Principles of Robot Autonomy I

Multi-sensor perception and sensor fusion

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Today's lecture

- Aim
  - Introduce the topic of multi-sensor perception and sensor fusion
  - Learn about Kalman filtering applied to sensor fusion
  - Devise a sensor fusion algorithm for position estimation (low-level fusion)
  - Introduce the topic of multi-object tracking
  - Discuss emerging sensor fusion techniques

- Readings
  - F. Gustafsson. Statistical Sensor Fusion. 2010.
  - C. Lundquist, Z. Sjanic, F. Gustafsson. Statistical Sensor Fusion: Exercises. 2015.
  - D. Simon. Optimal State Estimation: Kalman, H_∞, and Nonlinear Approaches. 2006.

Multi-sensor approach in robotics

- **Self-awareness** (localization and positioning)
- **Situational awareness** (detection and tracking)
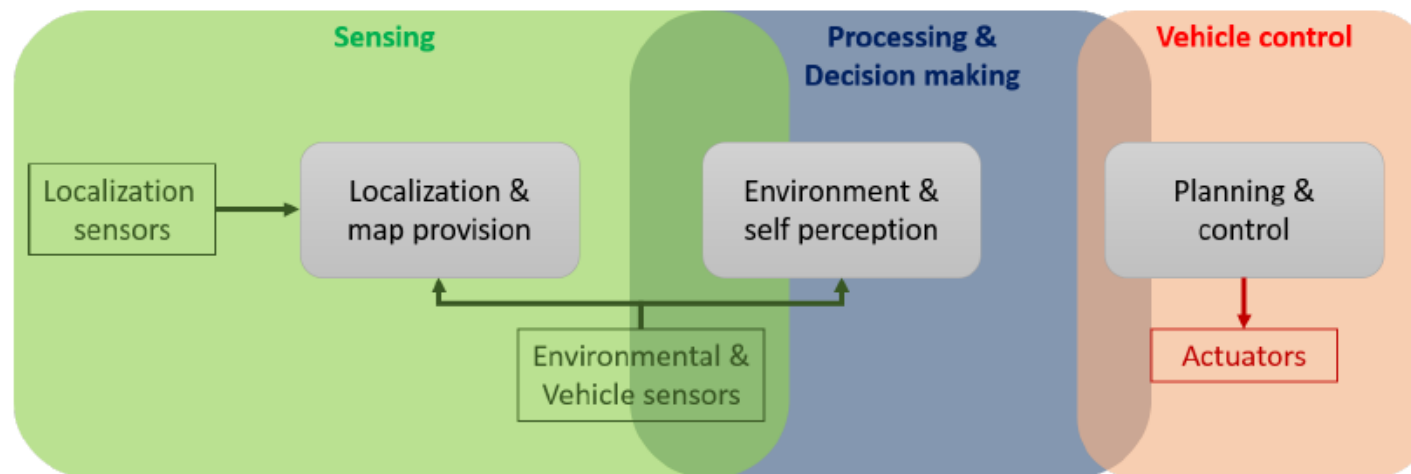
Single-sensor vs multi-sensor perception

- Drawbacks of **single-sensor** perception
  - Limited range and field of view
  - Performance is susceptible to common environmental conditions
  - Range determination is not as accurate as required
  - Detection of artefacts, so-called false positives
- **Multi-sensor perception** might compensate these, and provide:
  - Increased classification accuracy of objects
  - Improved state estimation accuracy
  - Improved robustness for instance in adverse weather conditions
  - Increased availability
  - Enlarged field of view

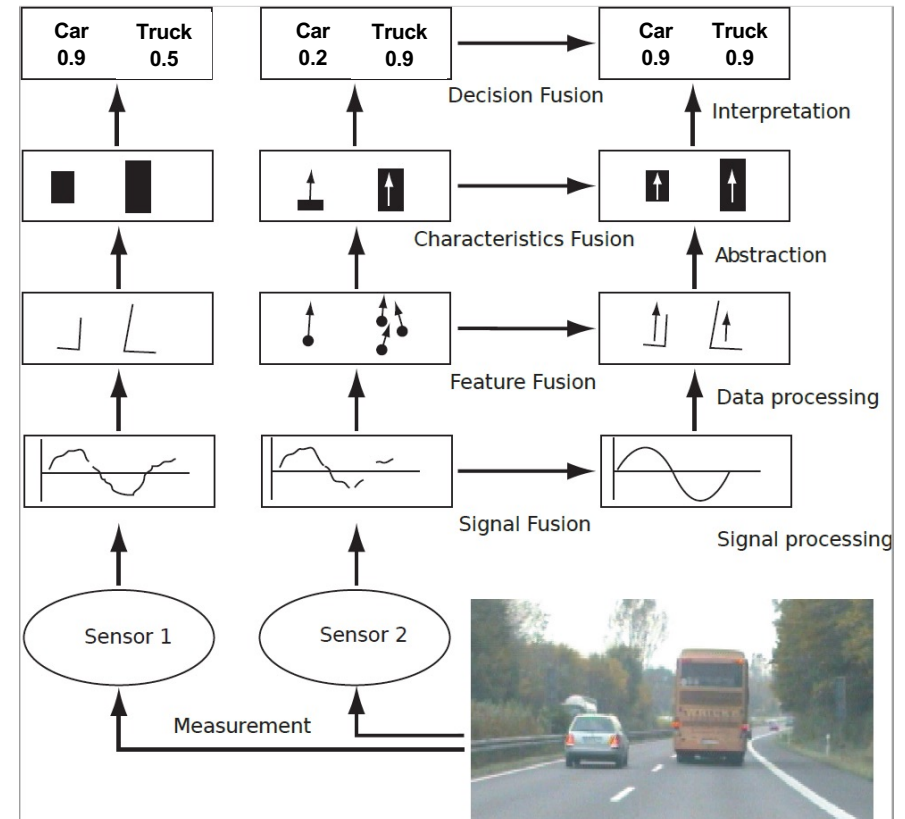# Using stationary sensors

700-800m

200-300m
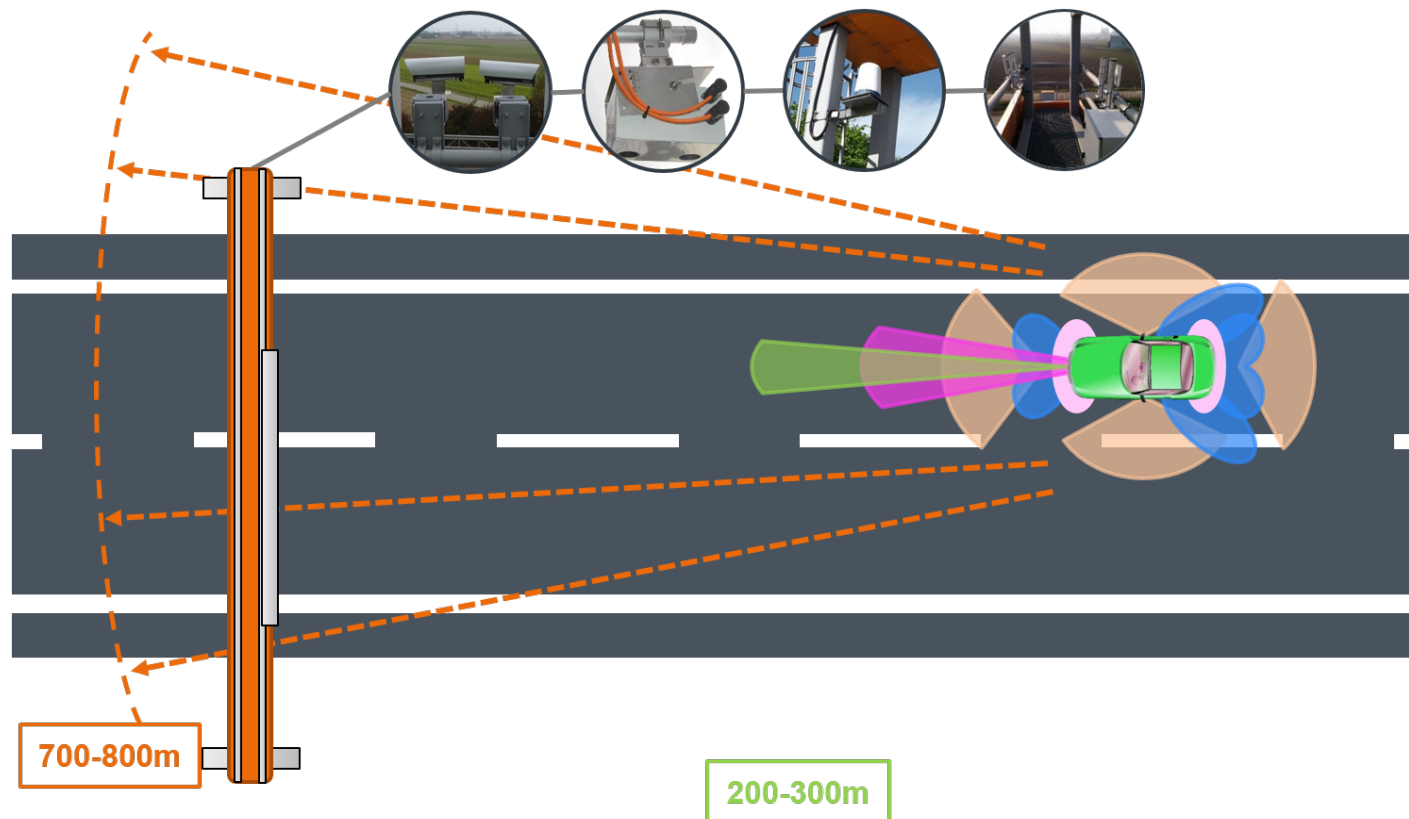
Sensor fusion taxonomies
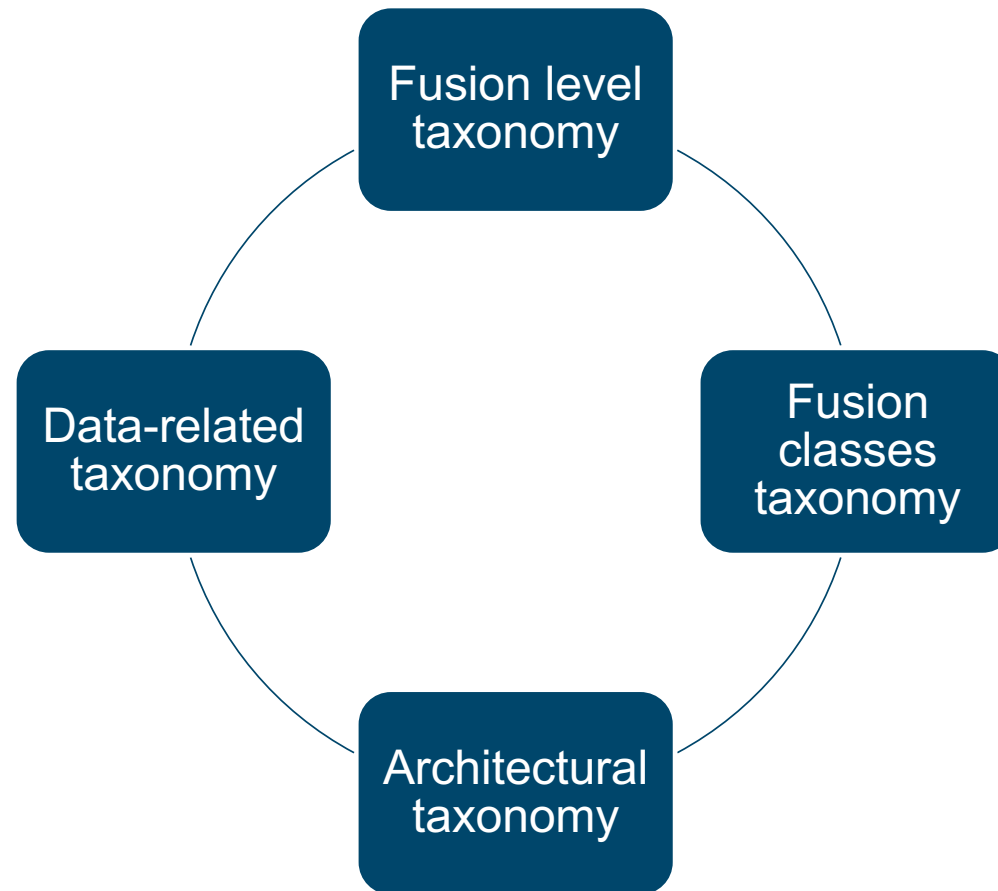
Fusion level taxonomy

- Fusion is typically divided into three levels of abstraction:
  - Low-level fusion
  - Intermediate-level fusion
  - High-level fusion

- They respectively fuse:
  - Signals (early)
  - Features and characteristics (deep)
  - Decisions (late)

Schematic depiction of fusion levels (Stüker, Heterogene Sensordatenfusion zur robusten Objektverfolgung im automobilen Straßenverkehr, 2016)

Comparison

Aspect	Early Fusion	Deep Fusion	Late Fusion
Definition	Raw sensor data (LiDAR, camera, radar) is combined before processing.	Features from sensors are fused at multiple processing stages.	Decisions from separate sensor models are combined at the end.
Advantages	Captures sensor interdependencies early and is better for low-latency tasks.	Balances early and late fusion benefits while adapting to complex scenarios.	Modular, flexible, and easier to implement.
Disadvantages	High computational cost and complexity with high-dimensional data.	More complex design and requires careful tuning.	Misses early sensor synergies and has limited cross-sensor learning.
Best Use Cases	Real-time sensor fusion for low-latency decisions.	Complex environments needing adaptive fusion.	Tasks with well-defined, independent sensor models.
Examples	Combining raw LiDAR and camera data for object detection.	Adaptive fusion of LiDAR, radar, and camera for dynamic driving scenarios.	Combining separate LiDAR and camera model outputs for final decisions.

