# Principles of Robot Autonomy I

Non-parametric filtering

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# Today's lecture

- Aim
  - Learn about non-parametric filters

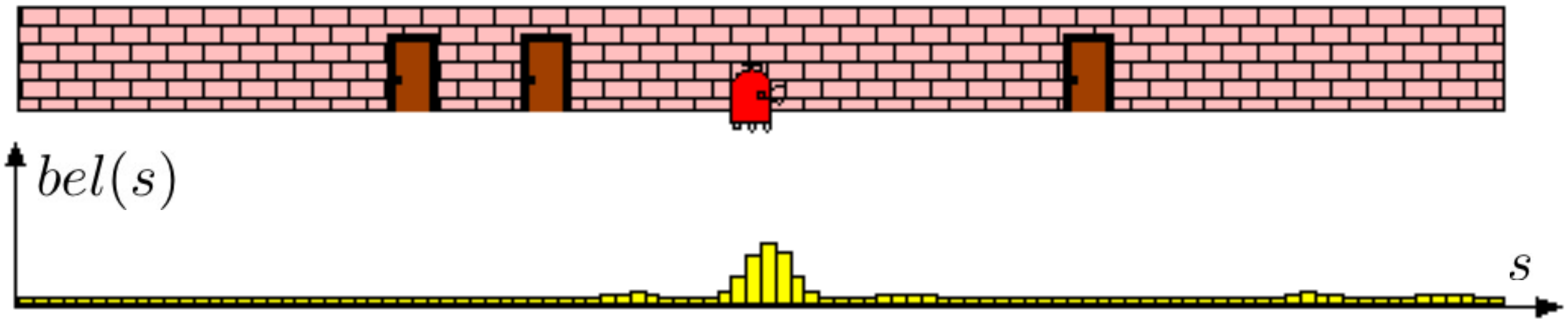
- Readings
  - S. Thrun, W. Burgard, and D. Fox. Probabilistic robotics. MIT press, 2005. Sections 3.1 – 3.4, 4.1, 4.3, 7.1

# Instantiating the Bayes' filter

- Tractable implementations of Bayes' filter exploit structure and / or approximations; two main classes
  - Parametric filters: e.g., KF, EKF, UKF, etc.
  - Non parametric filters: e.g., histogram filter, particle filter, etc.

# Histogram filter

- Key idea: use *discrete* Bayes' filter as an approximate inference tool for *continuous* state spaces



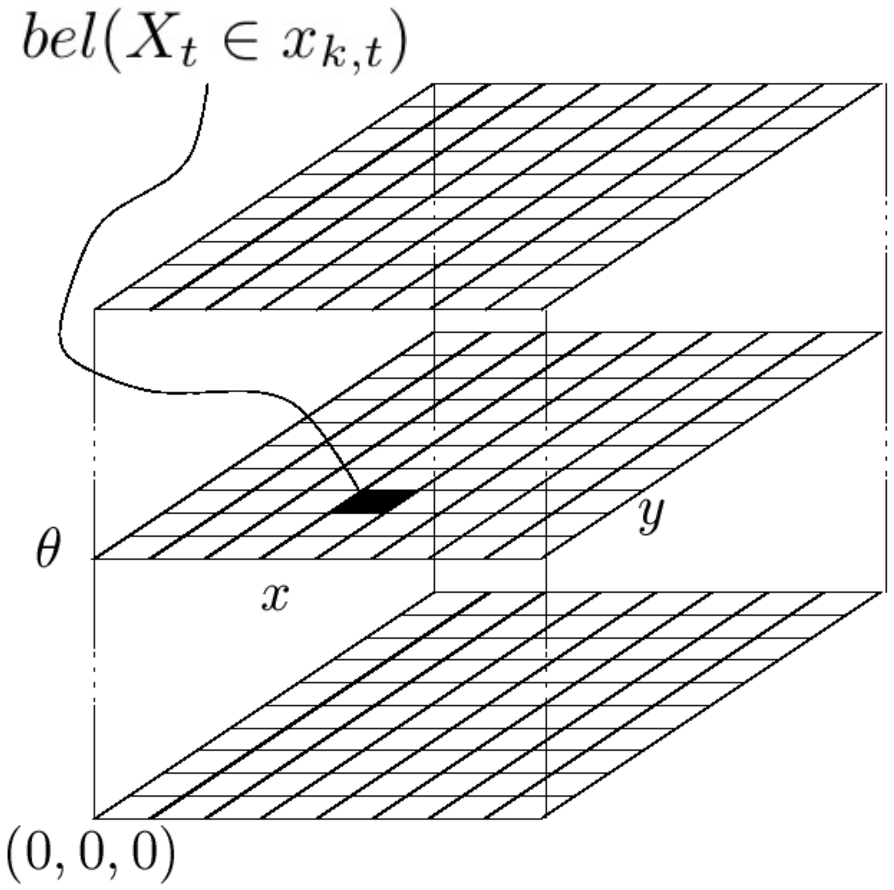

- Step #1: histogram filters decompose a continuous space into finitely many bins

