Principles of Robot Autonomy I

Information extraction

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Techniques for information extraction

- Aim
  - Learn how to extract information from sensor measurements

- Readings
  - Siegwart, Nourbakhsh, Scaramuzza. Introduction to Autonomous Mobile Robots. Sections: 4.1.3, 4.6.1 - 4.6.5, 4.7.1 - 4.7.4

Information extraction

- Next step is to extract *information* from images, such as
  - Geometric primitives (e.g., lines and circles): useful, for example, for robot localization and mapping
  - Object recognition and scene understanding: useful, for example, for localization within a topological map and for high-level reasoning

Geometric feature extraction

- Geometric feature extraction: extract geometric primitives from sensor data (e.g., range data)
- Examples: line, circles, corners, planes, etc.
- We focus on *line extraction* from range data (a quite common task); other geometric feature extraction tasks are conceptually analogous
- The two main problems of line extraction from range data
    1. Which points belong to which line? -> *segmentation*
    2. Given an association of points to a line, how to estimate line parameters? -> *fitting*

Step #2: line fitting

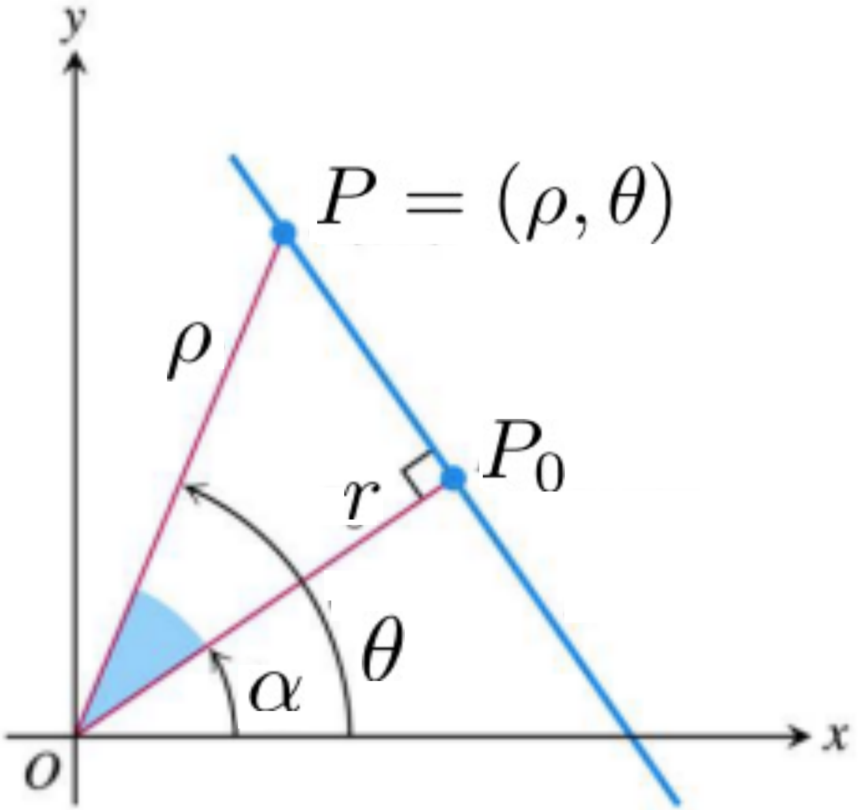
- Goal: fit a line to a set of sensor measurements
- It is useful to work in polar coordinates:

$$x = \rho \cos \theta, \quad y = \rho \sin \theta$$

- Equation of a line in polar coordinates
  - Let $P = (\rho, \theta)$ be an arbitrary point on the line
  - Since P, P_0, O determine a right triangle

$$\boxed{\rho \cos(\theta - \alpha) = r} \quad \text{or} \quad x \cos \alpha + y \sin \alpha = r$$

- (r, α) are the parameters of the line



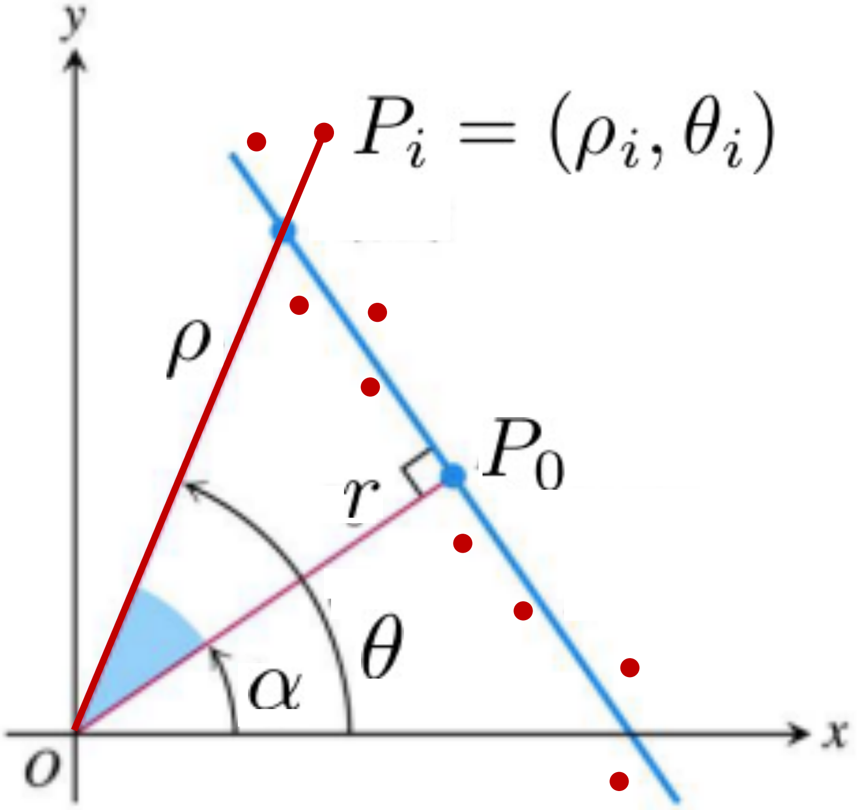
Step #2: line fitting

- Since there is measurement error, the equation of the line is only *approximately* satisfied

$$\rho_i \cos(\theta_i - \alpha) = r + d_i$$

Error

- Assume *n* ranging measurement points represented in polar coordinates as (ρ_i, θ_i)
- We want to find a line that best "fits" all the measurement points



Step #2: line fitting

- Consider, first, that all measurements are equally uncertain
- Find line parameters (r, α) that minimize squared error

$$S(r,\alpha) := \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} (\rho_i \cos(\theta_i - \alpha) - r)^2$$

- Unweighted least squares

Step #2: line fitting

- Consider, now, the case where each measurement has its own, unique uncertainty
- For example, assume that the variance for each range measurement ρ_i is σ_i
- Associate with each measurement a weight, e.g., $w_i = 1/\sigma_i^2$
- Then, one minimizes

$$S(r, \alpha) := \sum_{i=1}^{n} w_i \, d_i^2 = \sum_{i=1}^{n} w_i \left(\rho_i \cos(\theta_i - \alpha) - r\right)^2$$