[Source: K. Huang et al., Multi-modal Sensor Fusion for Auto Driving Perception: A Survey, CVPR, 2024, https://arxiv.org/abs/2202.02703]

Fusion class taxonomy

- Competitive fusion
  - is used when redundant sensors measure the same quantity, in order to reduce the overall uncertainty

- Complementary fusion
  - is used when sensors provide a complementary information about the environment, for instance distance sensors with different ranges

- Cooperative fusion
  - is used when the required information can not be inferred from a single sensor (e.g. GPS localization and stereo vision)
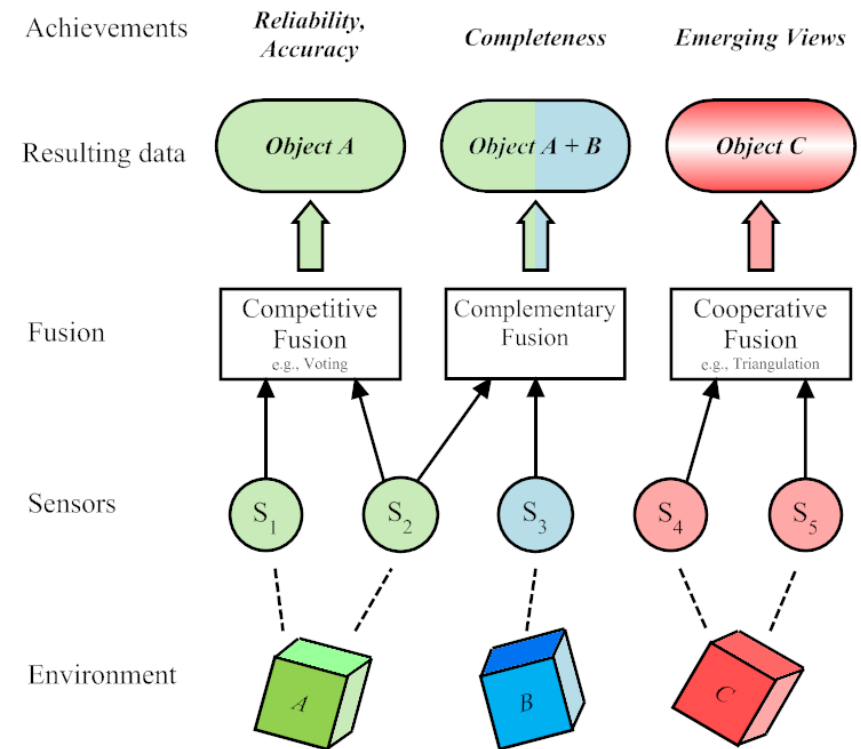
Architectural taxonomy

- The **centralized** architecture is theoretically optimal, but scales badly with respect to communication and processing
- The **decentralized** architecture is a collection of autonomous centralized systems, and has the same scaling issues
- The **distributed** architecture scales better, but can lead to information loss because each sensor processes its information locally
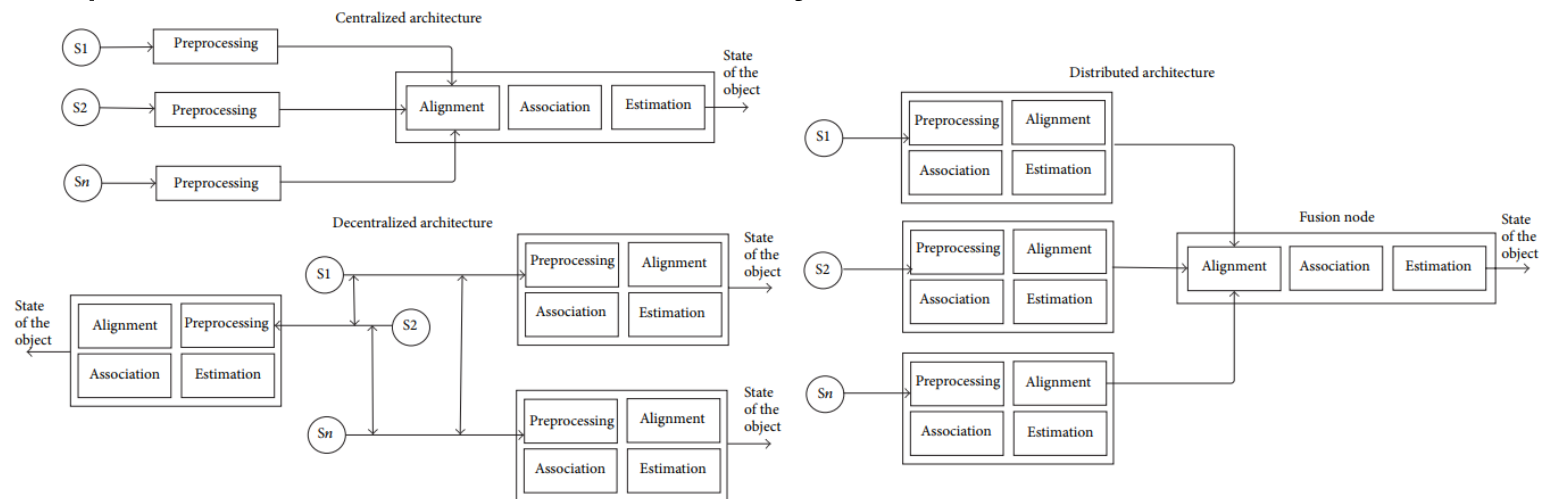
Data-related taxonomy

- Inherent imperfection of the sensory data
- The data-related taxonomy provides us with a checklist of underlying data issues and how to deal with them
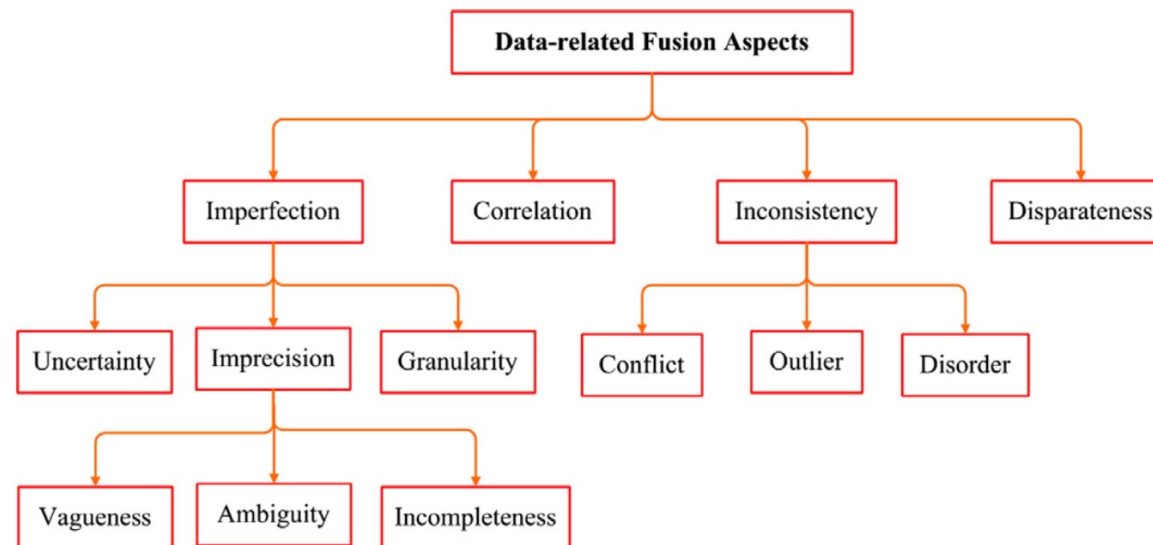
Data-related taxonomy

- Sensory data makes a statement about the environment
  - "The distance to the nearest car is 35.12 m"

- Due to the inherent data imprecision, we have to deal with:
  - **Uncertainty:** The distance to the nearest car is more than 20 m with 80% probability
  - **Vagueness:** The distance to the nearest car is more than 20 m with 80% probability, and we are 90% confident in this statement
  - **Ambiguity**
  - **Incompleteness**
- The underlying data can contain multiple imperfections at once

Bayesian statistics in multi-sensor data fusion

- **Basic premise**: all unknowns are treated as random variables and the knowledge of these quantities is summarized via a probability distribution
  - This includes the observed data, any missing data, noise, unknown parameters, and models
- Bayesian statistics provides
  - a framework for quantifying objective and subjective uncertainties
  - principled methods for model estimation and comparison and the classification of new observations
  - a natural way to combine different sensor observations
  - principle methods for dealing with missing information

Sensor fusion – a simple example

- Problem: determine the distance to n objects using measurements from two sensors

- Assumptions:
  - Both sensors have the same field of view
  - First sensor has a higher precision than the second sensor
  - Consider the simplest case (n=1)

- How to fuse these measurements properly?

Sensor fusion – a simple example

- Sensors provide redundant measurements of the same physical quantity (distance)
- To incorporate the precision information → measurements are assumed to be normally distributed random variables
- Specifically, the univariate Gaussian distributions are:

$$d_1(x) = (2\pi\sigma_1^2)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\frac{(x-\mu_1)^2}{\sigma_1^2}\right) \quad \sim \mathcal{N}(\mu_1, \sigma_1^2)$$

$$d_2(x) = (2\pi\sigma_2^2)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\frac{(x-\mu_2)^2}{\sigma_2^2}\right) \quad \sim \mathcal{N}(\mu_2, \sigma_2^2)$$

Sensor fusion – a simple example

- Assumption from before:
  - First sensor has a higher precision than the second sensor
- This can be captured as: $\sigma_1^2 < \sigma_2^2$
- Problem is to find $d(x) \sim \mathcal{N}(\mu, \sigma^2)$
- The idea is to combine the previous Gaussian distributions

$$d(x) = d_1(x) \cdot d_2(x) = (4\pi^2 \sigma_1^2 \sigma_2^2)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\left(\frac{(x - \mu_1)^2}{\sigma_1^2} + \frac{(x - \mu_2)^2}{\sigma_2^2}\right)\right)$$

Sensor fusion – a simple example

- Re-arranging the expression in the exponent and dividing the numerator and denominator by $(\sigma_1^2 + \sigma_2^2)$:

$$-\frac{1}{2}\left(\frac{(x-\mu_1)^2}{\sigma_1^2} + \frac{(x-\mu_2)^2}{\sigma_2^2}\right) = -\frac{1}{2}\frac{(\sigma_1^2+\sigma_2^2)x^2 - 2(\sigma_2^2\mu_1+\sigma_1^2\mu_2)x + (\sigma_2^2\mu_1^2+\sigma_1^2\mu_2^2)}{\sigma_1^2\sigma_2^2}$$

$$= -\frac{1}{2}\frac{x^2 - 2\frac{\mu_1\sigma_2^2+\mu_2\sigma_1^2}{\sigma_1^2+\sigma_2^2}x + \frac{\mu_1^2\sigma_2^2+\mu_2^2\sigma_1^2}{\sigma_1^2+\sigma_2^2}}{\frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}}$$

- To obtain an expression of form $x^2 - 2\mu x + \mu^2 = (x-\mu)^2$ in the numerator, it is necessary to add and subtract the square of the second term

Sensor fusion – a simple example

$$-\frac{1}{2}\frac{x^2 - 2\frac{\mu_1\sigma_2^2+\mu_2\sigma_1^2}{\sigma_1^2+\sigma_2^2}x + \left(\frac{\mu_1\sigma_2^2+\mu_2\sigma_1^2}{\sigma_1^2+\sigma_2^2}\right)^2 - \left(\frac{\mu_1\sigma_2^2+\mu_2\sigma_1^2}{\sigma_1^2+\sigma_2^2}\right)^2 + \frac{\mu_1^2\sigma_2^2+\mu_2^2\sigma_1^2}{\sigma_1^2+\sigma_2^2}}{\frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}}$$