$$\text{dom}(X_t) = x_{1,t} \cup x_{2,t} \cup \dots x_{K,t}$$

State space

$\{x_{k,t}\}$: convex regions forming a partition of state space (e.g., grid cell)

# Example



# Histogram filter

- Step #2: assign to each region $x_{k,t}$ a probability $p_{k,t}$; probabilities are then approximated according to a piecewise scheme

$$p(x_t) \equiv \frac{p_{k,t}}{|x_{k,t}|}, \quad \text{for all } x_t \in x_{k,t} \qquad \Rightarrow \qquad p(X_t \in x_{k,t}) = \int_{x_{k,t}} \frac{p_{k,t}}{|x_{k,t}|}\, dx_t = p_{k,t}$$

# Histogram filter

- Step #3: discretize motion and measurements models, i.e.,

$$p(x_t \mid u_t,\, x_{t-1}) \qquad \text{and} \qquad p(z_t \mid x_t)$$

1. Select mean state as representative state

$$\hat{x}_{k,t} = |x_{k,t}|^{-1} \int_{x_{k,t}} x_t \, dx_t$$

2. Approximate measurement model

$$p(z_t \mid x_{k,t}) \approx p(z_t \mid \hat{x}_{k,t})$$

3. Approximate transition model

$$p(x_{k,t} \mid u_t,\, x_{i,t-1}) \approx \eta \, |x_{k,t}| \, p(\hat{x}_{k,t} \mid u_t,\, \hat{x}_{i,t-1})$$

- Step #4: execute discrete Bayes' filter with discretized probabilities

# Histogram filter

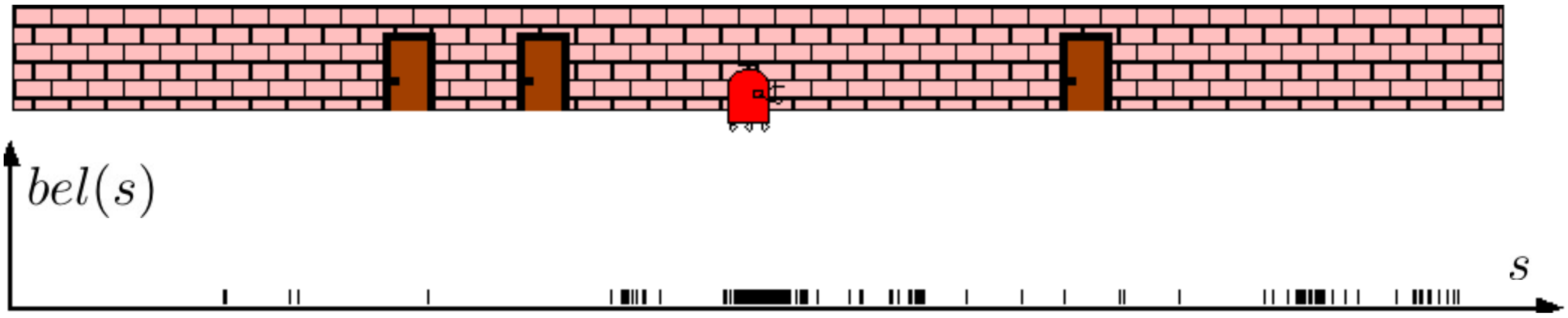
- Then one car run the usual discrete Bayes' filter

- Belief $bel(x_t)$ represented as pmf $\{p_{k,t}\}$

**Data:** $\{p_{k,t-1}\}, u_t, z_t$
**Result:** $\{p_{k,t}\}$
**foreach** $k$ **do**
  $\overline{p}_{k,t} = \sum_i p(X_t = x_k \mid u_t, X_{t-1} = x_i)\, p_{i,t-1};$
  $p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \overline{p}_{k,t};$
**end**
Return $\{p_{k,t}\}$

# Particle filter

- Key idea: represent posterior $bel(x_t)$ by a set of random samples




- Allows one to represent non-Gaussian distributions and handle nonlinear transformations in a direct way

