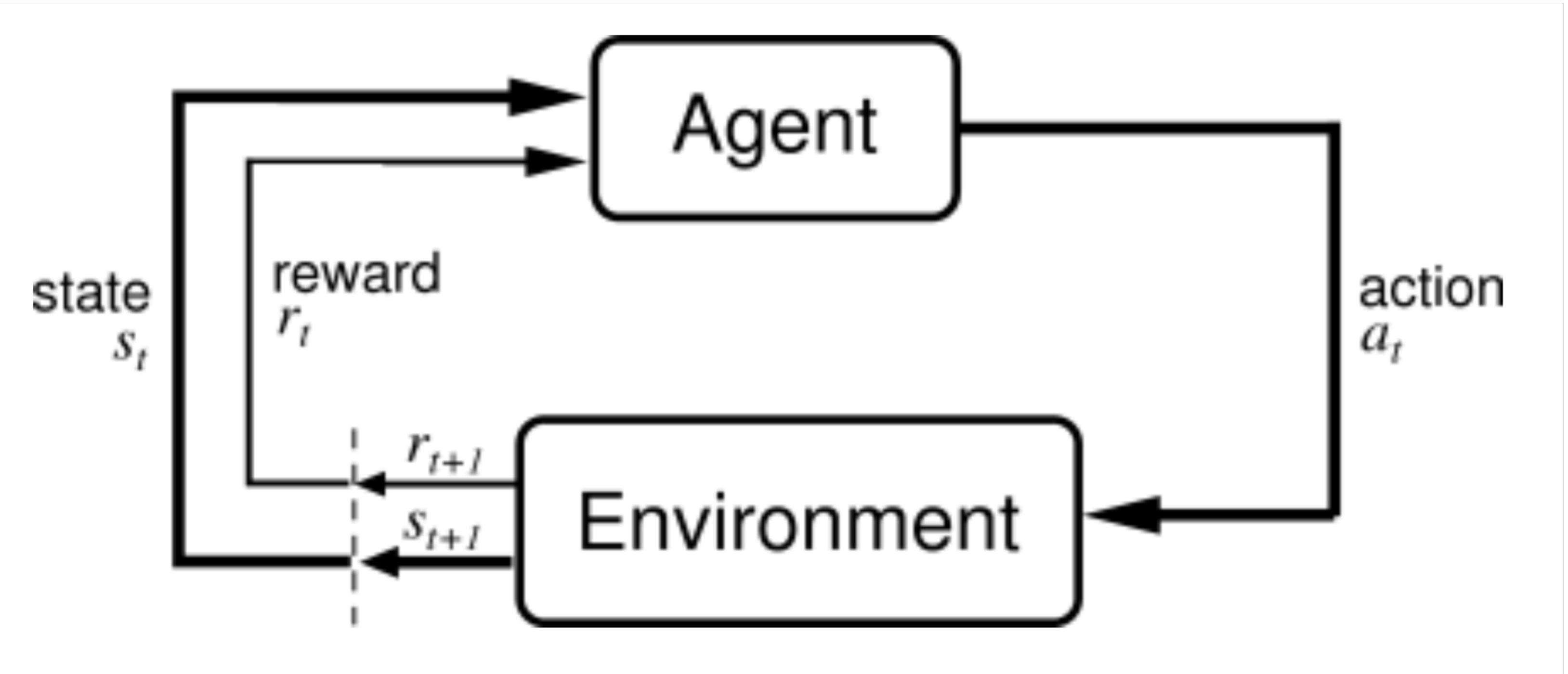
Reinforcement Learning

Stanford University

Autonomous Systems Lab

What is Reinforcement Learning?

Learning how to make good decisions by interaction.



Why Reinforcement Learning

- Only need to specify a **reward function**. Agent learns everything else!
- Successes in
  - Helicopter acrobatics
  - Superhuman Gameplay: Backgammon, Go, Atari
  - Investment portfolio management
  - Making a humanoid robot walk

Why Reinforcement Learning?

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- Successes in
  - Helicopter acrobatics
    - positive for following desired traj, negative for crashing
  - Superhuman Gameplay: Backgammon, Go, Atari
    - positive/negative for winning/losing the game
  - Investment portfolio management
    - positive reward for $$$
  - Making a humanoid robot walk
    - positive for forward motion, negative for falling

Outline

- Formalisms
- Algorithms
- Deep Reinforcement Learning
- RL in Robotics

Markov Decision Process (MDP)

State: $x \in \mathcal{X}$ (often $s \in \mathcal{S}$)

Action: $u \in \mathcal{U}$ (often $a \in \mathcal{A}$)

Transition Function: $T(x_t \mid x_{t-1}, u_{t-1}) = p(x_t \mid x_{t-1}, u_{t-1})$

Reward Function: $r_t = R(x_t, u_t)$

Discount Factor: γ

Horizon: H

MDP: $\mathcal{M} = (\mathcal{X}, \mathcal{U}, T, R, \gamma, H)$

Markov Decision Process (MDP)

MDP: $\mathcal{M} = (\mathcal{X}, \mathcal{U}, T, R, \gamma, H)$

Policy: $u_t = \pi(x_t)$

Goal: Choose policy that **maximizes cumulative reward.**

$$\pi^* = \arg\max_{\pi} \mathrm{E}\left[\sum_{t=0}^{H} \gamma^t R\big(x_t, \pi(x_t)\big)\right]$$