- The expression in the exponent becomes

$$-\frac{1}{2}\frac{(x-\mu)^2 - \mu^2 + s}{\sigma^2} = -\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2} + \frac{\mu^2 - s}{2\sigma^2}$$

Sensor fusion – a simple example

- Putting everything together leads to the final distribution which represents the fused information
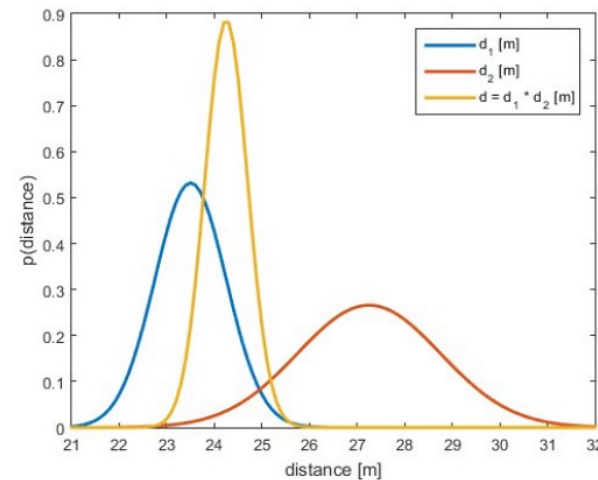
$$
\begin{aligned}
d(x) \quad &= (2\pi\sigma_1\sigma_2)^{-1} \exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2} + \frac{\mu^2-s}{2\sigma^2} \right) \\
&= (2\pi\sigma_1\sigma_2)^{-1} \exp\left(\frac{\mu^2-s}{2\sigma^2} \right) \exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2} \right) \\
&= C \cdot \exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2} \right)
\end{aligned}
$$

Sensor fusion – a simple example

- Mean value and variance are

$$\mu = \frac{\mu_1 \sigma_2^2 + \mu_2 \sigma_1^2}{\sigma_1^2 + \sigma_2^2}$$

$$\sigma^2 = \frac{\sigma_1^2 \cdot \sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

- The fused value is the weighted average of the measurements
- The weighting favors the sensor with higher precision
- The overall uncertainty decreases

Kalman filter (KF) – again

- Assumption #1: linear dynamics $x_t = A_t x_{t-1} + B_t u_t + \epsilon_t$
  - i.i.d. process noise ϵ_t is $\mathcal{N}(0, R_t)$
  - Assumption #1 implies that the probabilistic generative model is Gaussian

- Assumption #2: linear measurement model $z_t = C_t x_t + \delta_t$
  - i.i.d. measurement noise δ_t is $\mathcal{N}(0, Q_t)$
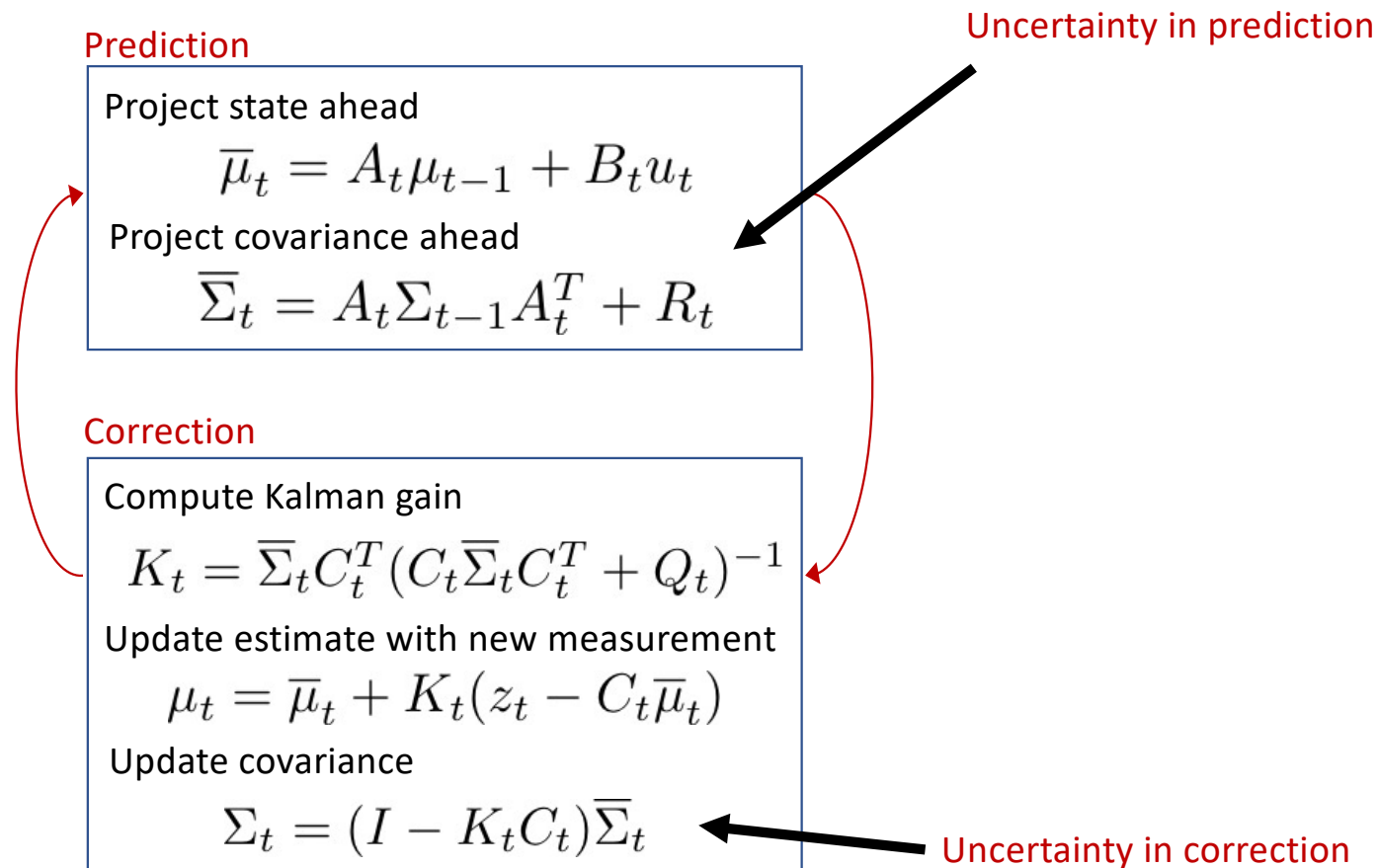 - Assumption #2 implies that the measurement probability is Gaussian

Kalman filter (KF)
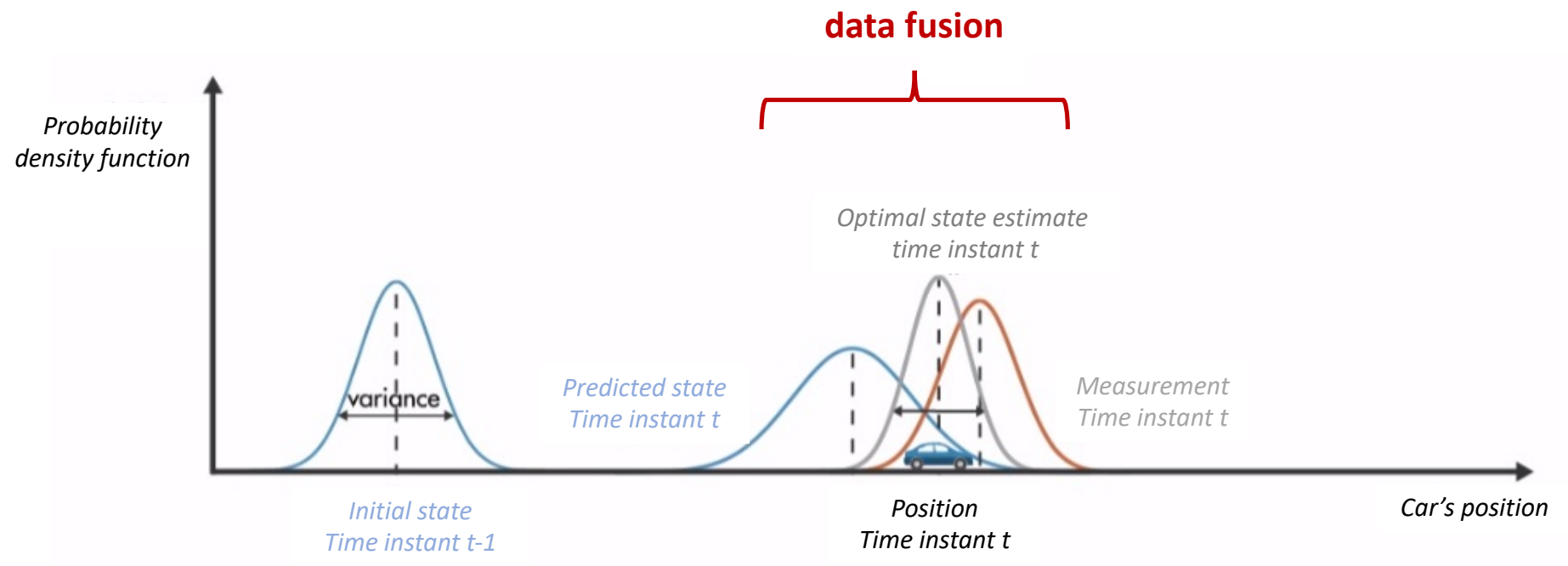
- Assumption #3: the initial belief is Gaussian

$$bel(x_0) = p(x_0) = \det(2\pi\Sigma_0)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(x_0 - \mu_0)^T \Sigma_0^{-1}(x_0 - \mu_0)\right)$$

- Key fact: These three assumptions ensure that the posterior $bel(x_t)$ is Gaussian for all t, i.e., $bel(x_t) = \mathcal{N}(\mu_t, \Sigma_t)$
- Note:
  - KF implements a belief computation for continuous states
  - Gaussians are unimodal → commitment to single-hypothesis filtering

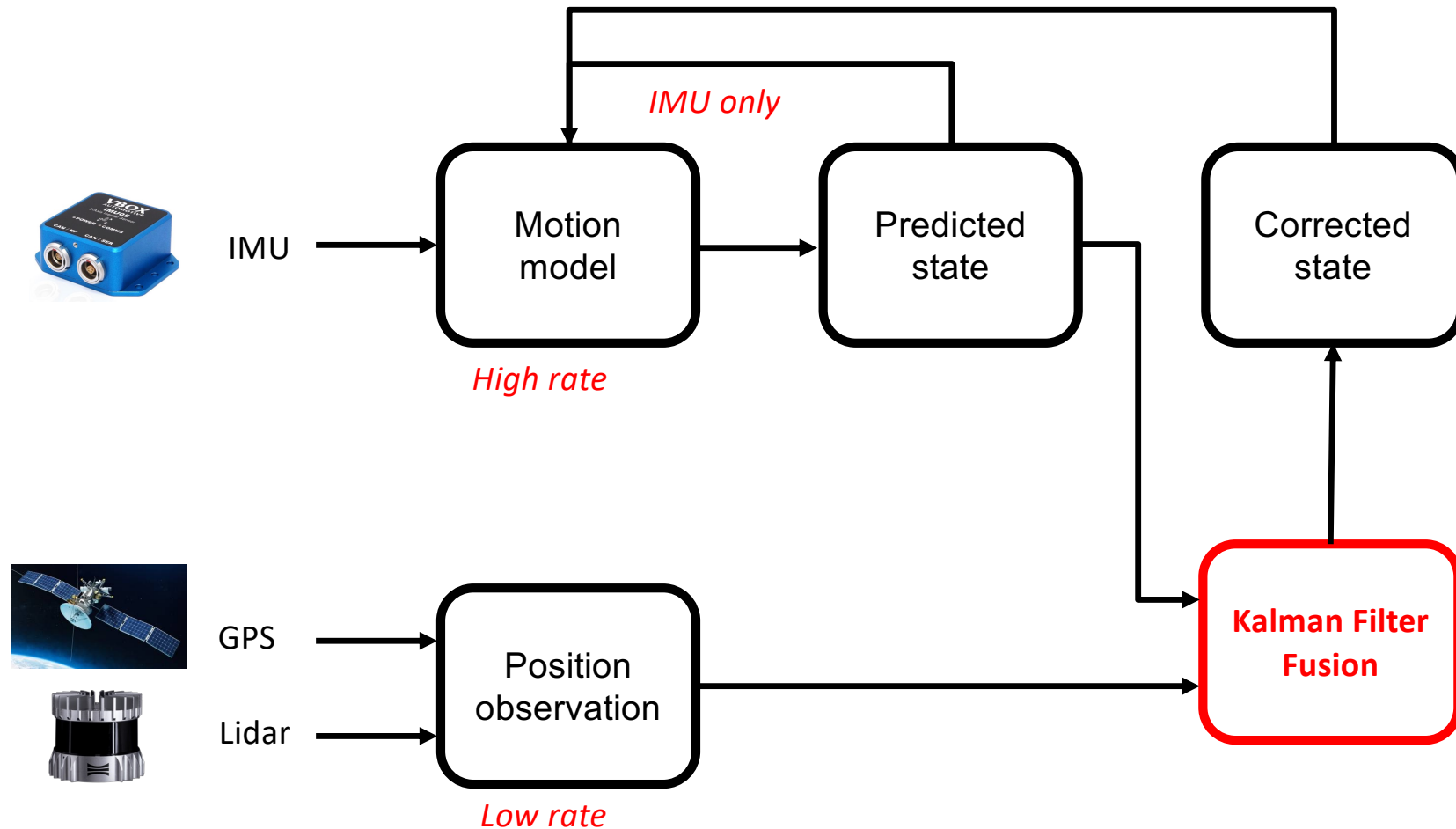# Kalman filter: algorithm revisited

Uncertainty in prediction

Prediction

Project state ahead
$$\overline{\mu}_t = A_t \mu_{t-1} + B_t u_t$$

Project covariance ahead
$$\overline{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$$