# Particle filter

- Samples of posterior distribution are called *particles*, denoted as

$$\mathcal{X}_t := x_t^{[1]}, x_t^{[2]}, \ldots, x_t^{[M]}$$

- A particle represents a hypothesis about what the true world state might be at time $t$
- Ideally, particles should be distributed according to

$$x_t^{[m]} \sim p(x_t \mid z_{1:t},\, u_{1:t}) = bel(x_t)$$

- Matching exact only as $M \to \infty$, but $M \approx 1000$ usually good enough
- A particle filter constructs the particle set $\mathcal{X}_t$ from the particle set $\mathcal{X}_{t-1}$ recursively, with the goal of matching the distribution $bel(x_t)$

# Particle filter: algorithm

- The temporary particle set $\overline{\mathcal{X}}_t$ represents the belief $\overline{bel}(x_t)$
- The particle set $\mathcal{X}_t$ represents the belief $bel(x_t)$
- Importance factors are used to incorporate measurement $z_t$ in the particle set
- After resampling, particles are (as $M \rightarrow \infty$) distributed as

$$bel(x_t) = \eta\, p(z_t \mid x_t^{[m]})\, \overline{bel}(x_t)$$

$bel(x_{t-1})$

**Data:** $\mathcal{X}_{t-1}, u_t, z_t$
**Result:** $\mathcal{X}_t$
$\overline{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$;
**for** $i = 1$ **to** $M$ **do**

Prediction: $\overline{bel}(x_t)$

  Sample $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$;
  $w_t^{[m]} = p(z_t \mid x_t^{[m]})$;  ← Importance factor
  $\overline{\mathcal{X}}_t = \overline{\mathcal{X}}_t \cup \left(x_t^{[m]}, w_t^{[m]}\right)$;