- Weighted least squares

Step #2: line fitting solution

- Assume that the *n* ranging measurements are independent
- Solution:

$$\alpha = \frac{1}{2}\text{atan2}\left(\frac{\sum_i w_i \rho_i^2 \sin 2\theta_i - \frac{2}{\sum_i w_i}\sum_i \sum_j w_i w_j \rho_i \rho_j \cos\theta_i \sin\theta_j}{\sum_i w_i \rho_i^2 \cos 2\theta_i - \frac{1}{\sum_i w_i}\sum_i \sum_j w_i w_j \rho_i \rho_j \cos(\theta_i + \theta_j)}\right) + \frac{\pi}{2}$$

$$r = \frac{\sum_i w_i \rho_i \cos(\theta_i - \alpha)}{\sum_i w_i}$$

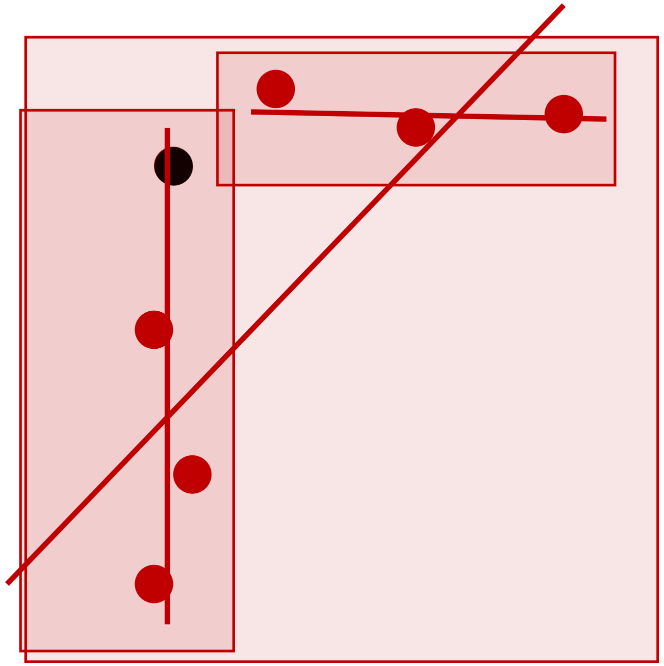
Step #1: line segmentation

- Several algorithms are available
- We will consider three popular algorithms
  1. Split-and-merge
  2. RANSAC
  3. Hough-Transform

Split-and-merge algorithm

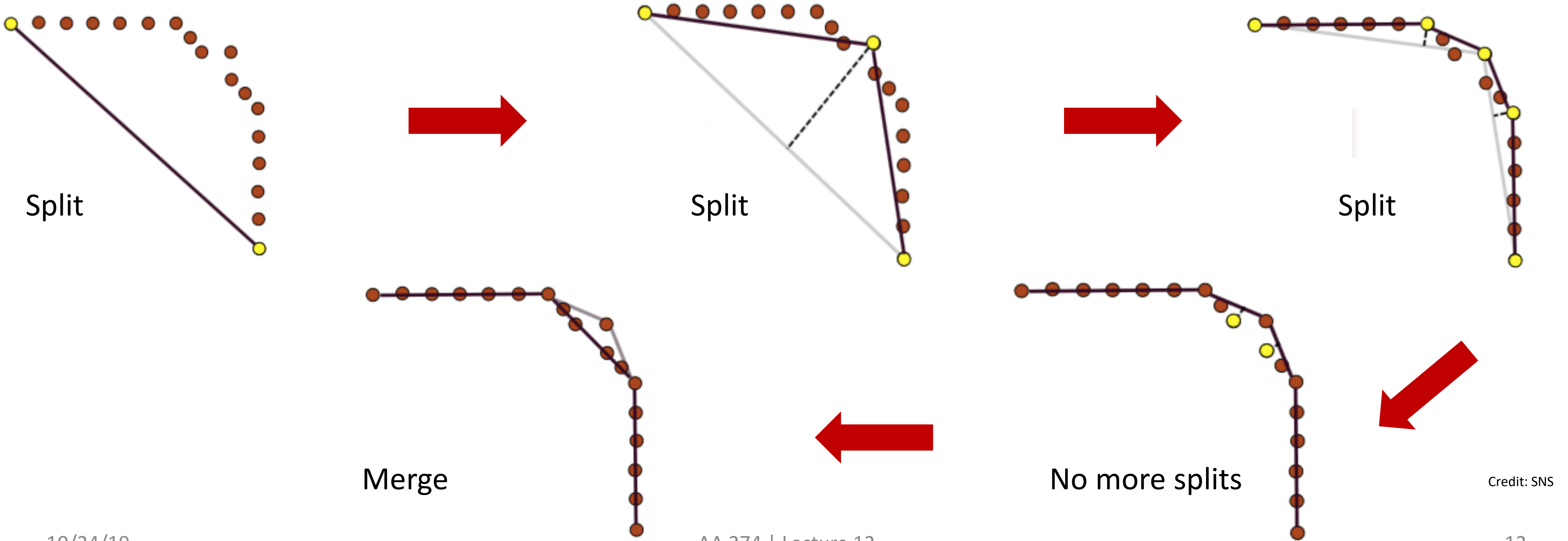
- Most popular line extraction algorithm

Data: Set S consisting of all N points, a distance threshold $d > 0$
Result: L, a list of sets of points each resembling a line
$L \leftarrow (S)$, $i \leftarrow 1$;
while $i \leq len(L)$ **do**
  fit a line (r, α) to the set L_i;
  detect the point $P \in L_i$ with the maximum distance D to the line (r, α);
  if $D < d$ **then**
    $i \leftarrow i + 1$
  else
    split L_i at P into S_1 and S_2;
    $L_i \leftarrow S_1$; $L_{i+1} \leftarrow S_2$;
  end
end
Merge collinear sets in L;



Split-and-merge: iterative-end-point-fit variant

- Iterative-end-point-fit: split-and-merge where the line is constructed by simply connecting the first and last points