Correction

Compute Kalman gain
$$K_t = \overline{\Sigma}_t C_t^T (C_t \overline{\Sigma}_t C_t^T + Q_t)^{-1}$$

Update estimate with new measurement
$$\mu_t = \overline{\mu}_t + K_t(z_t - C_t \overline{\mu}_t)$$

Update covariance
$$\Sigma_t = (I - K_t C_t)\overline{\Sigma}_t$$

Uncertainty in correction

Kalman filter (KF) – pose estimation

Kalman filter (KF) – multi-sensor fusion

IMU → Motion model → Predicted state → Kalman Filter Fusion → Corrected state

IMU only

High rate

GPS, Lidar → Position observation → Kalman Filter Fusion

Low rate

Sensor fusion example

- Problem: Estimate position, velocity, and acceleration of a vehicle from noisy position and acceleration measurements
- Assumptions:
  - Single track model for the vehicle
  - Lidar provides position measurements with low precision
  - GPS provides position measurements with high precision
  - IMU provides acceleration measurements
- Sensor fusion is done using the Kalman filter

Sensor fusion algorithm Kalman filter
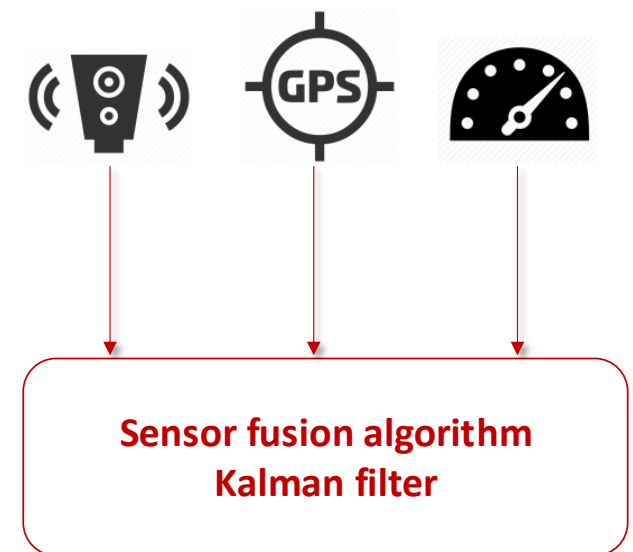
Sensor fusion example: Motion model

- State vector: $\mu_t = \begin{bmatrix} p & v & a \end{bmatrix}^T$
- Change of the state over time is captured by the motion model

$$p_t = p_{t-1} + T_s v_{t-1} + \frac{T_s^2}{2} a_{t-1} + \epsilon_{pt}$$

$$v_t = \qquad\qquad\qquad v_{t-1} + T_s a_{t-1} + \epsilon_{vt}$$

$$a_t = \qquad\qquad\qquad\qquad\qquad a_{t-1} + \epsilon_{at}$$

- T_s represents sampling time

Sensor fusion example: Motion model

- The motion model can be represented in matrix form

$$\begin{bmatrix} p \\ v \\ a \end{bmatrix}_t = \begin{bmatrix} 1 & T_s & \frac{T_s^2}{2} \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ v \\ a \end{bmatrix}_{t-1} + \begin{bmatrix} \epsilon_p \\ \epsilon_v \\ \epsilon_a \end{bmatrix}_t$$

State vector   State transition matrix   Process noise

$$\mu = A_t \mu_{t-1} + \epsilon_t$$

where ϵ_t is i.i.d. process noise distributed as $\mathcal{N}(0, R_t)$

Sensor fusion example: Measurement model

- The measurement model defines a mapping from the state space to the measurement space

- For this example, two possible fusion scenarios will be considered:

    1. Lidar + IMU

    2. Lidar + GPS + IMU

- In the first scenario, only measurements from Lidar and IMU are available
    - Assumption: Lidar provides low precision measurements (noisy data)

- In the second scenario, high precision GPS measurements are also available

Sensor fusion example: Measurement model

- First scenario – measurement model is given by

$$
\begin{bmatrix} p_{lidar} \\ a_{imu} \end{bmatrix}_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ v \\ a \end{bmatrix}_t + \begin{bmatrix} \delta_{lidar} \\ \delta_{imu} \end{bmatrix}_t
$$

Measurement vector   *Measurement matrix*   *State vector*   *Measurement noise*

$$z_t = C_t \mu_t + \delta_t$$

where δ_t is i.i.d. measurement noise distributed as $\mathcal{N}(0, Q_t)$

Sensor fusion example: Initialization

- Choosing the initial state vector μ_0 - depends on available information
  - If there is *a-priori* knowledge – initialization is done with known values
  - If there is a lack of information – initial state is chosen to be zero
  - For this example the initial state vector is set to zero

$$\mu_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- Choosing the initial covariance matrix Σ_0 - should be defined based on the initialization error
  - If the initial state is not very close to the correct state - Σ_0 will have large values
  - If the initial state is close to the correct state - Σ_0 will have small values

$$\Sigma_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sensor fusion example: Noise model tuning

- The process noise covariance matrix R_t - describes the confidence in the system model
  - Small values indicate higher confidence – predicted values are more weighted
  - Large values indicate lower confidence – measurements become dominant
- The measurement noise covariance matrix Q_t - describes the confidence in the measurements
  - Has a similar interpretation as R_t
- Both matrices need to be symmetric and positive definite

$$R_t = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.05 \end{bmatrix} \qquad Q_t = \begin{bmatrix} \sigma_{lidar}^2 & 0 \\ 0 & \sigma_{imu}^2 \end{bmatrix}$$

Sensor fusion example: Position estimation
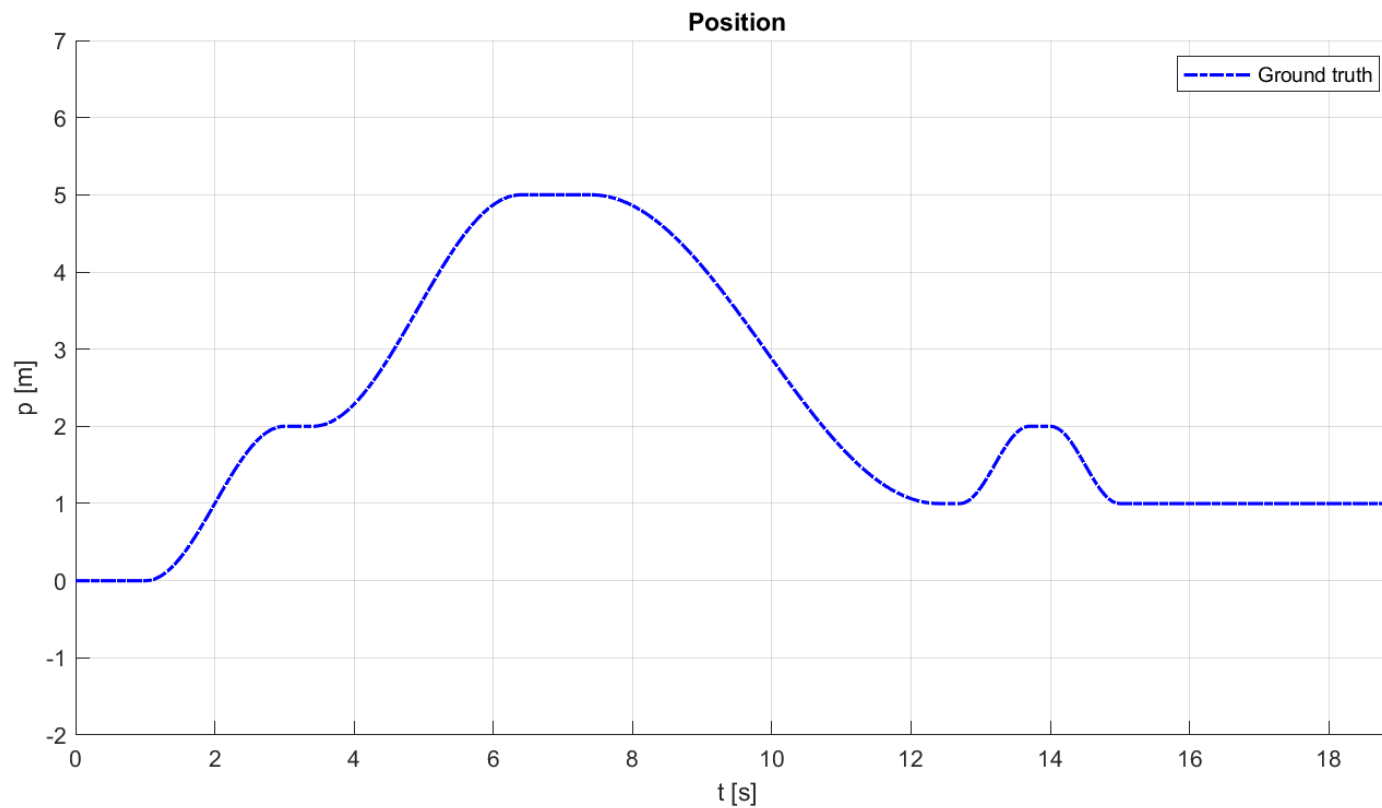
Sensor fusion example: Position estimation