Solving MDPs

If you know the model, use dynamic programming

- Value Iteration / Policy Iteration

RL: Learning from interaction

- Model-Based
- Model-free
  - Value based
  - Policy based

Dynamic Programming in MDPs

Define a policy's **value function** as the expected cumulative discounted reward when acting according to the policy from a given state.

$$V_k^\pi(x) = \mathrm{E}\left[\sum_{t=0}^{k} \gamma^t R\big(x_t, \pi(x_t)\big) | x_0 = x\right]$$

Value with k steps to go

$$V_k^\pi(x) = R\big(x, \pi(x)\big) + \gamma \sum_{x' \in \mathcal{X}} T(x' | x, \pi(x))\, V_{k-1}^\pi(x')$$

Optimality in MDPs

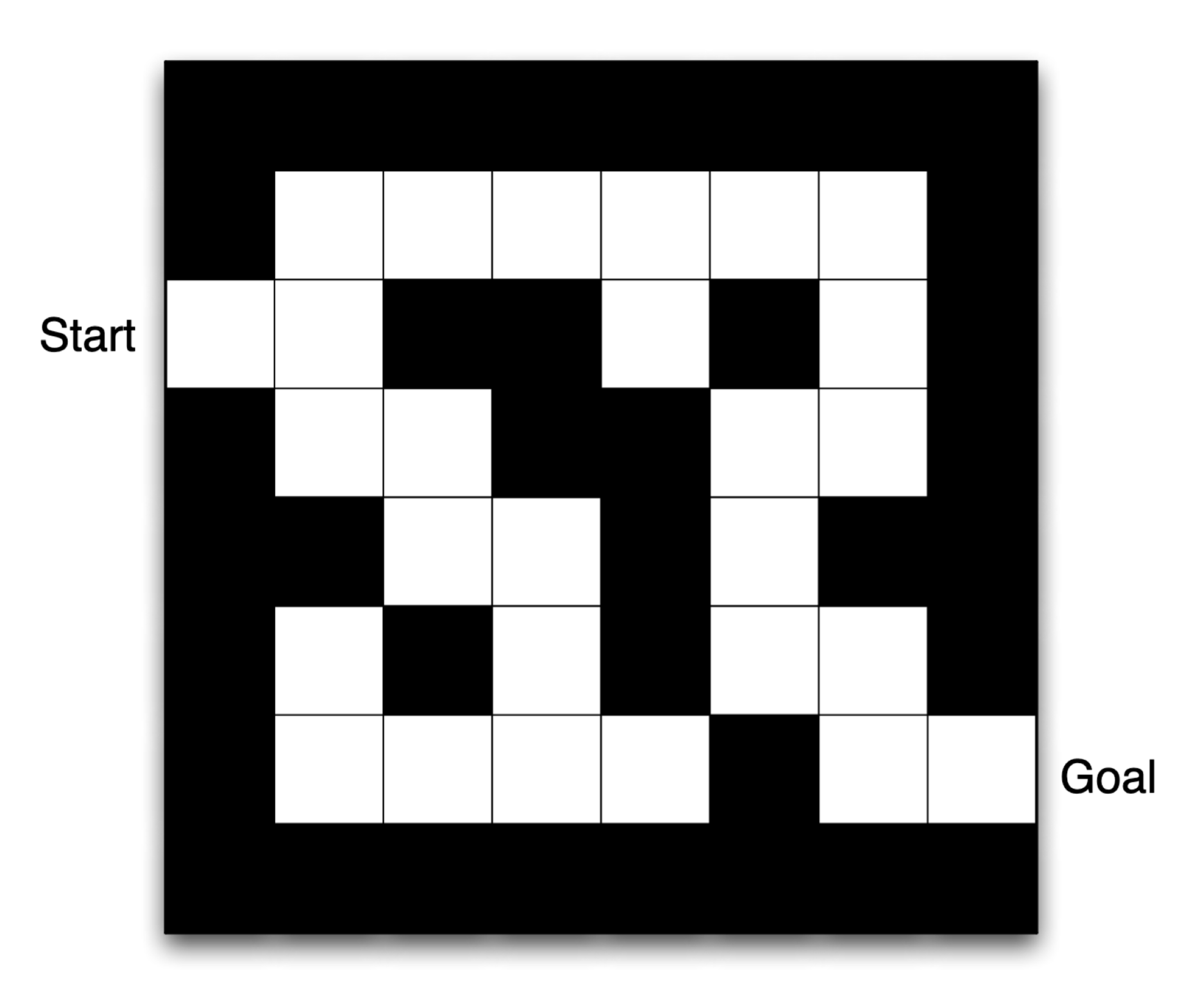
The optimal policy π^* is a policy that has the highest value.

$$\pi_t^*(x) = \arg\max_{\pi} V_t^{\pi}(x)$$

There exists a unique **optimal value function**:

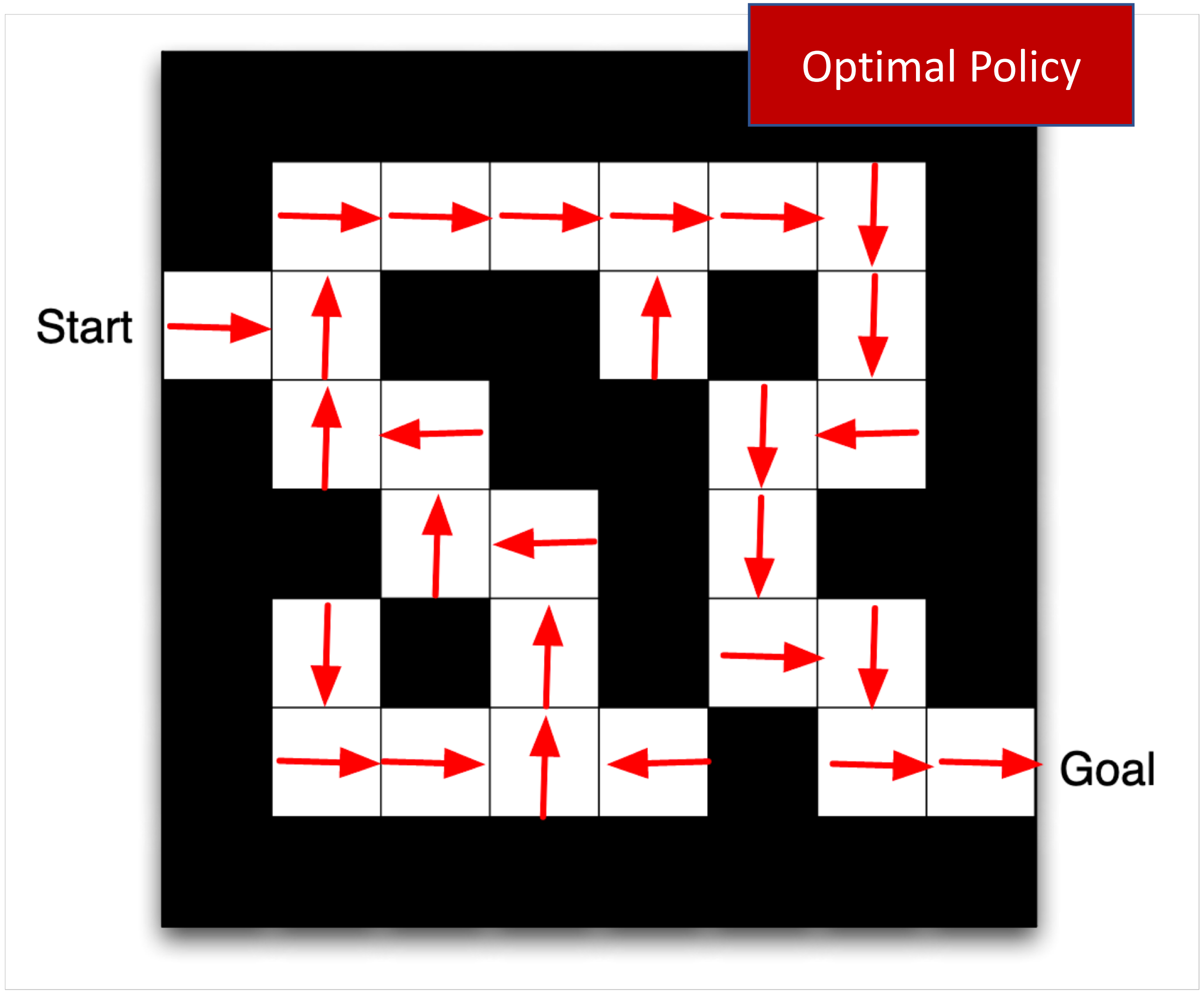
$$V_t^*(x) = V_t^{\pi^*}(x)$$

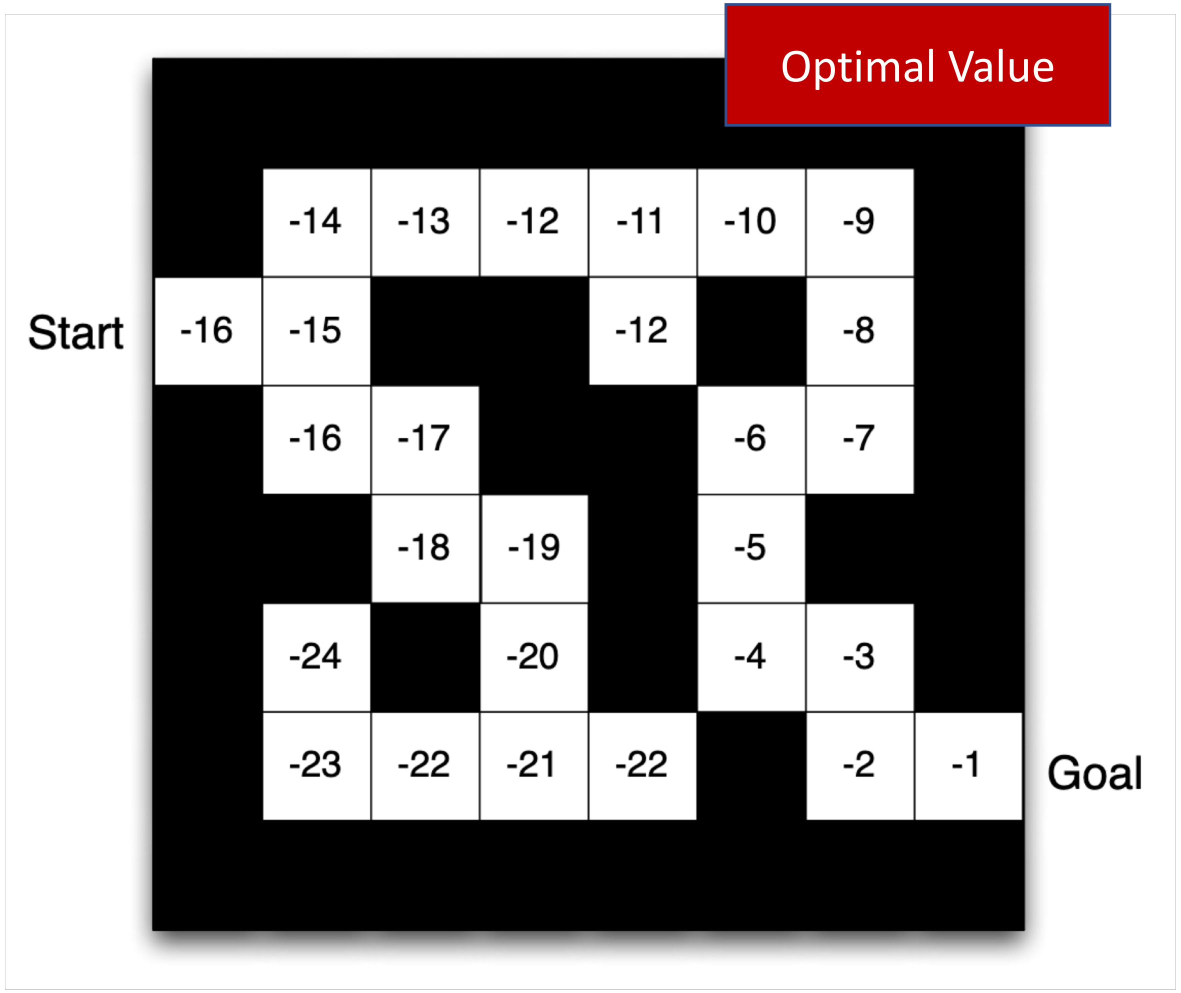
Gridworld Example



- Reward: -1 at each timestep
- Actions: N/S/E/W
- State: 2D location

Gridworld Example





Value Iteration

- Dynamic programming for MDPs
- Initialize $V_0^*(x) = 0$ for all states x
- Loop until finite horizon / convergence:

$$V_{k+1}^* = \max_u \left(R(x,u) + \gamma \sum_{x' \in \mathcal{X}} T(x'|x,u)\, V_k^*(x') \right)$$

Q-functions

Another related function in MDPs is the Q function, which is a function of state and action, and corresponds to the value of taking a given action and then acting according to the given policy:

$$Q_k^{\pi}(x,u) = R(x,u) + \gamma \sum_{x' \in \mathcal{X}} T(x'|x,u)\, V_{k-1}^{\pi}(x')$$

Similarly, we can define the optimal Q function:

$$Q_k^{*}(x,u) = R(x,u) + \gamma \sum_{x' \in \mathcal{X}} T(x'|x,u)\, V_{k-1}^{*}(x')$$

Q functions

$$V_{k+1}^* = \max_u \left(R(x,u) + \gamma \sum_{x' \in \mathcal{X}} T(x'|x,u)\, V_k^*(x') \right)$$

$$V_{k+1}^*(x) = \max_u Q_{k+1}^*(x,u)$$

Policy Iteration

Suppose we have a policy $\pi_k(x)$

We can use DP to compute $Q^{\pi_k}(x, u)$

Define $\pi_{k+1}(x) = \arg\max_u \; Q^{\pi_k}(x, u)$

Proposition: $V^{\pi_{k+1}}(x) \geq \; V^{\pi_k}(x) \; \forall \; x \in \mathcal{X}$

Inequality is strict if π is suboptimal.

Use this procedure to iteratively improve policy until convergence.

Recap

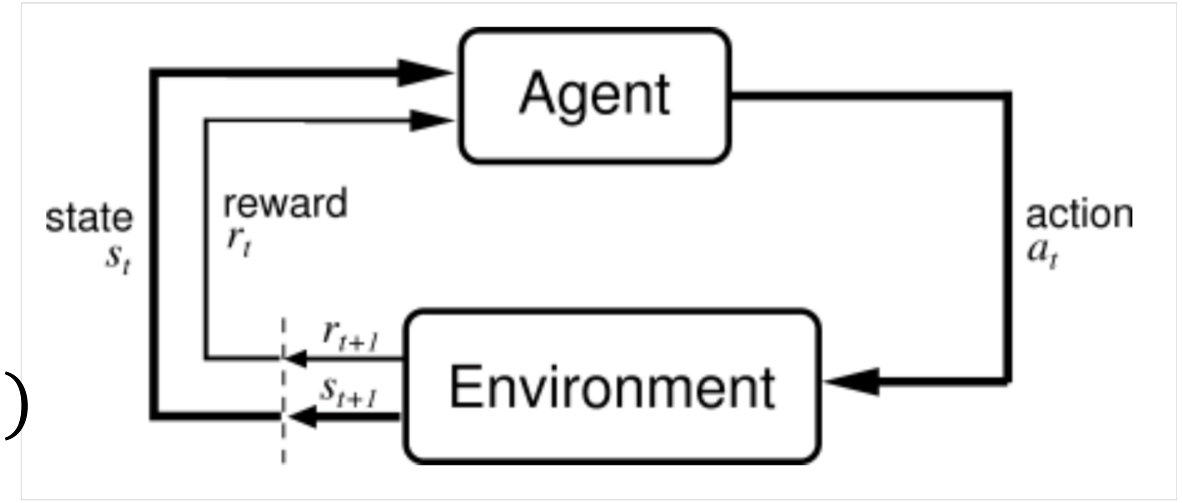
- Value Iteration
  - Estimate Optimal Value Function
  - Compute optimal policy from optimal value function

- Policy Iteration
  - Start with random policy
  - Iteratively improve it until convergence to optimal policy

- Require **model of MDP** to work!