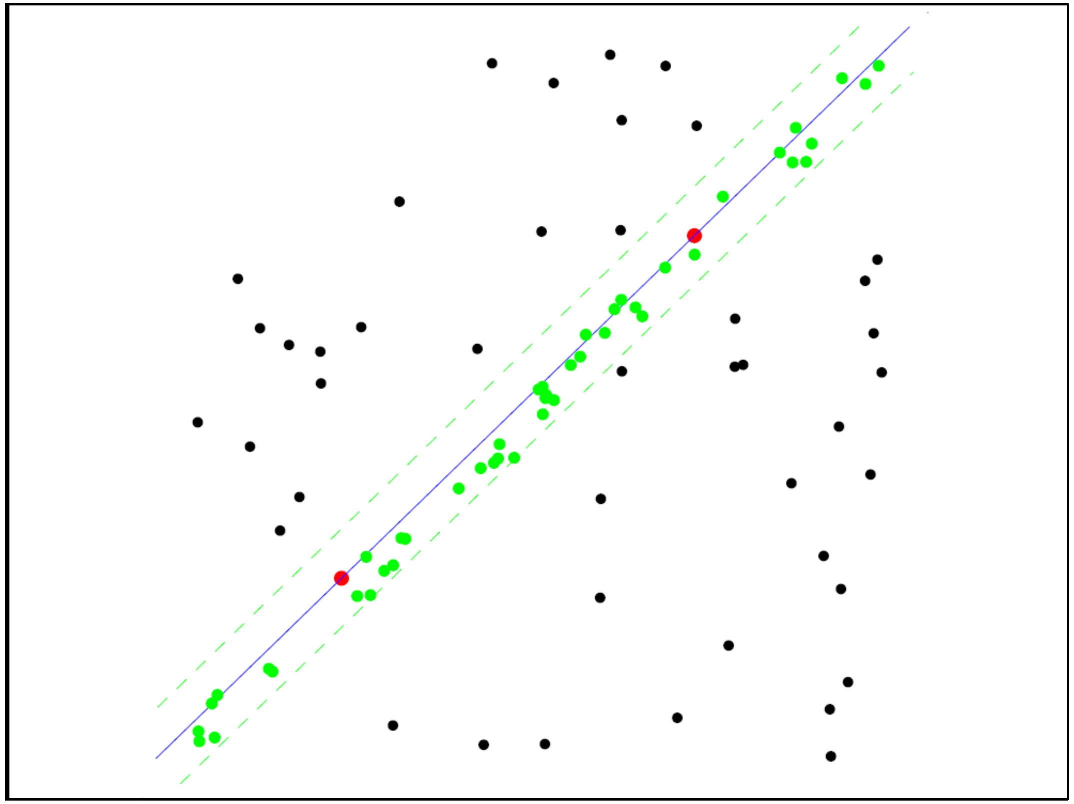


RANSAC

- RANSAC: **Ran**dom **Sa**mple **C**onsensus
- General method to estimate parameters of a model from a set of observed data in the presence of outliers, where outliers should have no influence on the estimates of the values
- Typical applications in robotics: line extraction from 2D range data, plane extraction from 3D point clouds, feature matching for structure from motion, etc.
- RANSAC is *iterative* and *non-deterministic*: the probability of finding a set free of outliers increases as more iterations are used

RANSAC

Data: Set S consisting of all N points
Result: Set with maximum number of inliers
(and corresponding fitting line)
while $i \leq k$ **do**
  randomly select 2 points from S;
  fit line l_i through the 2 points;
  compute distance of all other points to line l_i ;
  construct *inlier* set, i.e., count number of
    points with distance to the line less than γ;
  store line l_i and associated set of inliers;
  $i \leftarrow i + 1$
end
Choose set with maximum number of inliers



RANSAC iterations

- In principle, one would need to check all possible combinations of 2 points in dataset
- If $|S| = N$, number of combinations is $N(N-1)/2$-> too many
- However, if we have a rough estimate of the percentage of inliers, we do not need to check all combinations…

RANSAC iterations: statistical characterization

- Let w be the percentage of inliers in the dataset, i.e.,

$$w = \frac{\text{number of inliers}}{N}$$

- Let p be the desired probability of finding a set of points free of outliers (typically, $p = 0.99$)
- Assumption: 2 points chosen for line estimation are selected independently
    - $P(\text{both points selected are inliers}) = w^2$
    - $P(\text{at least one of the selected points is an outlier}) = 1 - w^2$
    - $P(\text{RANSAC nevers selects two points that are both inliers}) = (1 - w^2)^k$

RANSAC iterations: statistical characterization

- Then minimum number of iterations $\bar{k}$ to find an outlier-free set with probability at least p is:

$$1 - p = (1 - w^2)^{\bar{k}} \Rightarrow \bar{k} = \frac{\log(1-p)}{\log(1-w^2)}$$

- Thus if we know w (at least approximately), after $\bar{k}$ iterations RANSAC will find a set free of outliers with probability p
- Note:
  - $\bar{k}$ depends only on w, not on N!
  - More advanced versions of RANSAC estimate w adaptively

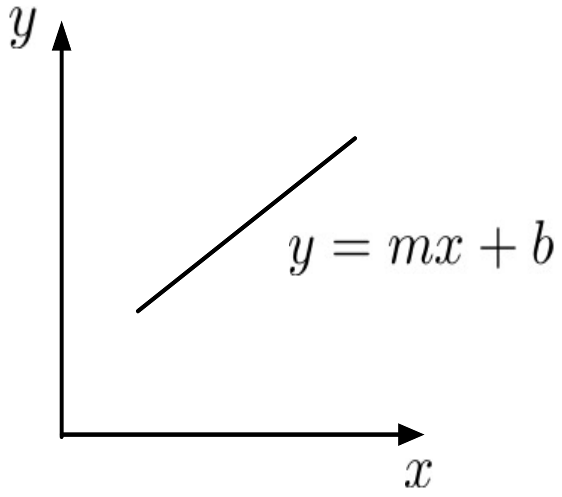
Hough transform

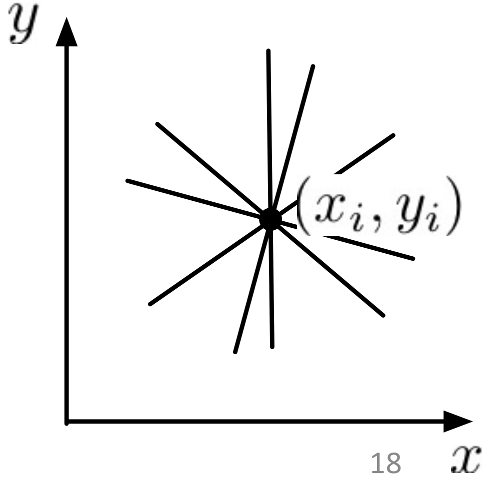
- Key idea: each point votes for a *set* of plausible line parameters

- A line has two parameters: (m, b)

- Given a point (x_i, y_i), the lines that could pass through this point are all (m, b) satisfying

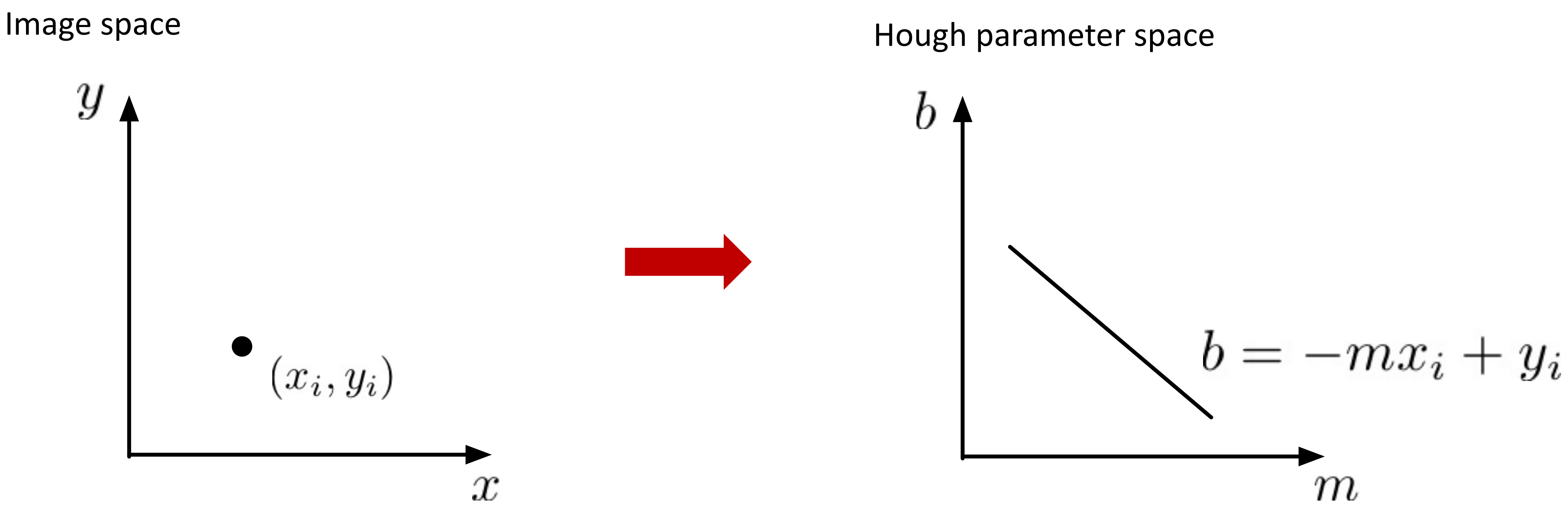
$$y_i = mx_i + b, \quad \text{or } b = -mx_i + y_i$$