Sensor fusion example: Position estimation

Position

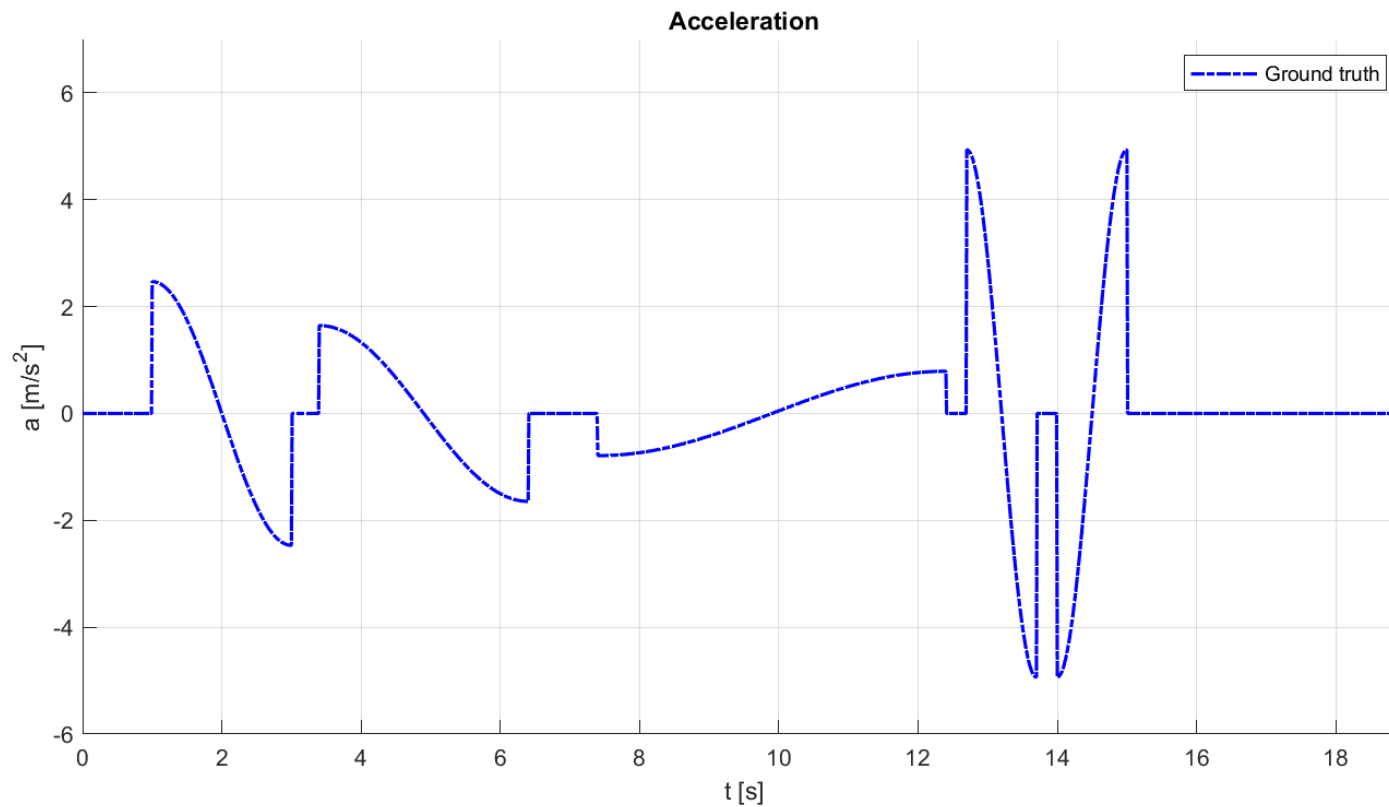Estimate is very noisy because of the poor Lidar measurements
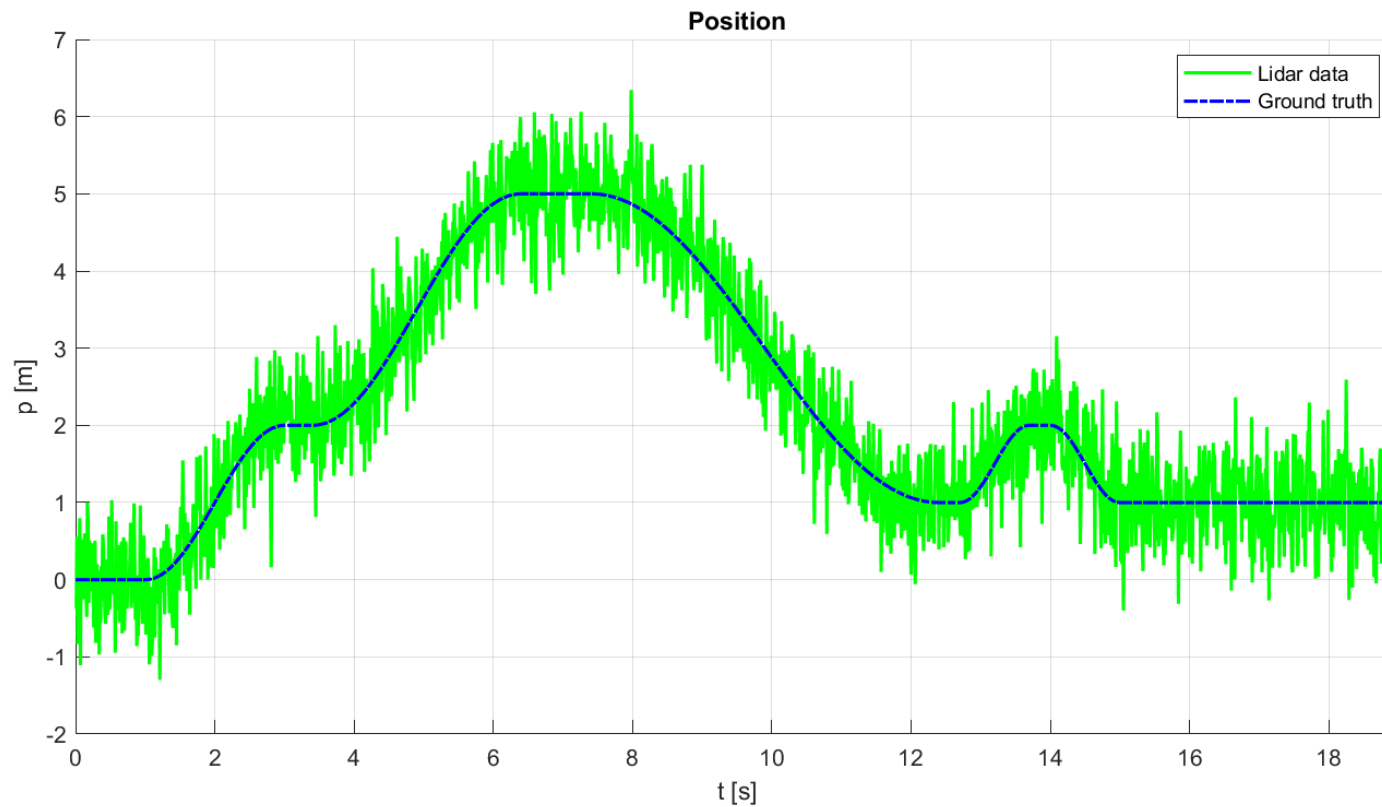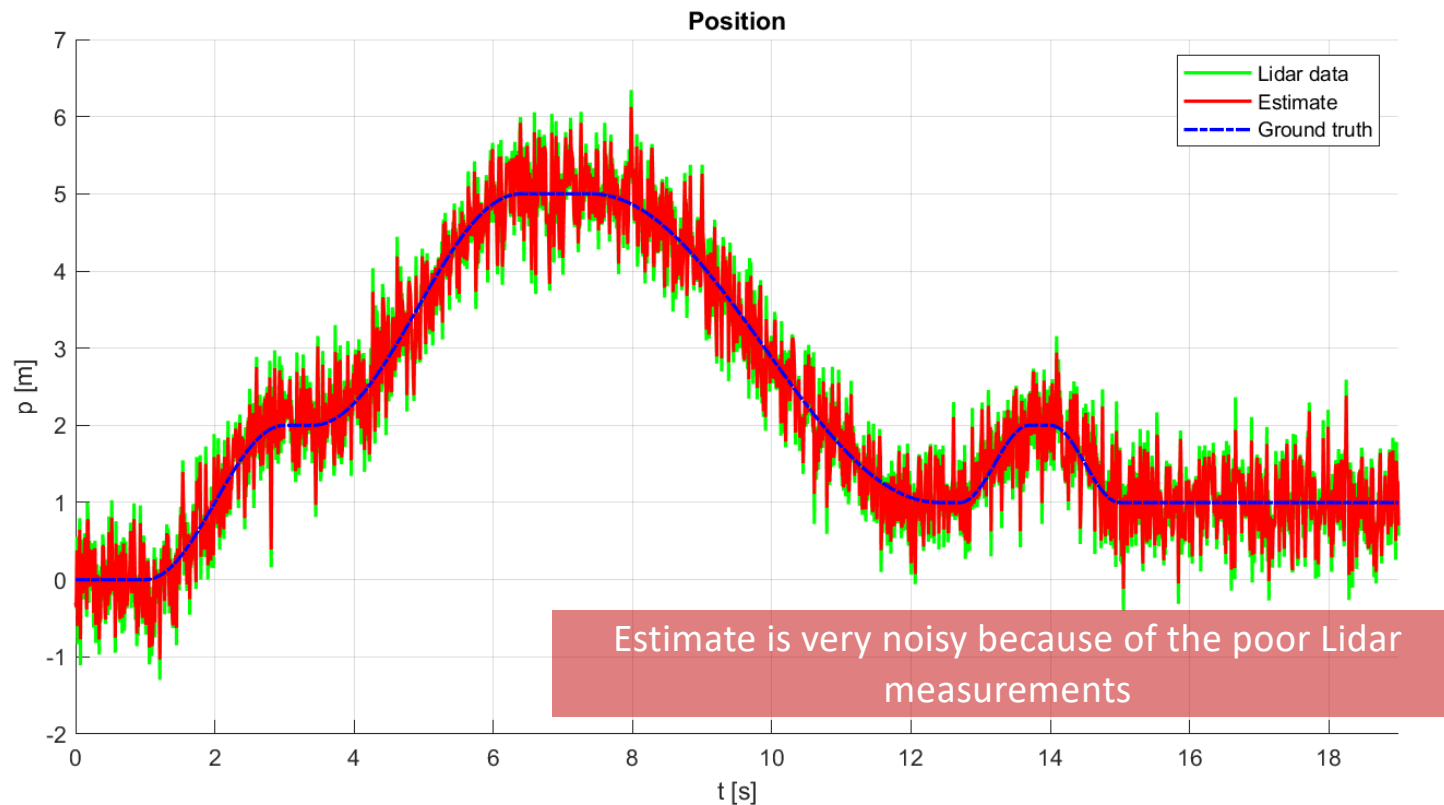