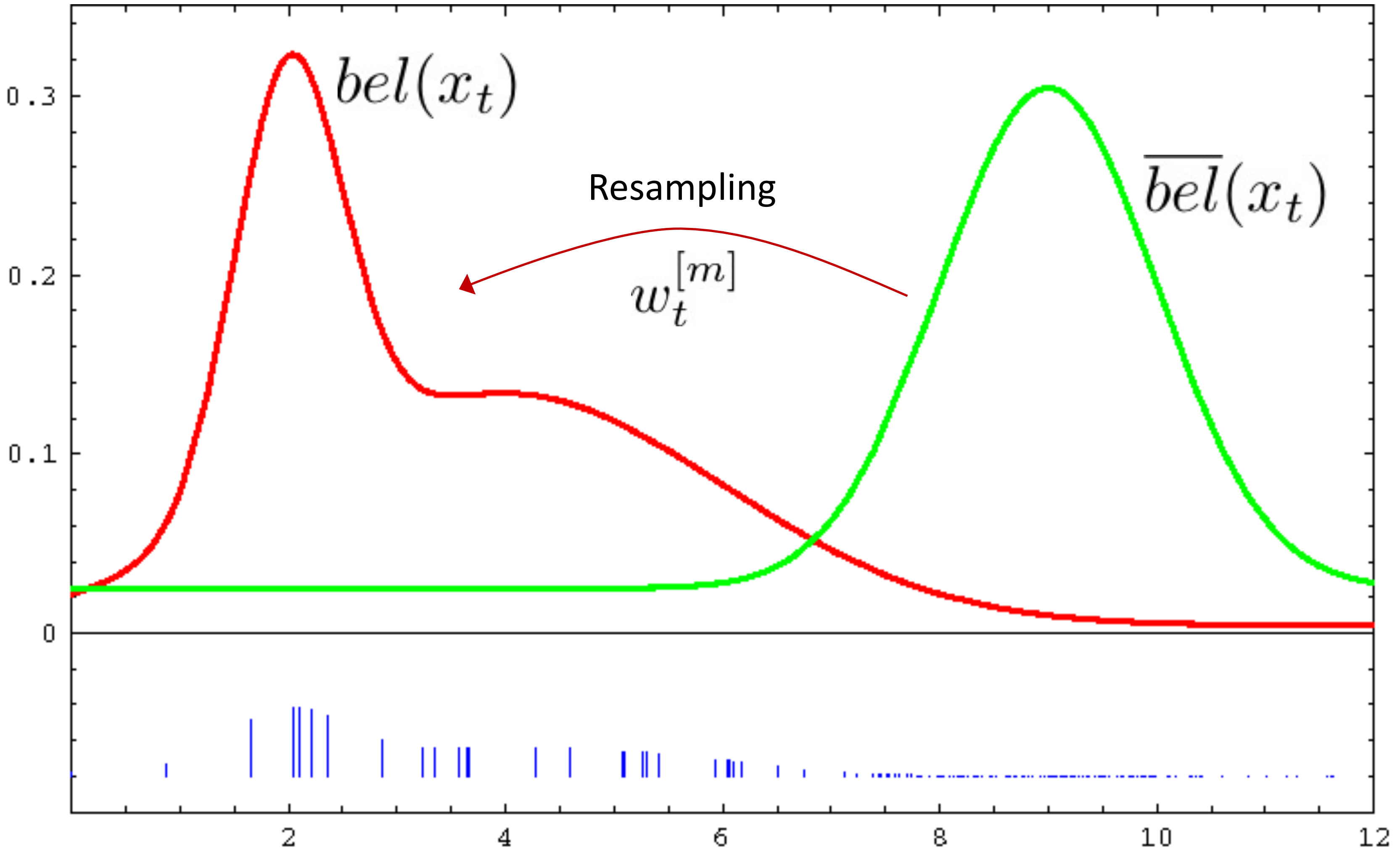
**end**

Correction: $bel(x_t)$

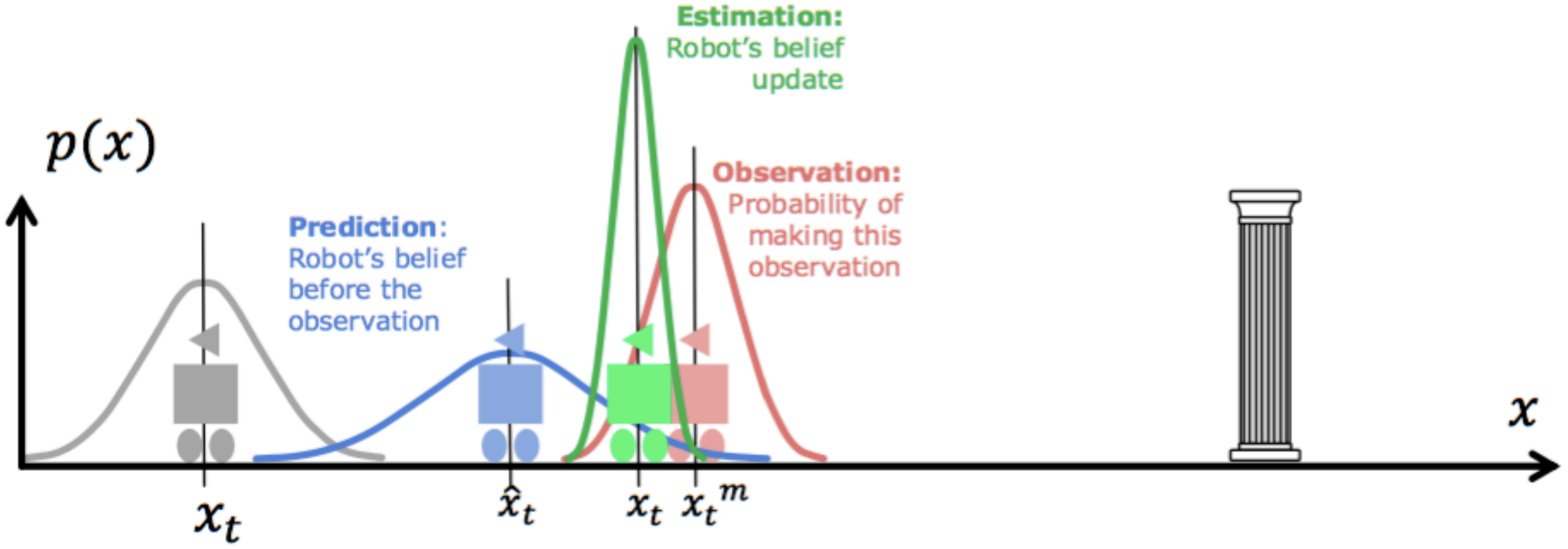
**for** $m = 1$ **to** $M$ **do**
  Draw $i$ with probability $\propto w_t^{[i]}$;
  Add $x_t^{[i]}$ to $\mathcal{X}_t$;
**end**
Return $\mathcal{X}_t$

$bel(x_t)$

# More on resampling



# Next time



p(x)
Estimation:
Robot's belief update
Observation:
Probability of making this observation
Prediction:
Robot's belief before the observation
x
$x_t$
$\hat{x}_t$
$x_t$
$x_t^m$