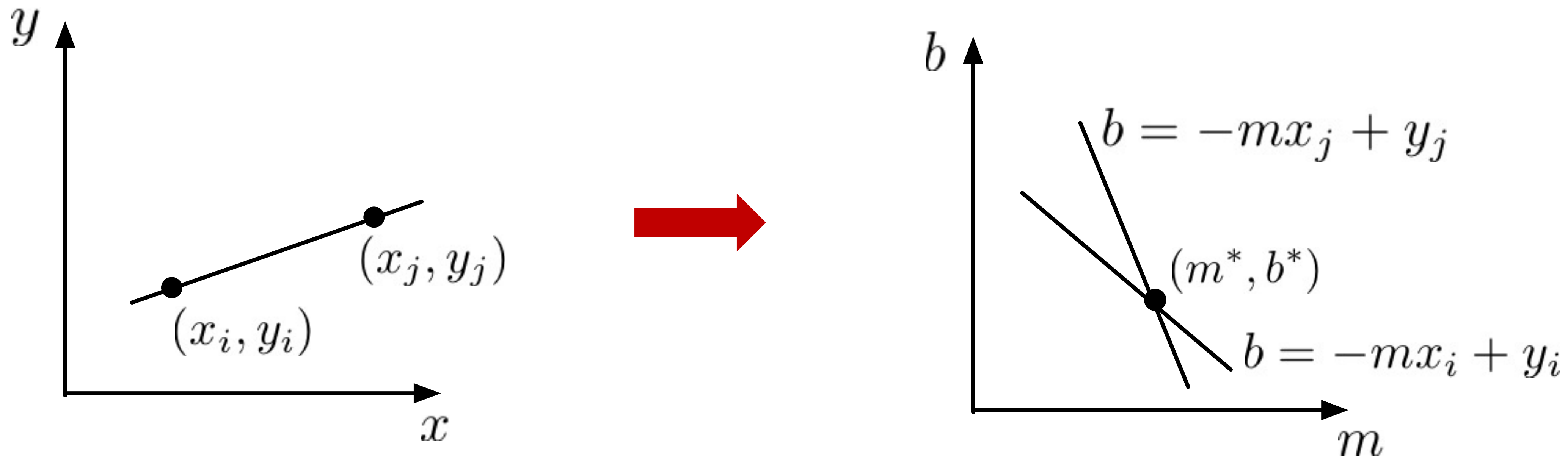
Hough transform

- A point in image space maps into a line in *Hough space*



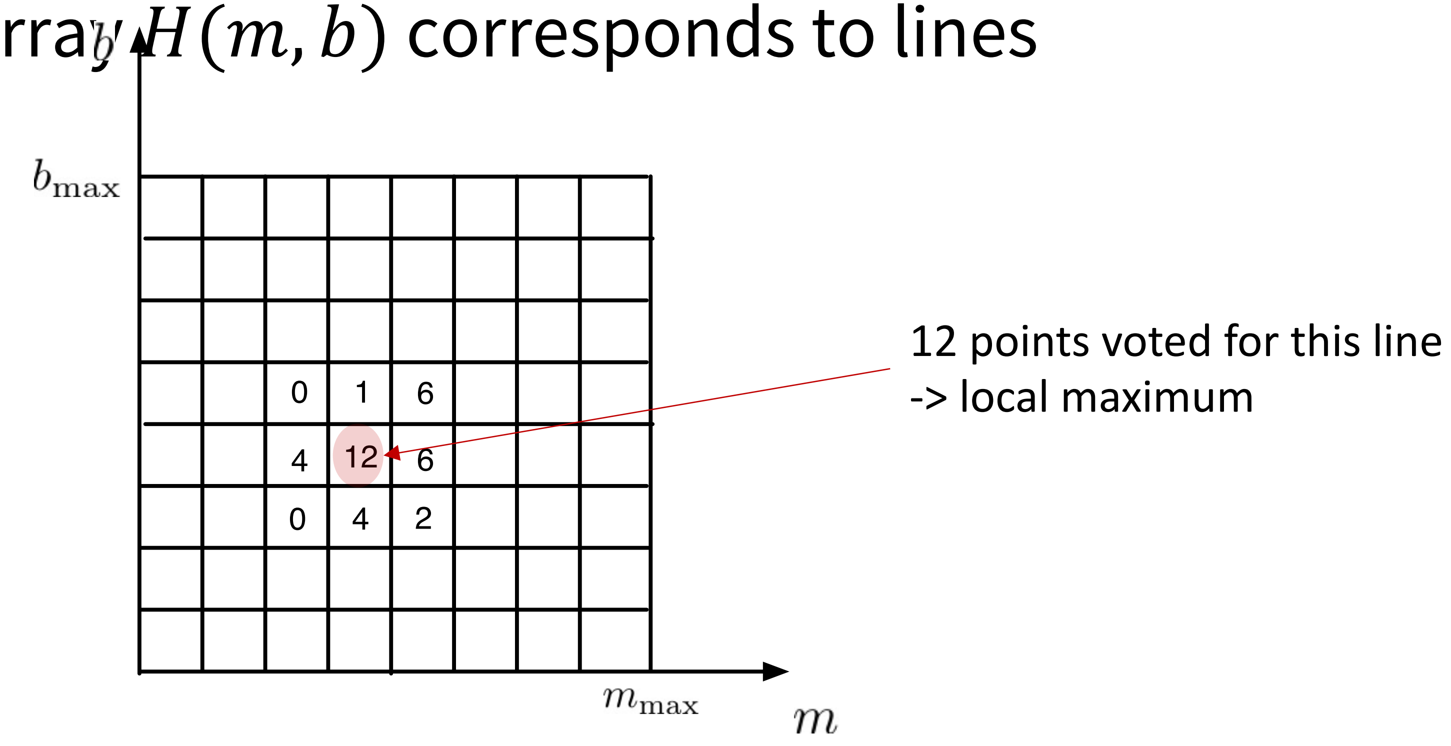
Hough transform

- Key fact: all points on a line in image space yield lines in parameter space which intersect at a *common point*, (m^*, b^*)



Hough transform algorithm

1. Initialize an accumulator array $H(m, b)$ to zero
2. For each point (x_i, y_i), increment all cells that satisfy $b = -x_i m + y_i$
3. Local maxima in array $H(m, b)$ corresponds to lines