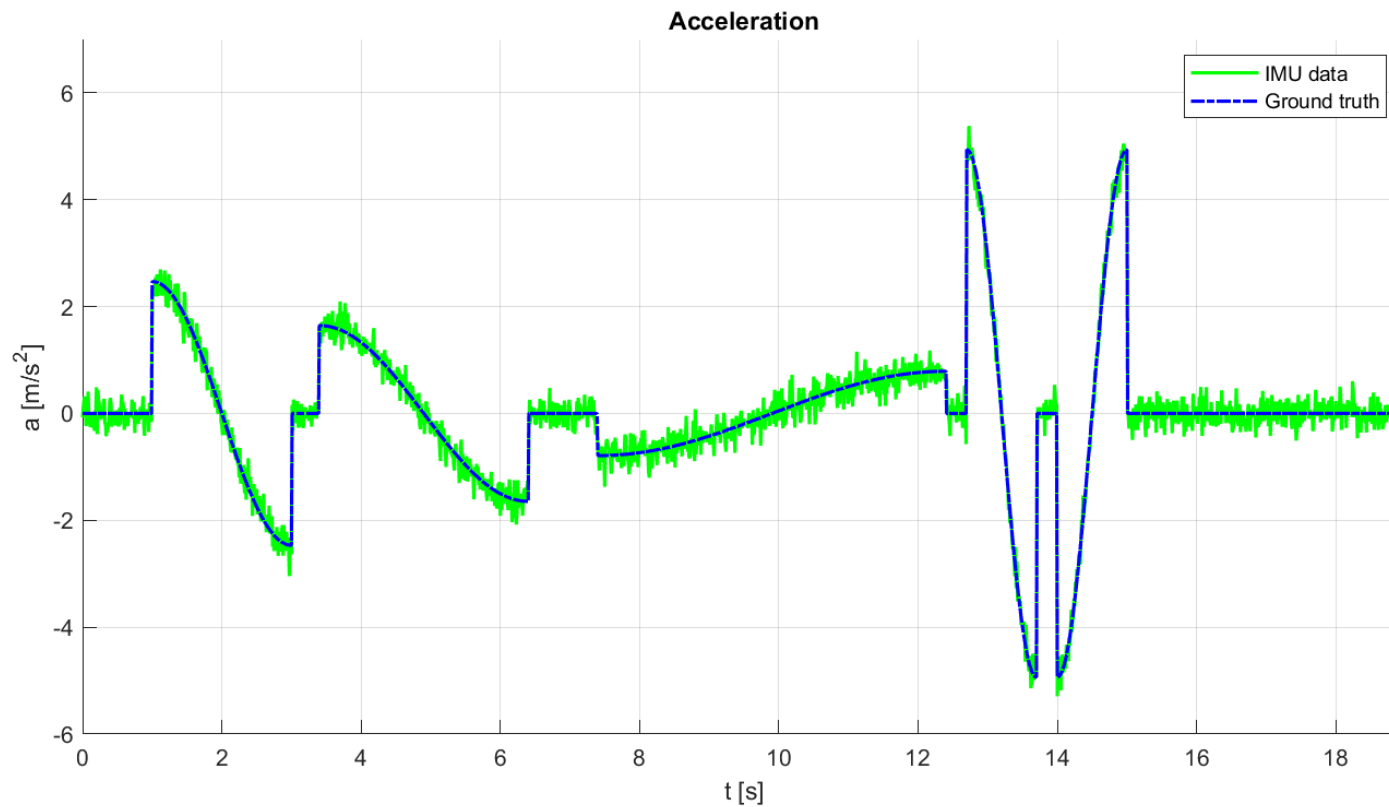# Sensor fusion example: Velocity estimation
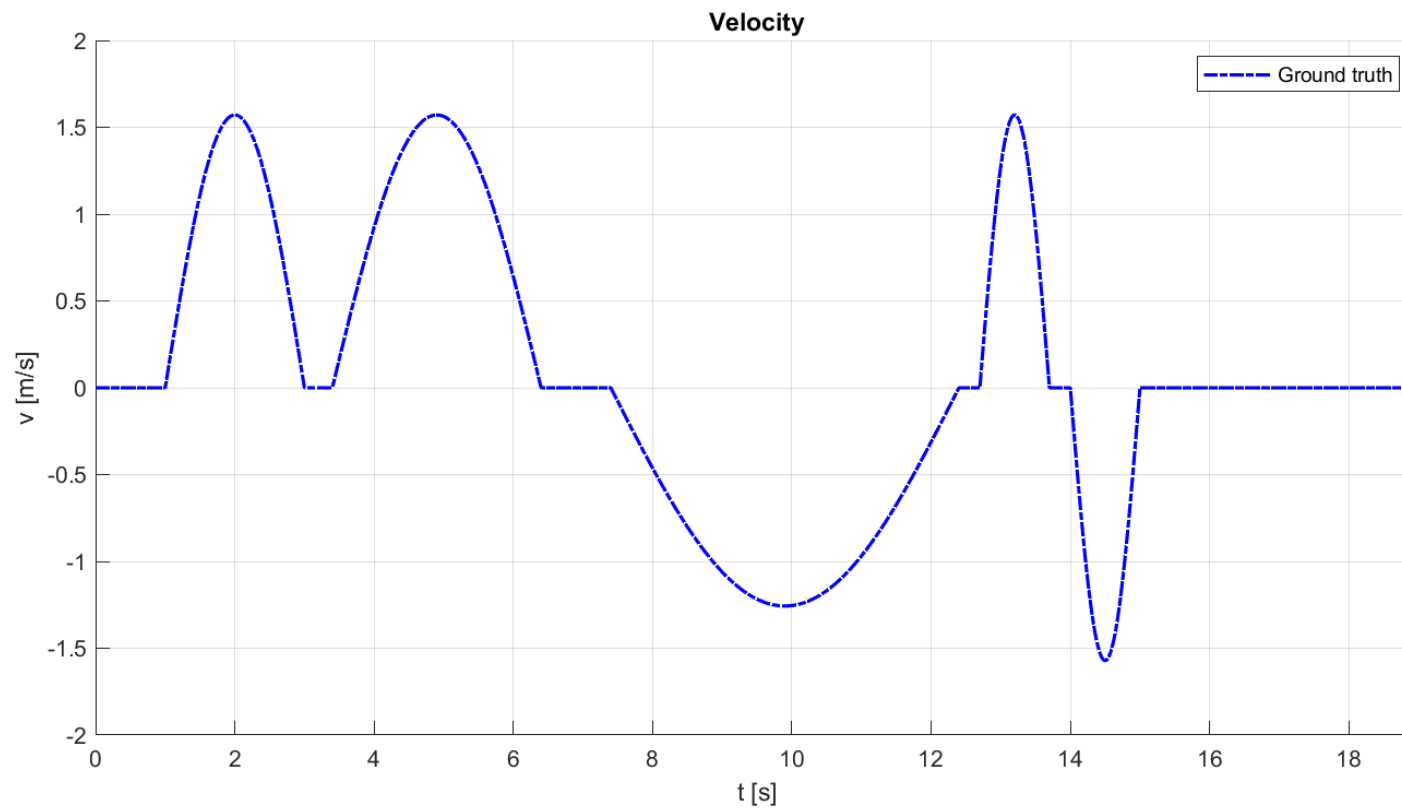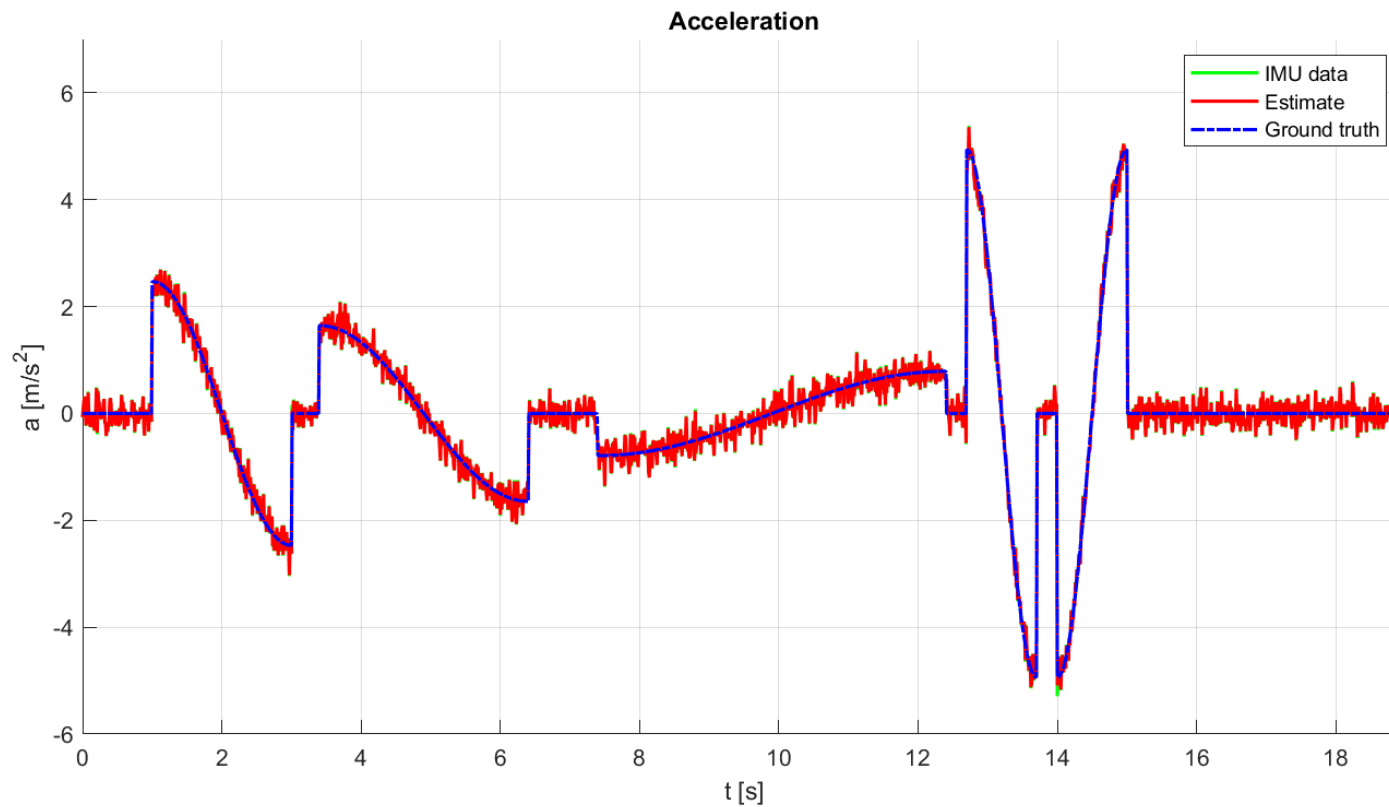
Sensor fusion example: Velocity estimation
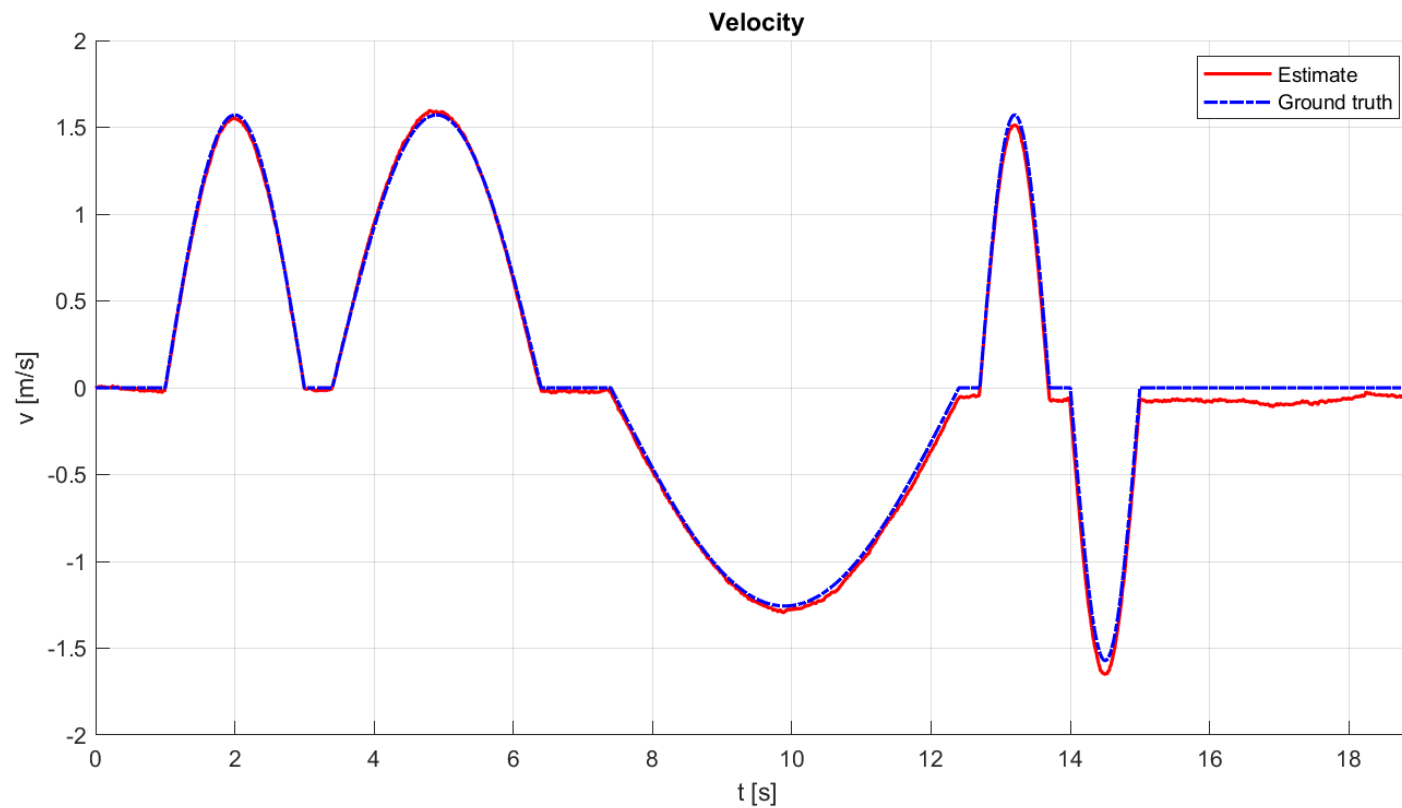
Sensor fusion example: Acceleration estimation