Learning from Experience

- Without access to the model, agent needs to optimize a policy from interaction with an MDP
- Only have access to trajectories in MDP:
- $\tau = (x_0, u_0, r_0, x_1, \ldots, u_{H-1}, r_{H-1}, x_H)$



Learning from Experience

How to use trajectory data?

- Model based approach: estimate $T(x'|x, u)$, then use model to plan
- Model free:
  - Value based approach: estimate optimal value (or Q) function from data
  - Policy based approach: use data to determine how to improve policy
  - Actor Critic approach: learn both a policy and a value/Q function

Exploration vs Exploitation

In contrast to standard machine learning on fixed data sets, in RL we **actively gather the data we use to learn.**

- We can only learn about states we visit and actions we take
- Need to **explore** to ensure we get the data we need
- Efficient exploration is a fundamental challenge in RL!

Simple strategy: add noise to the policy.

ϵ-greedy exploration:

- With probability ϵ, take a random action; otherwise take the most promising action

Model-free, value based: Q Learning

For simplicity, let's assume $H = \infty$, so optimal value and policy don't depend on time. Why?

Optimal Q function satisfies

$$Q^*(x,u) = R(x,u) + \gamma \sum_{x' \in \mathcal{X}} T(x'|x,u) \max_{u'} Q^*(x',u')$$

So, in expectation,

$$\mathrm{E}\left[\underbrace{Q^*(x_t,u_t) - \left(r_t + \gamma \max_{u'} Q^*(x_{t+1},u') \right)}_{\text{Temporal Difference (TD) error}} \right] = 0$$

Q Learning

Initialize $Q(x, u)$ for all states and actions.

Let $\pi(x)$ be an ϵ-greedy policy according to Q.

Loop:

Take action: $u_t \sim \pi(x_t)$.

Observe reward and next state: (r_t, x_{t+1}).

Update Q to minimize TD error:

$$Q(x_t, u_t) \leftarrow Q(x_t, u_t) + \alpha \left(r + \max_u Q(x_{t+1}, u) - Q(x_t, u_t) \right)$$

$$t = t + 1$$

Fitted Q Learning

Large / Continuous Action Space?

Use parametric model for Q function: $Q_\theta(x, u)$

Gradient ascent on θ:

$$\theta \leftarrow \theta + \alpha \left(r_t + \gamma \max_u Q_\theta(x_{t+1}, u) - Q_\theta(x_t, u_t) \right) \nabla_\theta Q_\theta(x_t, u_t)$$

learning rate

$$\frac{d(\textit{Squared TD Error})}{dQ}$$

$$\frac{dQ}{d\theta}$$

Q Learning Recap

Pros:

- Can learn Q function from any interaction data, not just trajectories gathered using the current policy (“**off-policy” algorithm**)
- Relatively data-efficient (can reuse old interaction data)

Cons:

- Need to optimize over actions: hard to apply to continuous action spaces
- Optimal Q function can be complicated, hard to learn
- Optimal policy might be much simpler!

Model-free, policy based: Policy Gradient

Instead of learning the Q function, learn the policy directly!

Define a class of policies π_θ where θ are the parameters of the policy.

Can we learn the optimal θ from interaction?

Goal: use trajectories to estimate a gradient of policy performance w.r.t parameters θ.

Policy Gradient

A particular value of θ induces a distribution of possible trajectories.

Objective function:

$$J(\theta) = E_{\tau \sim p(\tau;\theta)}[r(\tau)]$$

$$J(\theta) = \int_{\tau} r(\tau) p(\tau;\theta) d\tau$$

where $r(\tau)$ is the total discounted cumulative reward of a trajectory.

Policy Gradient

Gradient of objective w.r.t. parameters:

$$\nabla_\theta J(\theta) = \int_\tau r(\tau)\nabla_\theta p(\tau;\theta)d\tau$$

Trick: $\nabla_\theta p(\tau;\theta) = p(\tau;\theta)\frac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta)\nabla_\theta \log p(\tau;\theta)$

$$\nabla_\theta J(\theta) = \int_\tau (r(\tau)\nabla_\theta \log p(\tau;\theta))p(\tau;\theta)\, d\tau$$

$$\nabla_\theta J(\theta) = E_{\tau\sim p(\tau;\theta)}[r(\tau)\nabla_\theta \log p(\tau;\theta)]$$

Policy Gradient

$$\nabla_\theta J(\theta) = E_{\tau \sim p(\tau;\theta)}[r(\tau)\nabla_\theta \log p(\tau;\theta)]$$

$$\log p(\tau;\theta) = \log\left(\prod_{t\geq 0} T(x_{t+1}|x_t,u_t)\pi_\theta(u_t|x_t)\right)$$

$$= \sum_{t\geq 0} \log T(x_{t+1}|x_t,u_t) + \log \pi_\theta(u_t|x_t)$$

$$\nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \log \pi_\theta(u_t|x_t)$$

We don't need to know the transition model to compute this gradient!

Policy Gradient

If we use π_θ to sample a trajectory, we can approximate the gradient:

$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_\theta \log \pi_\theta(u_t | x_t)$$

Intuition: adjust theta to:

- Boost probability of actions taken if reward is high
- Lower probability of actions taken if reward is low

Learning by trial and error.

Policy Gradient Recap

Pros:

- Learns policy directly – often more stable
- Works for continuous action spaces
- Converges to local maximum of $J(\theta)$

Cons:

- Needs data from current policy to compute gradient – data inefficient
- Gradient estimates can be very noisy

Actor Critic

Actor: Learned Policy, π_θ

Critic: Estimated Q function of Actor, V_ϕ

Critic helps **reduce variance** in gradient estimates for the actor.