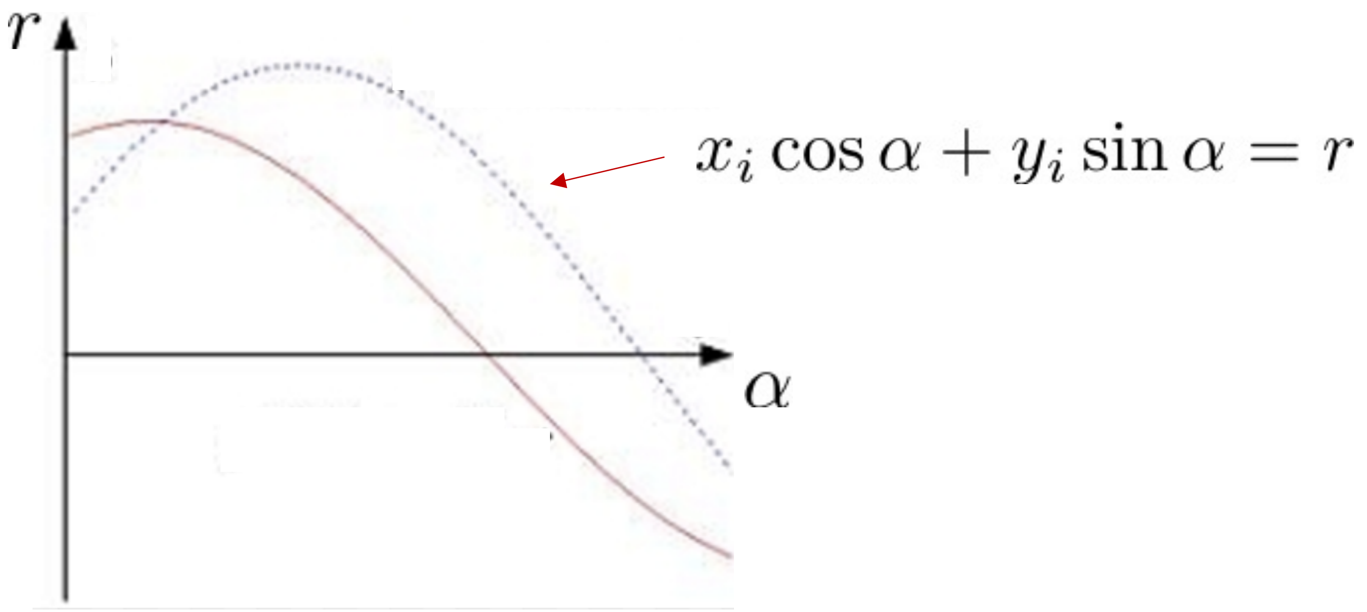


Hough transform algorithm: polar coordinate representation

- Equation of a line in polar coordinates

$$x \cos \alpha + y \sin \alpha = r$$

- The parameter space transform of a point is a sinusoidal curve

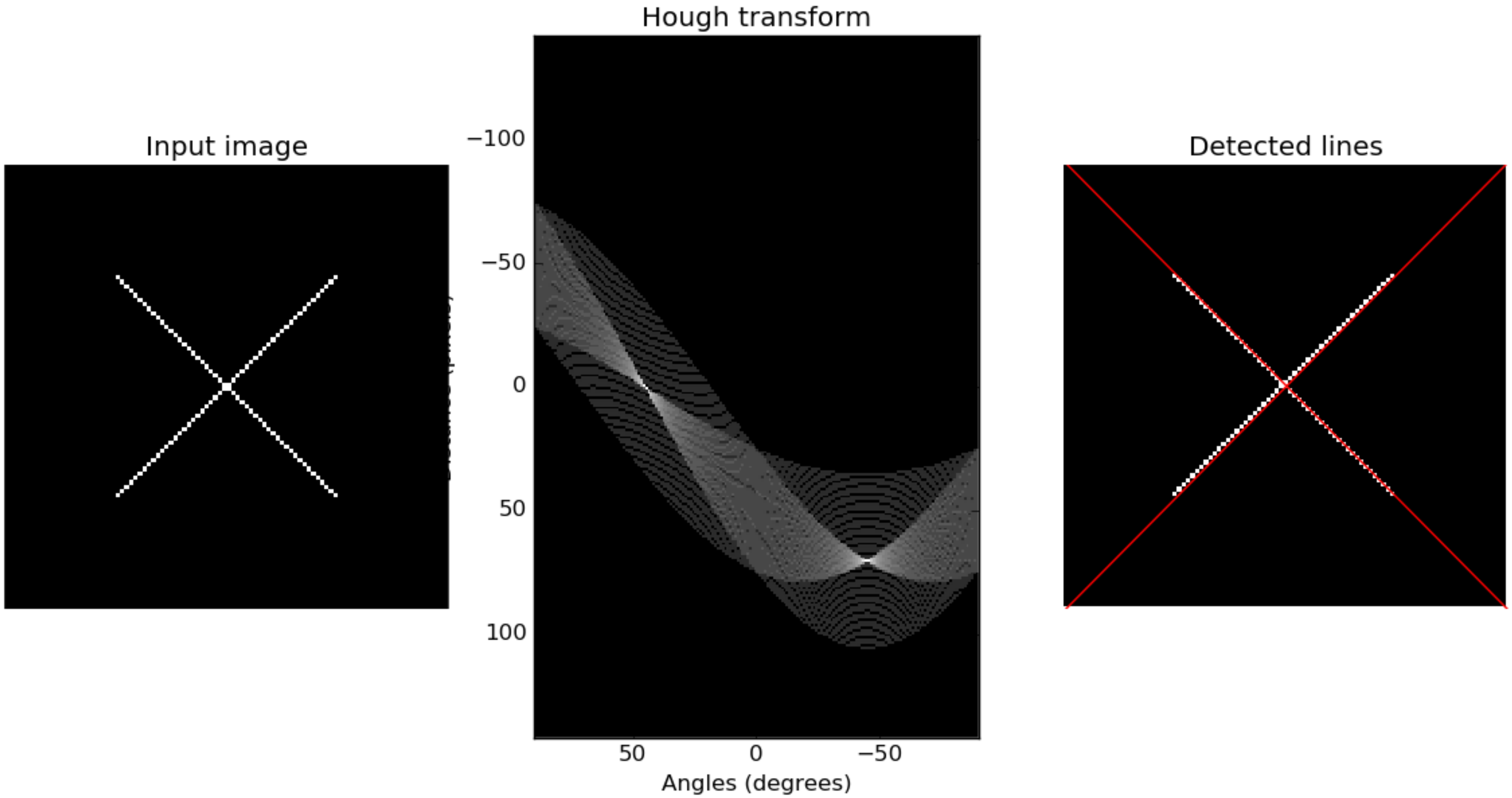


- Avoids infinite slope
- Constant resolution

Hough transform algorithm, revised

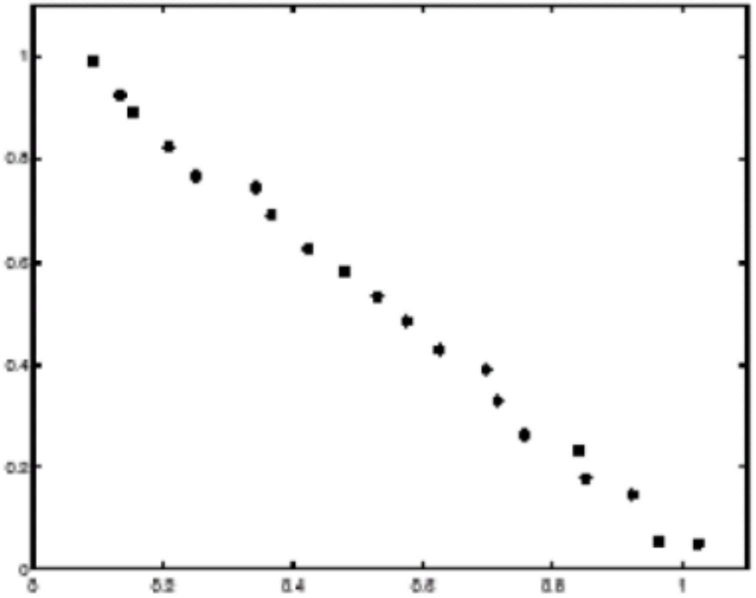
Data: Set S containing N points
Result: Line fitting the points in S
Initialize $n_\alpha \times n_r$ accumulator H with zeros;
foreach $(x_i, y_i) \in S$ **do**
  foreach $\alpha \in \{\alpha_1, \ldots, \alpha_{n_\alpha}\}$ **do**
    compute $r = x_i \cos\alpha + y_i \sin\alpha$;
    $H[\alpha, r] \leftarrow H[\alpha, r] + 1$;
  end
end
Choose (α^*, r^*) that corresponds to largest count in H;
Return line defined by (α^*, r^*)