Sensor fusion example: Acceleration estimation

Sensor fusion example: Acceleration estimation

Sensor fusion example: New measurement model

- In the previous scenario – the position estimate is quite noisy (because of the low precision of the Lidar measurements)
- Now: position is measured with Lidar <u>and</u> GPS

$$
\begin{bmatrix} p_{lidar} \\ p_{gps} \\ a_{imu} \end{bmatrix}_t = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ v \\ a \end{bmatrix}_t + \begin{bmatrix} \delta_{lidar} \\ \delta_{gps} \\ \delta_{imu} \end{bmatrix}_t
$$

$$
\color{red} z_t = C_t \mu_t + \delta_t
$$

Sensor fusion example: Noise model tuning

- The measurement noise covariance matrix Q_t for this scenario has an additional GPS variance

$$Q_t = \begin{bmatrix} \sigma_{lidar}^2 & 0 & 0 \\ 0 & \sigma_{gps}^2 & 0 \\ 0 & 0 & \sigma_{imu}^2 \end{bmatrix} = \begin{bmatrix} 0.5^2 & 0 & 0 \\ 0 & 0.1^2 & 0 \\ 0 & 0 & 0.2^2 \end{bmatrix}$$
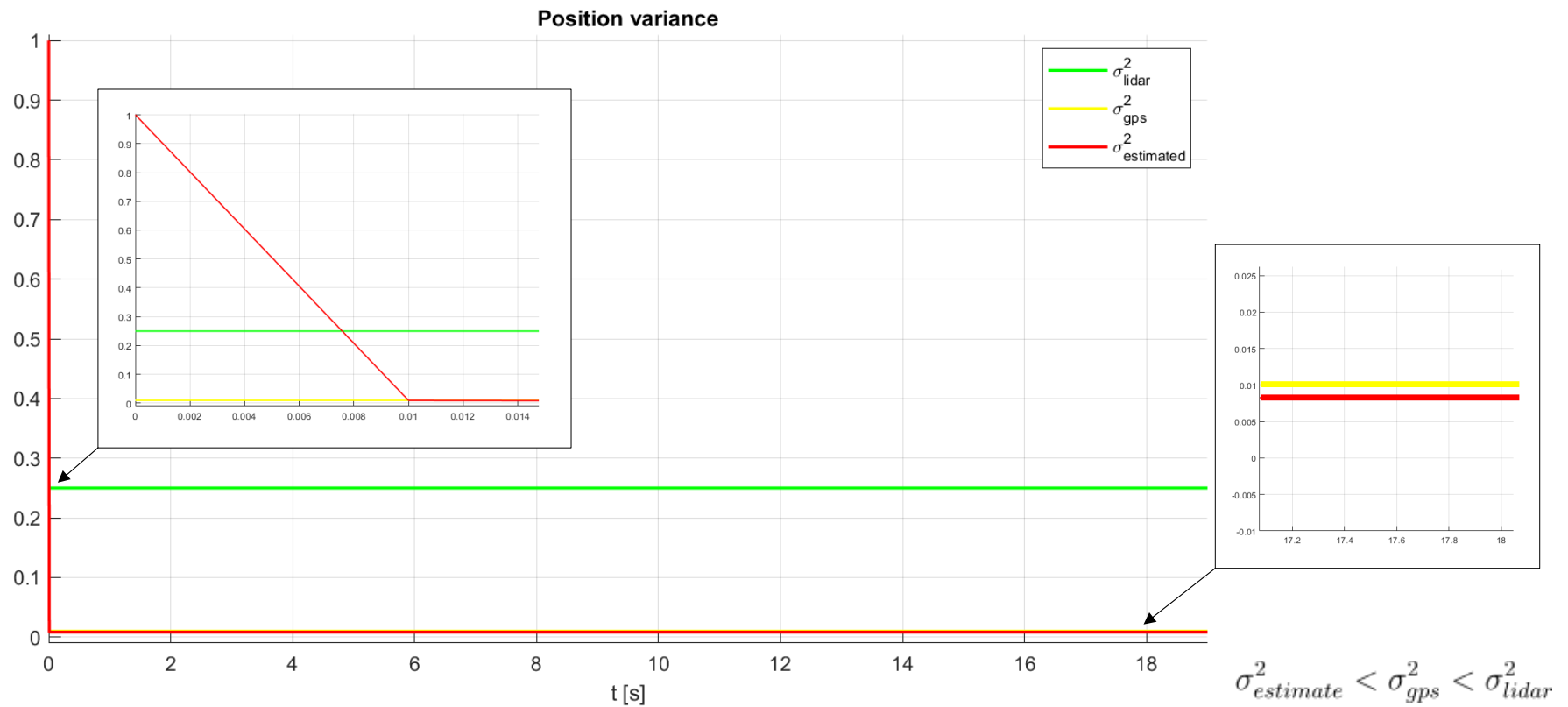
Sensor fusion example: Position estimation

Sensor fusion example: Position estimation

Combining Lidar and GPS data results in a more accurate estimation
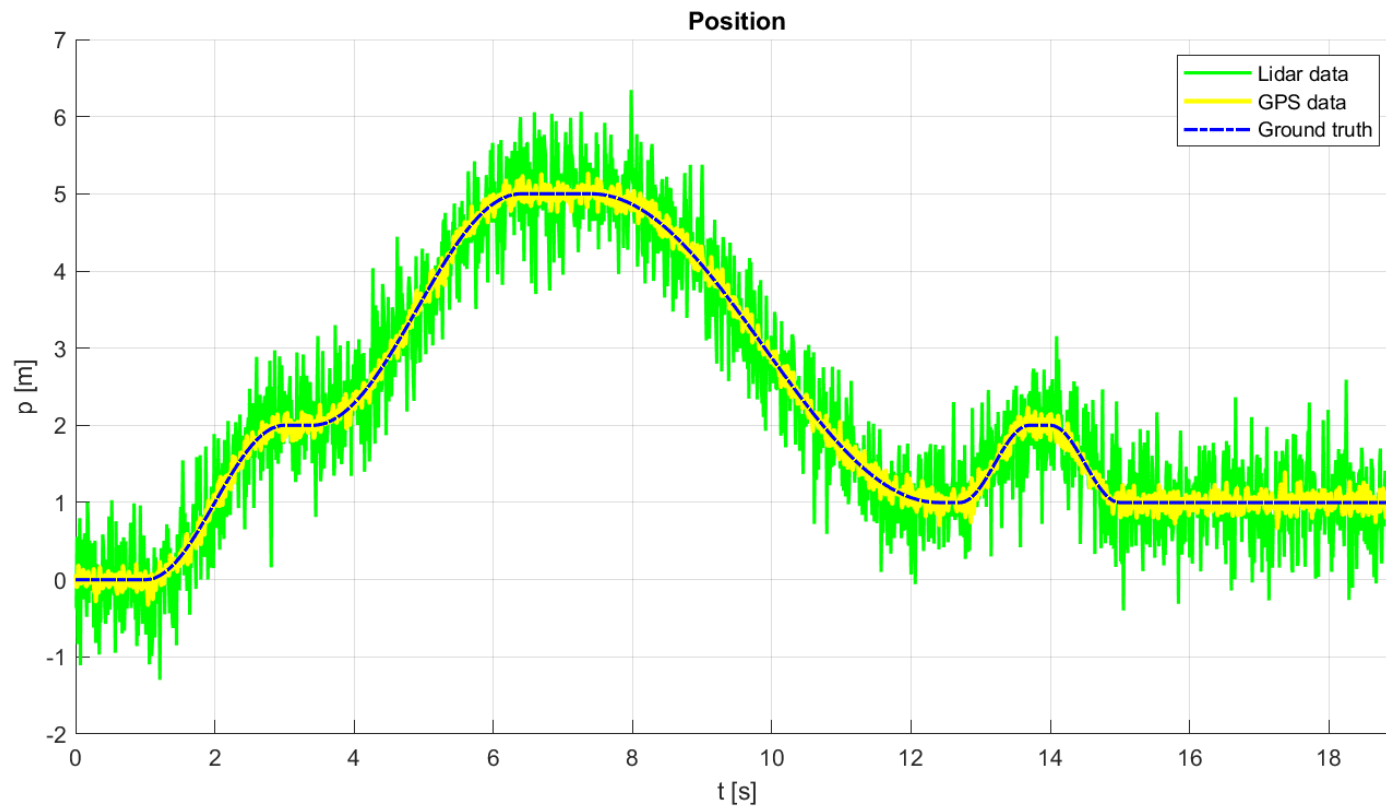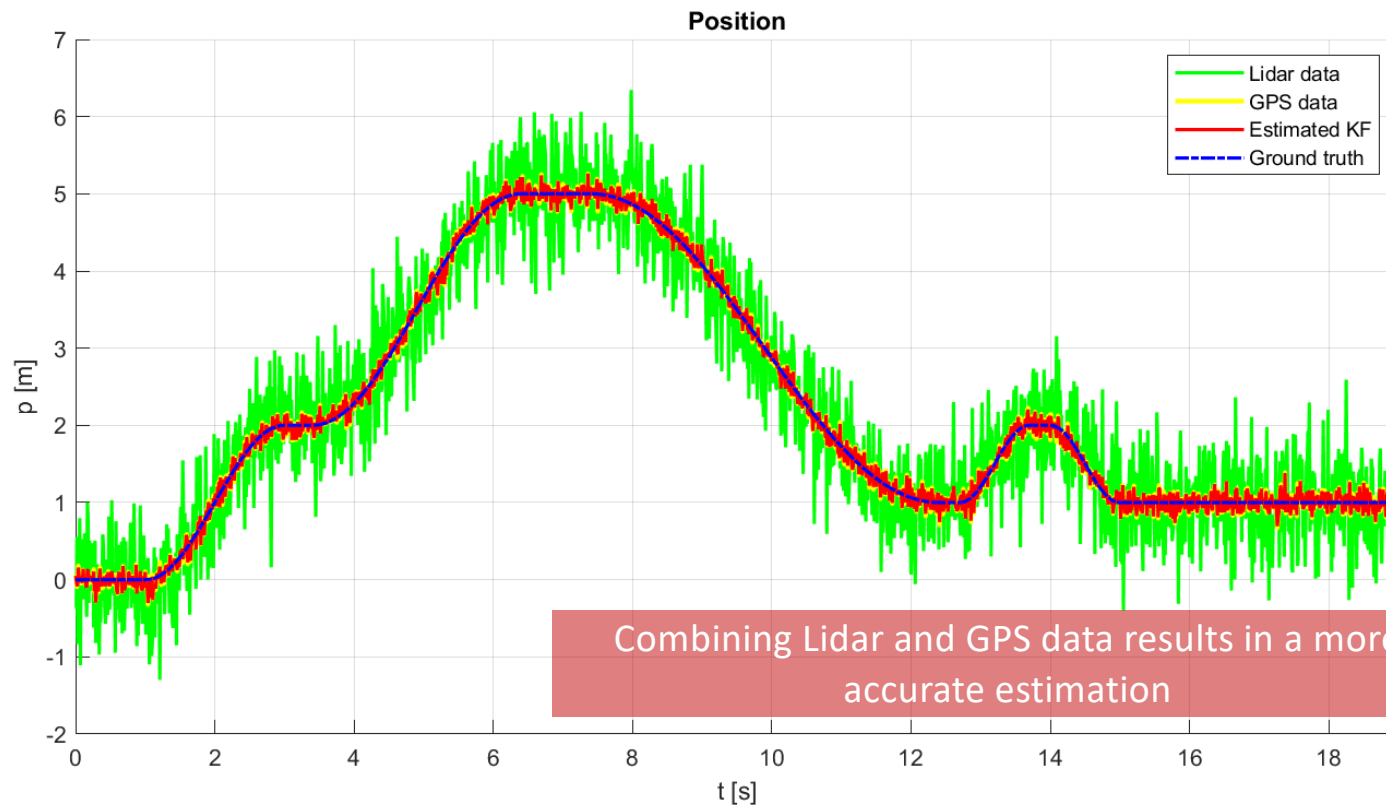
Sensor fusion example: Position variance

Position variance

$$\sigma_{estimate}^2 < \sigma_{gps}^2 < \sigma_{lidar}^2$$

Sensor fusion example: Conclusion

- Problem: Vehicle state estimation using EKF
- The example pointed out:
  - How to create a motion model and a measurement model
  - How to fuse the data from different types of sensors
  - How to set the initial state vector and the initial covariance matrix
  - How to chose appropriate values for process noise and measurement noise covariance matrices
  - How to achieve a more accurate state estimation by adding more sensors
 - How fusion of data decreases the overall estimation variance

Useful trick

- Augment the state vector with some auxiliary states and then apply the EKF to the augmented state space model

- What can we handle?
  - Colored state noise
  - Colored measurement noise
  - Sensor offset and drifts
  - Sensor faults (sudden offset)
  - Actuator fault (sudden offset)

Live demo / Autoware by M. Schratter

1. Lidar- and GNSS-based localization
2. Parameter variation (noisy data, bias, drift)

Multi-sensor approach in robotics

- Self-awareness (localization and positioning)
- **Situational awareness** (detection and tracking)
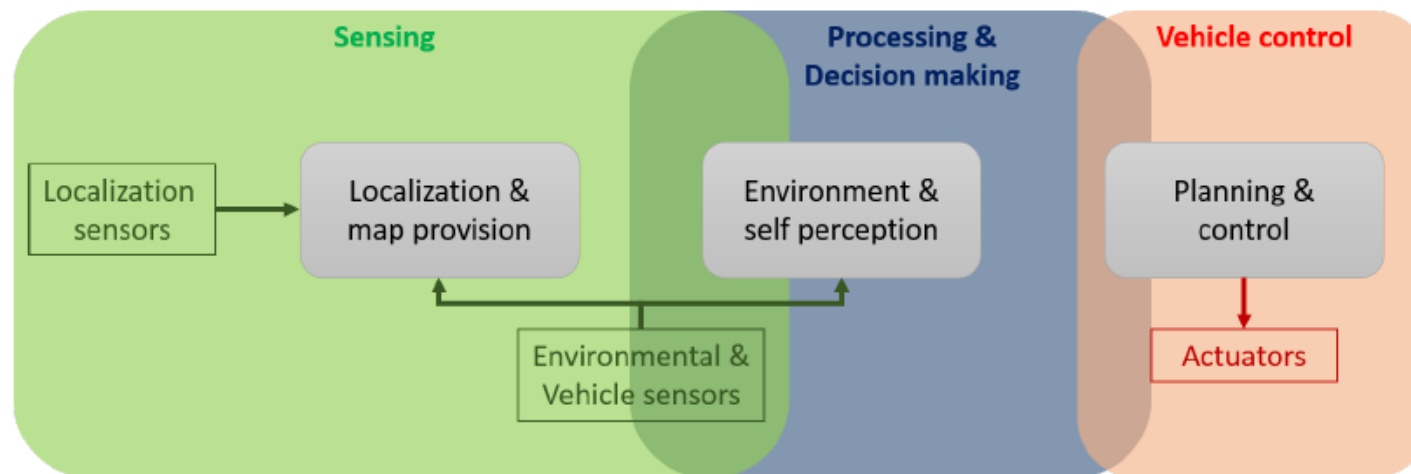
Modeling the environment