$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} \left[r(\tau) - V_\phi(x_0) \right] \nabla_\theta \log \pi_\theta(u_t | x_t)$$

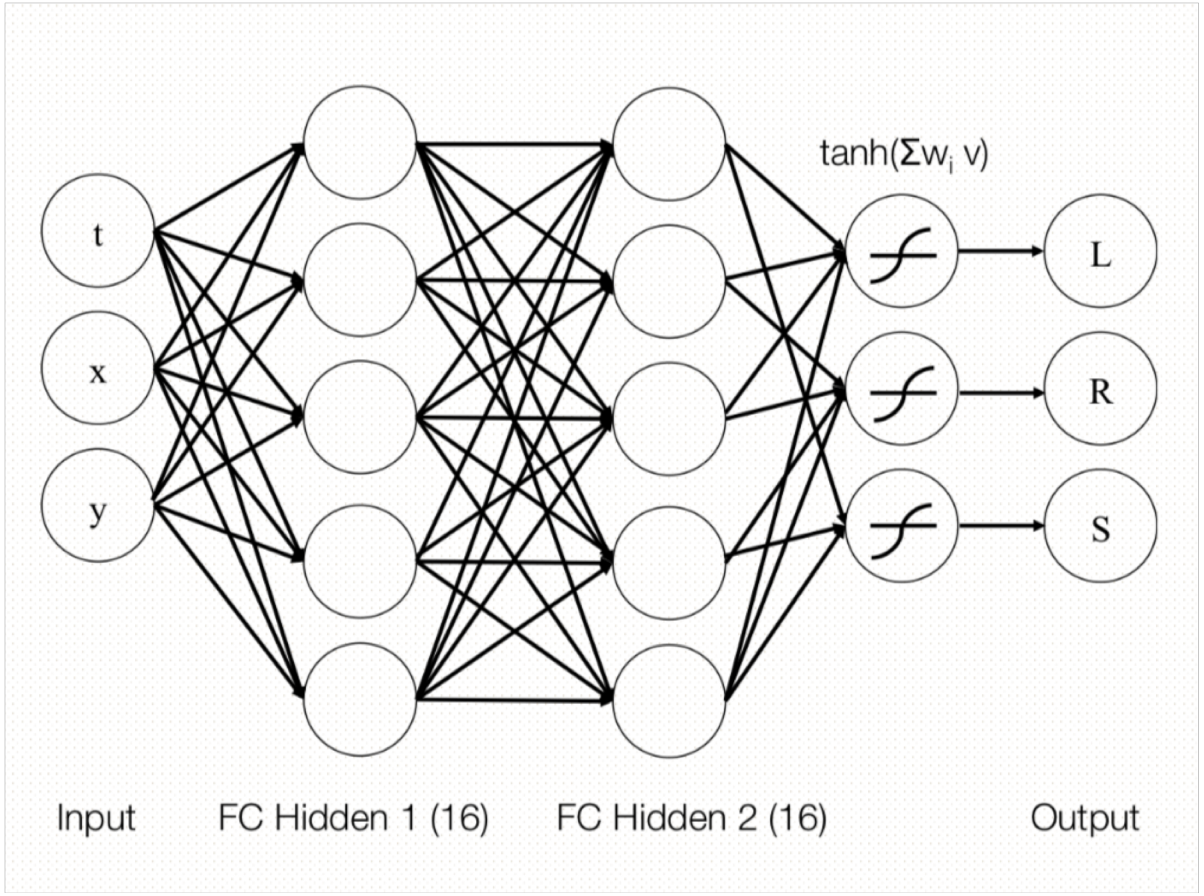
Learn ϕ by minimizing TD error, as before.

Result: learning is **more data-efficient**.