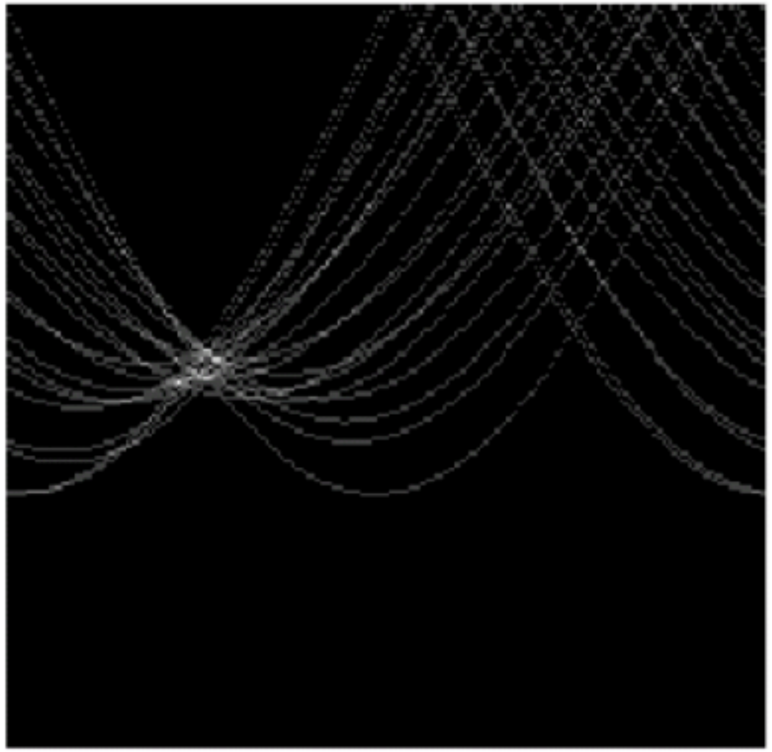
Hough transform: example



Hough transform: example

- With noise, peaks may be hard to detect



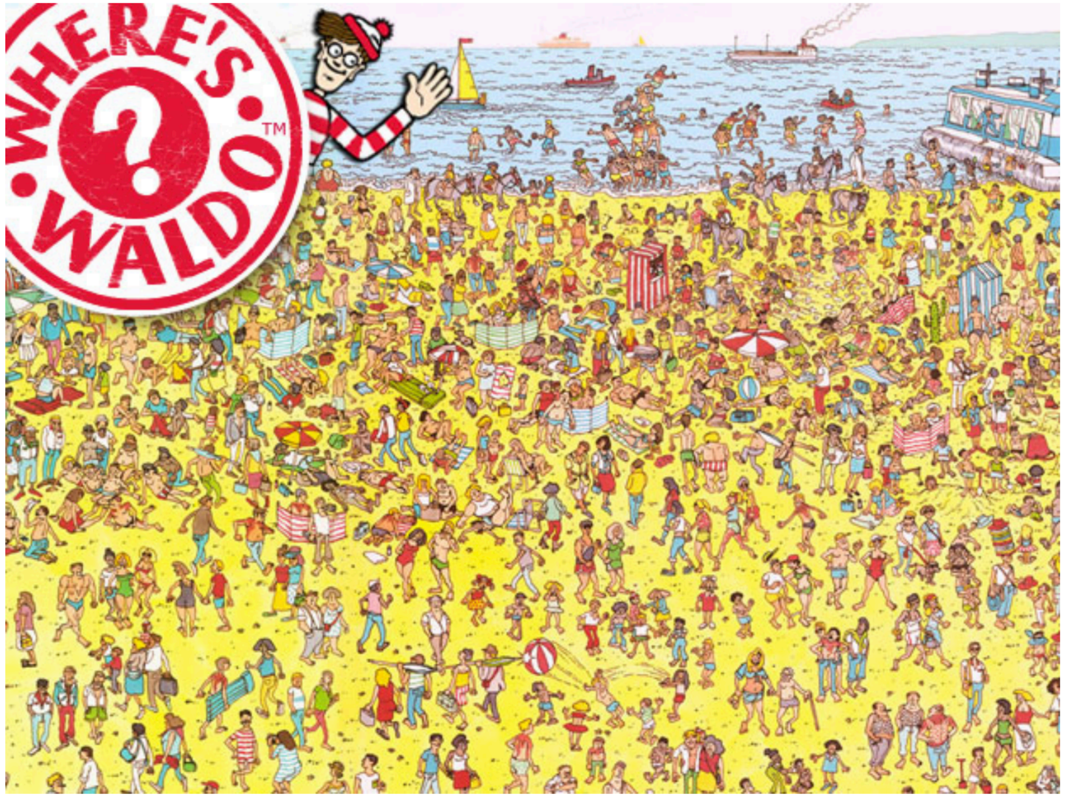


Object recognition

- Object recognition: capability of naming discrete objects in the world
- Why is it hard? Many reasons, including:
    1. Real world is made of a jumble of objects, which all occlude one another and appear in different poses
    2. There is a lot of variability intrinsic within each class (e.g., dogs)
- In this class, we will look at three methods:
    1. Template matching
    2. Bag of visual words
    3. Neural network methods (treated as a black box, take AA274B for details)

Template matching

- How can we find Waldo?



Source: Sanja Fidler

Template matching

- Slide and compare!

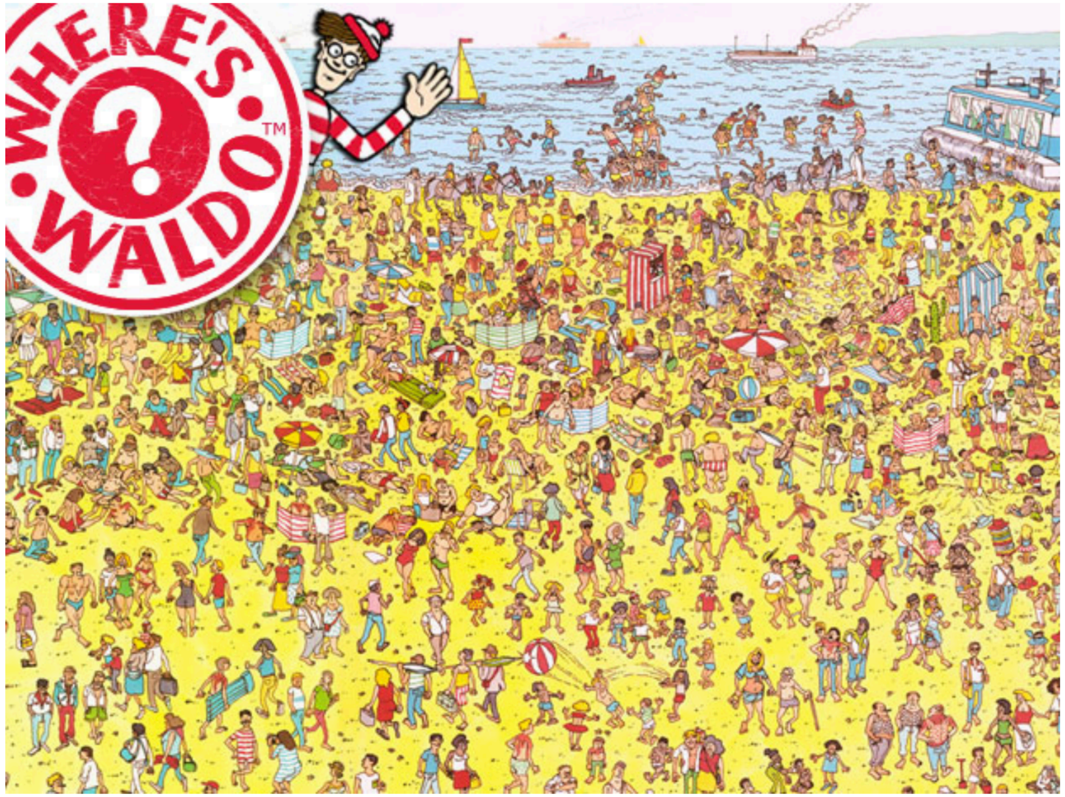


Image I

Filter F

Source: Sanja Fidler

Template matching

- In practice, remember correlation:

$$I'(x,y) = F \circ I = \sum_{i=-N}^{N} \sum_{j=-M}^{M} F(i,j) I(x+i, y+j)$$

- One can equivalently write: $I'(x, y) = \mathbf{f}^{\mathrm{T}} \cdot \mathbf{t}_{\mathrm{ij}}$

Vector representation of filter

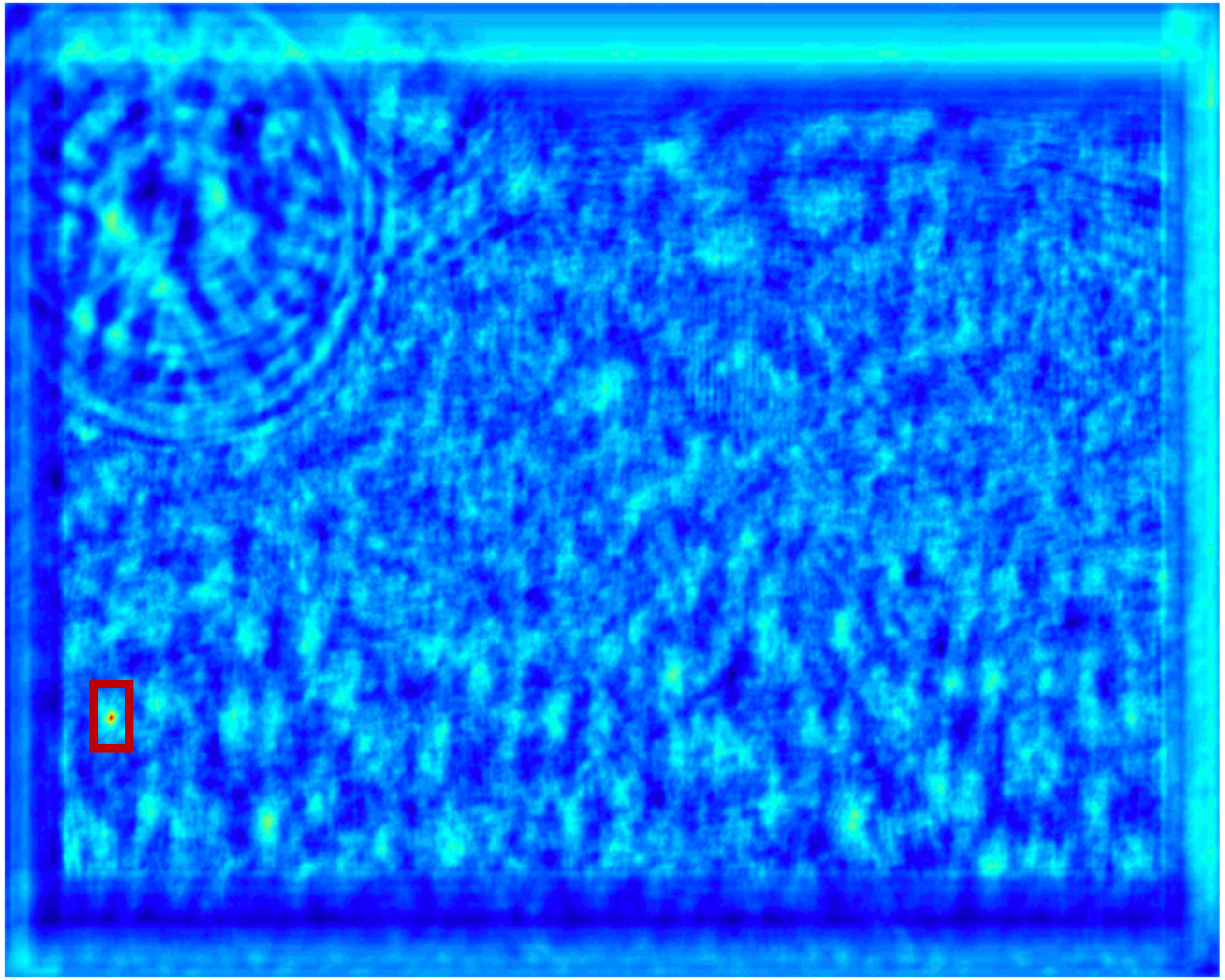
Vector representation of neighborhood patch

- To ensure that perfect matching yields one, we consider *normalized* correlation, that is

$$I'(x,y) = \frac{\mathbf{f}^{\mathrm{T}} \cdot \mathbf{t}_{\mathrm{ij}}}{\|\mathbf{f}\| \|\mathbf{t}_{\mathrm{ij}}\|}$$

Template matching

Result:



Source: Sanja Fidler

Template matching

- Problem: what if the object in the image is much larger or much smaller than our template?
- Solution: re-scale the image multiple times, and do correlation on every size!
- This leads to the idea of *image pyramids*