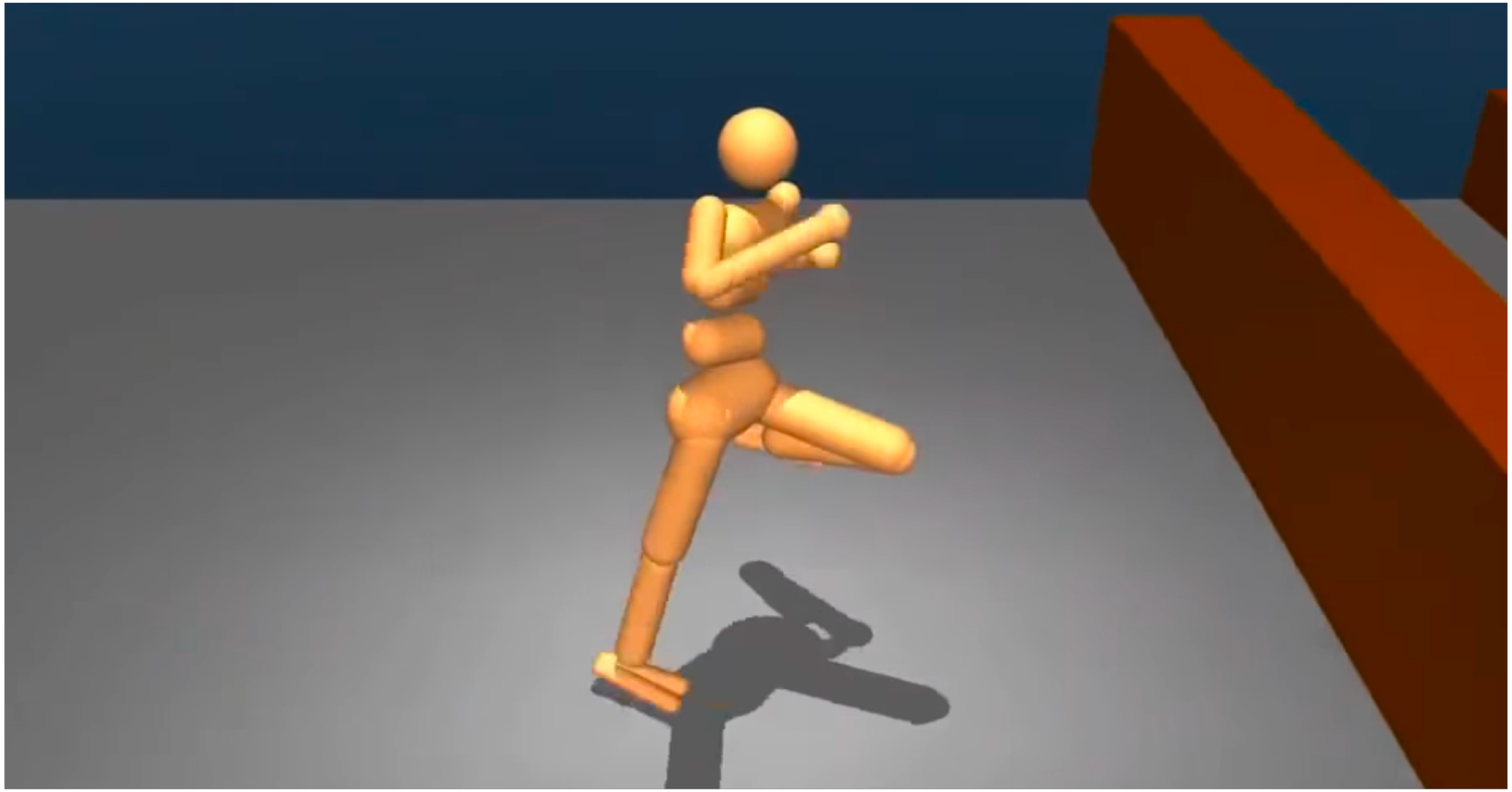
Deep Reinforcement Learning

- Deep Q learning:
  - Use neural network as Q function
  - Works in nonlinear, continuous state space domains

- Deep Policy Gradient:
  - Parameterize policy as deep neural network
  - Policy can act on high dimensional input, e.g. directly from visual feedback

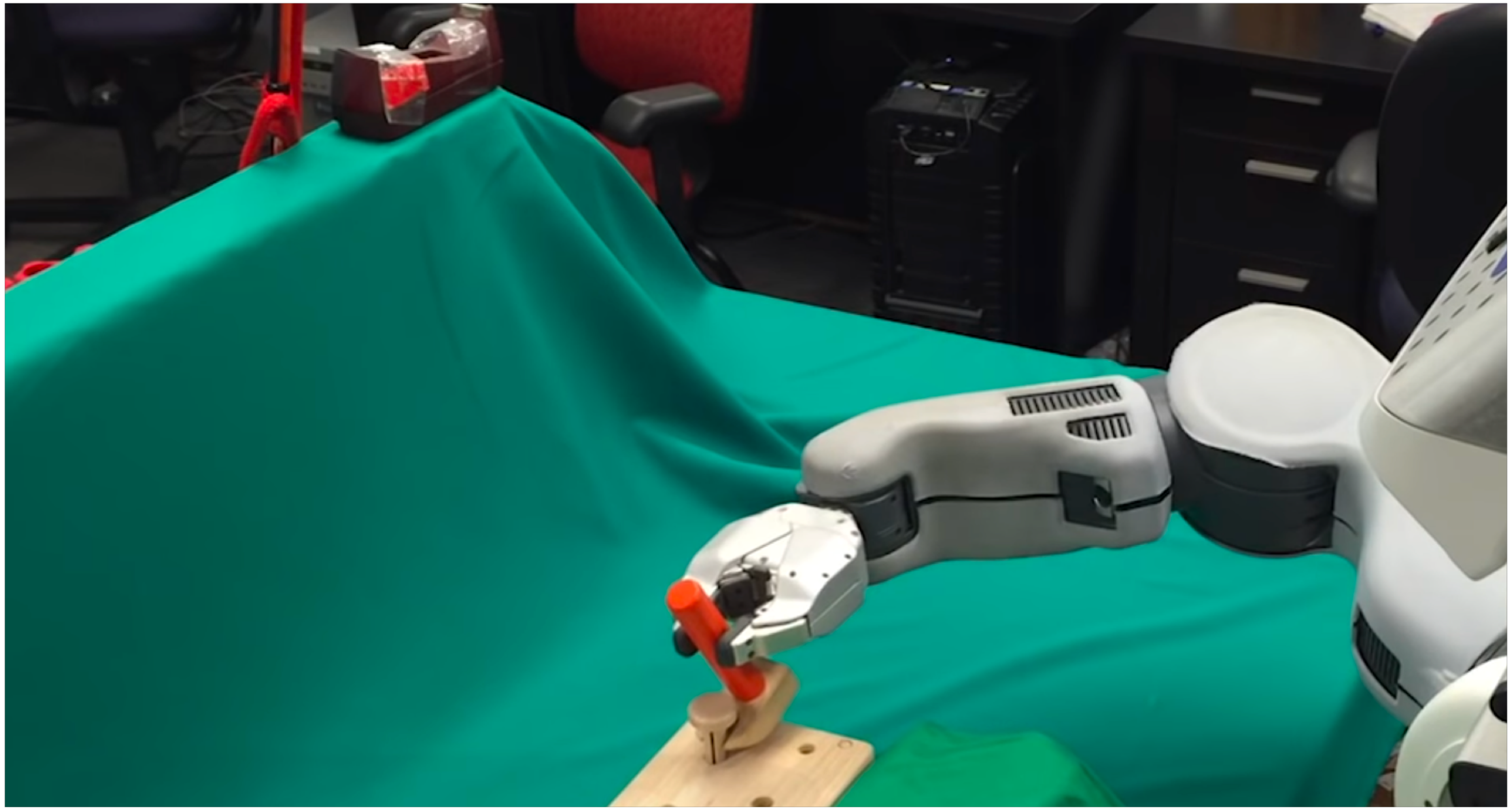


Results in simulation



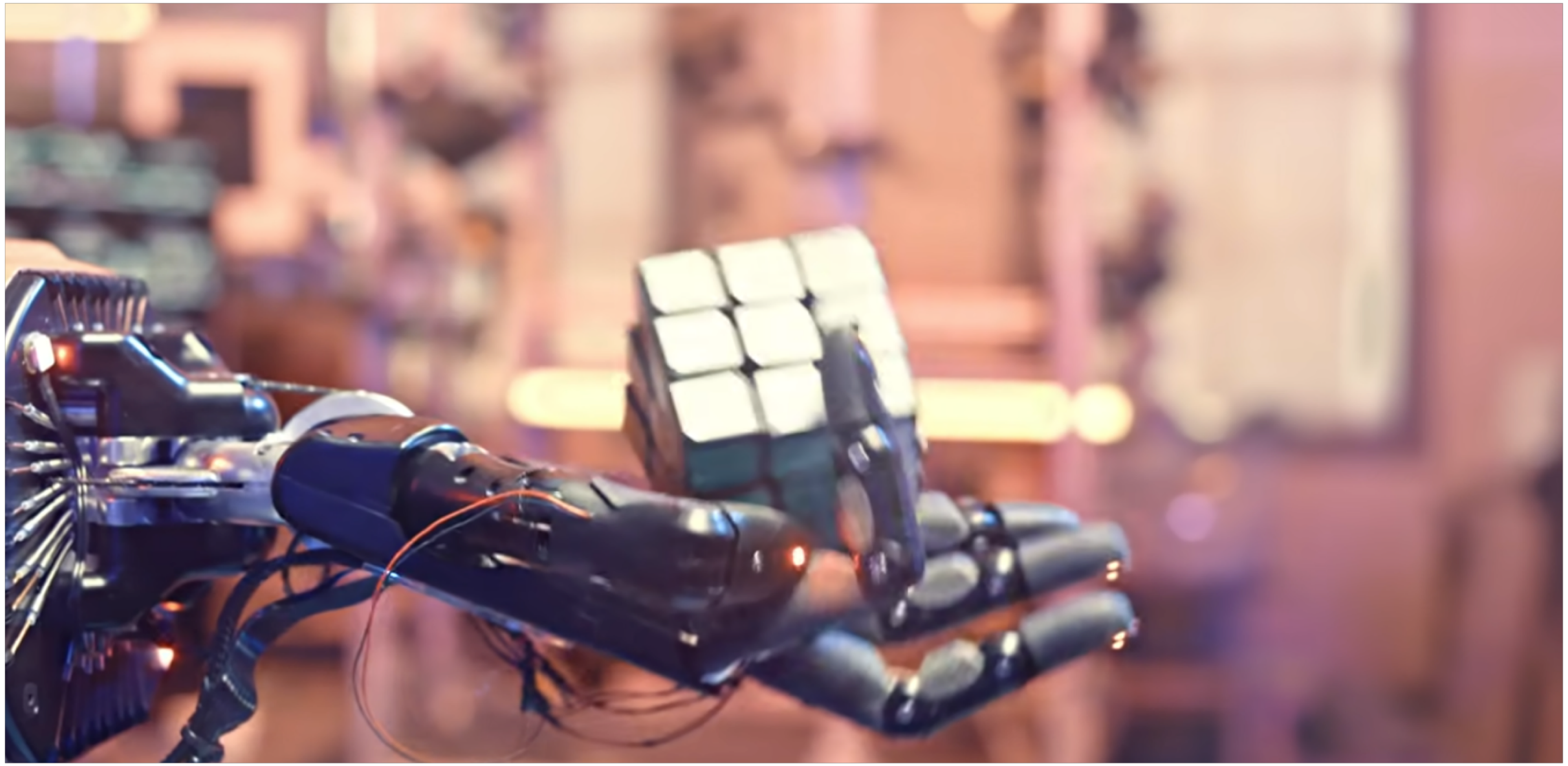
[Heess et al., "Emergence of Locomotion Behaviours in Rich Environments"]

Results in Robotics



Levine et al., "End-to-End Training of Deep Visuomotor Policies"

Results in Robotics



OpenAI, "Solving Rubik's Cube with a Robot Hand"

Challenges in RL for Robotics

Data-efficiency

Sim-to-real

Exploration

Reward design

Further Reading

Sutton and Barto, *Reinforcement Learning: an Introduction*

Bertsekas, *Reinforcement Learning and Optimal Control*

Courses at Stanford

- CS 234 Reinforcement Learning
- CS 332 Advanced Survey of Reinforcement Learning
- MS&E 338 Reinforcement Learning

Demo Day Tomorrow

Thanks for a great quarter!