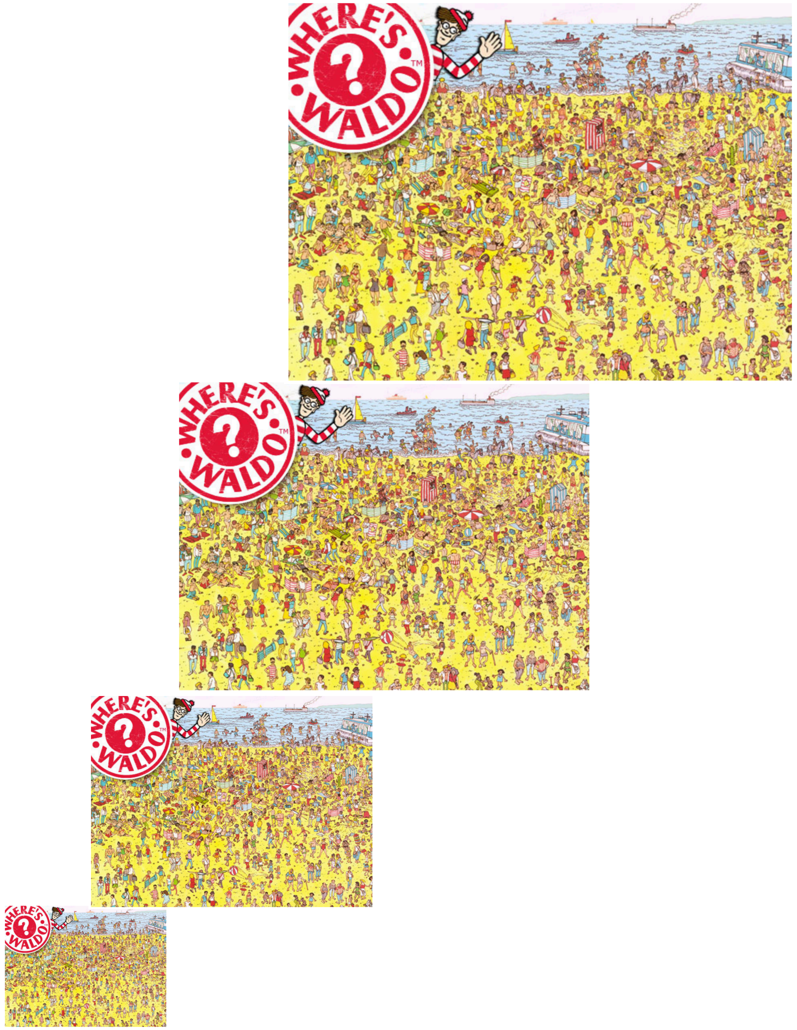


Image pyramids: scaling down

- Naïve solution: keep only some rows and columns
- E.g.: keep every other column to reduce image by 1/2 in width direction

Source:
Sanja Fidler

Image pyramids: scaling down

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- E.g.: keep every other column to reduce image by 1/2 in width direction

Source:
Sanja Fidler

Image pyramids: scaling down

- Solution: blur the image via Gaussian, *then* subsample
- Intuition: remove high frequency content in the image

Source:
Sanja Fidler

Image pyramids: scaling down

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Image pyramids: scaling down

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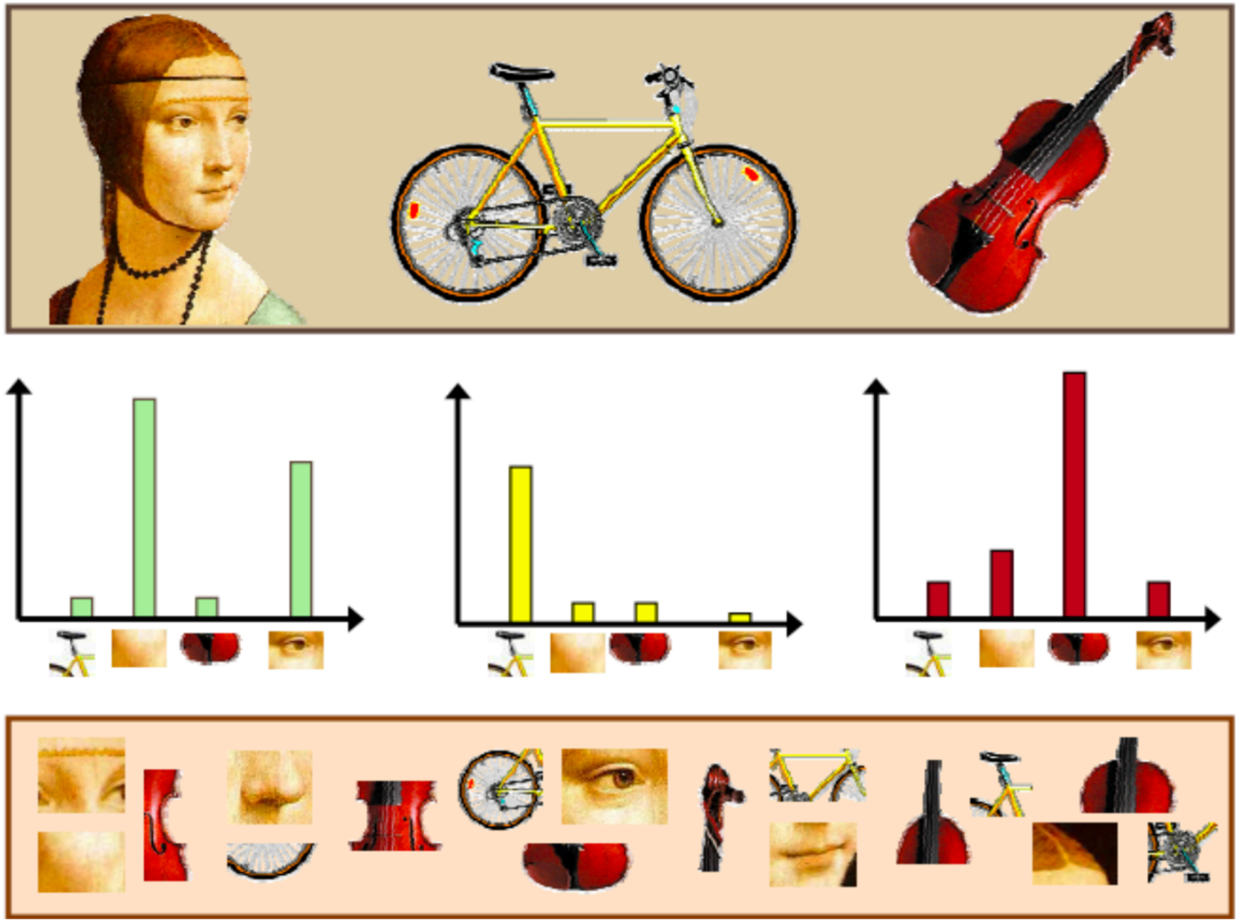
Source:
Sanja Fidler

Image pyramids

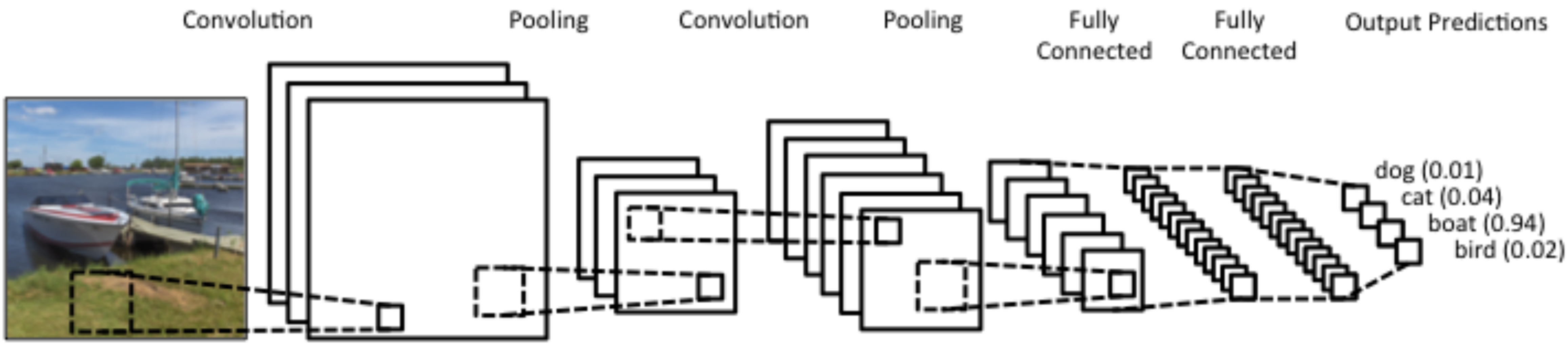
- A sequence of images created with Gaussian blurring and down-sampling is called a Gaussian pyramid
- The other step is to perform up-sampling (nearest neighbor, bilinear, bicubic, etc), see Extra Problem in pset

Bags of Visual Words

- Key idea: compute the distribution (histogram) of *visual words* found in the query image
- Compare this distribution to those found in the training images in order to perform classification



A different paradigm: using CNNs for recognition



Nest time