- Two types of algorithms are typically used (multiple sensors)
  - Object tracking algorithms
  - Occupancy grid algorithms
- Goal of object tracking algorithms
  - to determine the list of objects, which are currently present in the environment
  - to estimate their state variables
- Occupany grid approach
  - we describe the environment in a form of a discrete grid with certain height and width of the cells (fixed resolution step size)
  - each cell has a probability that it is occupied (or not), defined by sensor observations
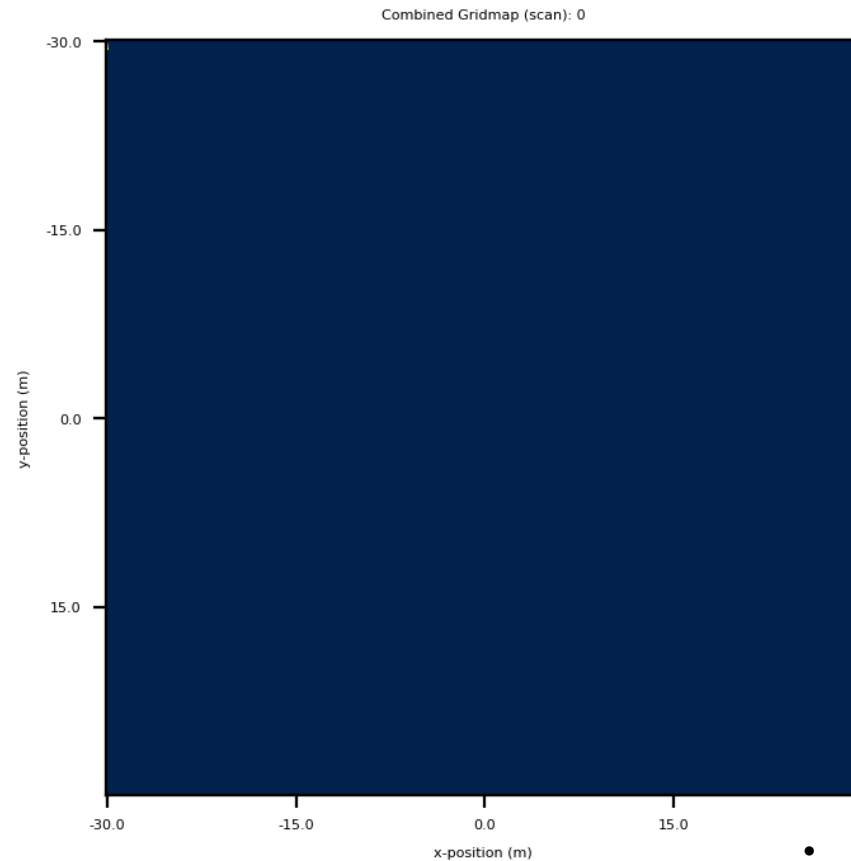
Tracking vs. occupancy grids
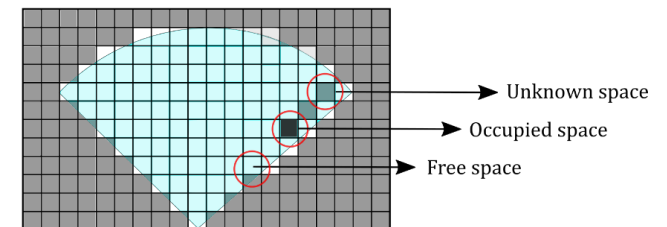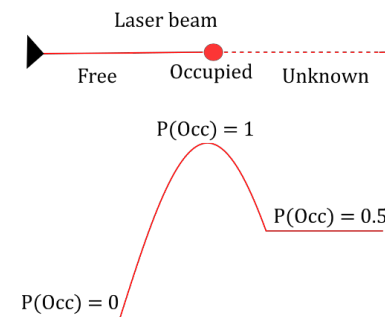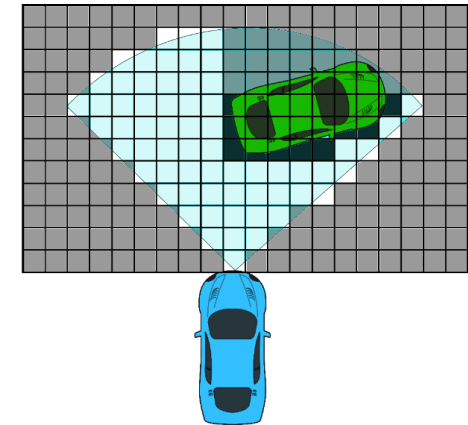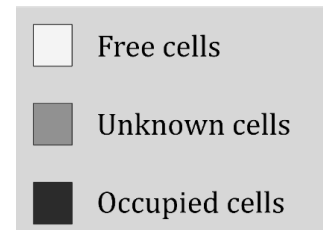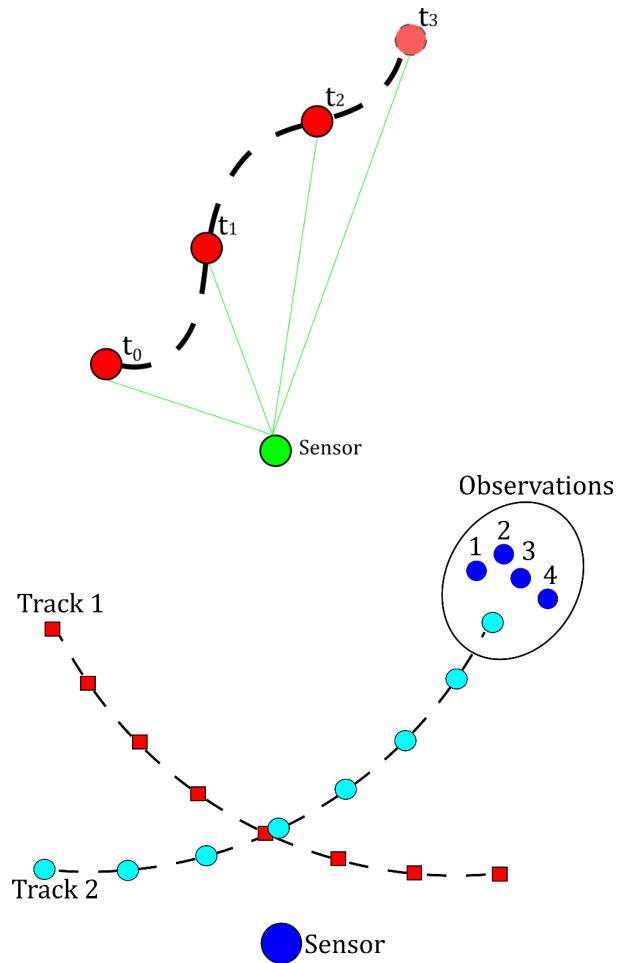
Occupancy grids
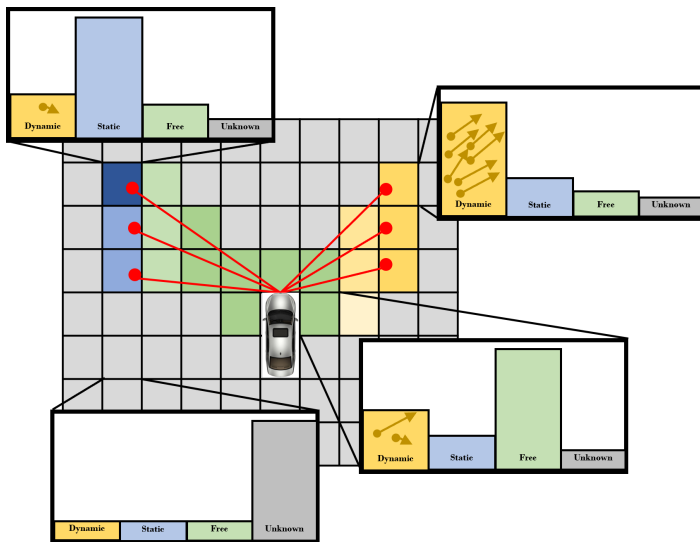
- Unknown: not observed by the sensor
- Free: not occupied by any object
- Static: occupied by a non-moving object
- Dynamic: occupied by a moving object (information like velocity and heading)

Combined Gridmap (scan): 0

- Blue: unknown
- Gray: free
- Brown: static
- Yellow: dynamic

Single object tracking

- Single model estimation filter (Kalman filter type)

Description of the system and the measurement models:

$$x_t = A_t x_{t-1} + B_t u_t + \epsilon_t$$

- Process noise ϵ_t is $\mathcal{N}(0, R_t)$

$$z_t = C_t x_t + \delta_t$$

- Measurement noise δ_t is $\mathcal{N}(0, Q_t)$

Prediction

Project state ahead

$$\overline{\mu}_t = A_t \mu_{t-1} + B_t u_t$$

Project covariance ahead

$$\overline{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$$

Correction

$$K_t = \overline{\Sigma}_t C_t^T (C_t \overline{\Sigma}_t C_t^T + Q_t)^{-1}$$
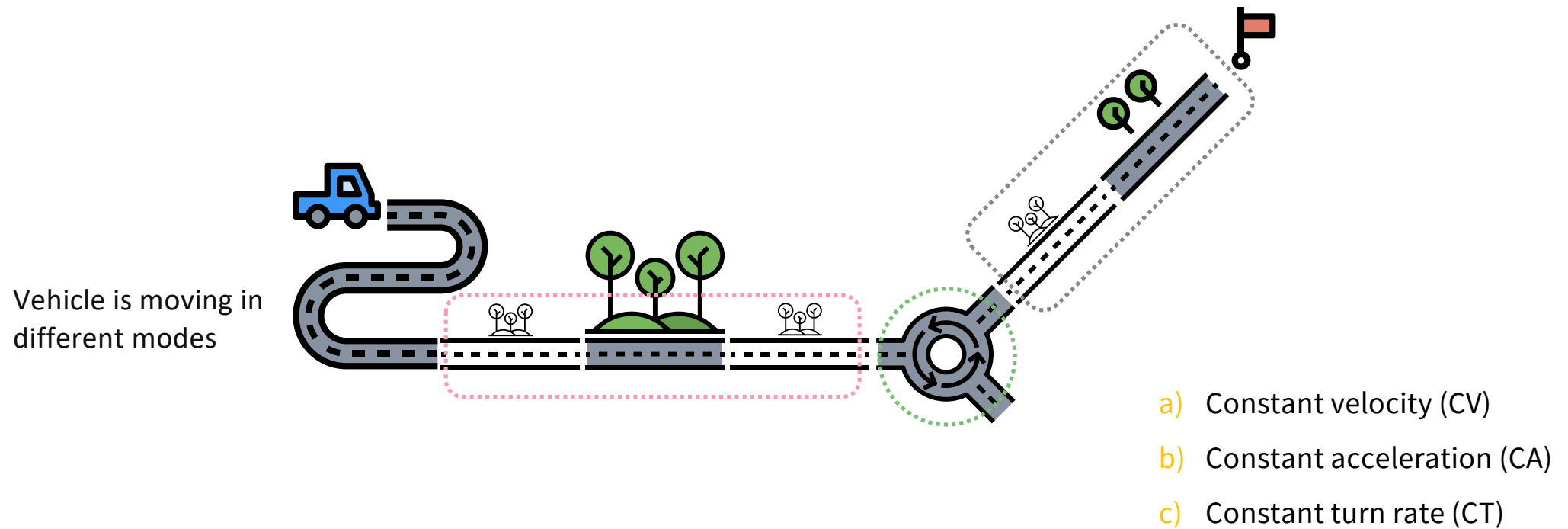
Update estimate with new measurement

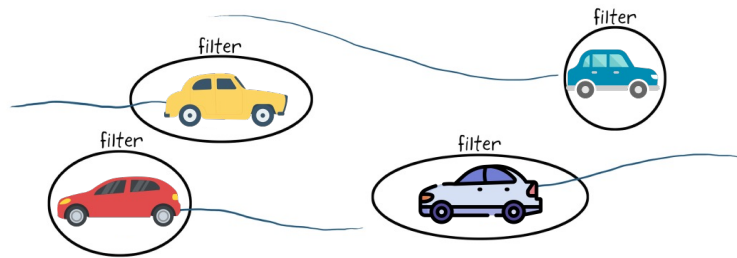$$\mu_t = \overline{\mu}_t + K_t(z_t - C_t \overline{\mu}_t)$$

Update covariance

$$\Sigma_t = (I - K_t C_t)\overline{\Sigma}_t$$

Single object tracking

- Vehicle model?

Vehicle is moving in
different modes

a) Constant velocity (CV)

b) Constant acceleration (CA)

c) Constant turn rate (CT)

Multi-object tracking

2. Data association problem

Match radar **detection** with closest **object**

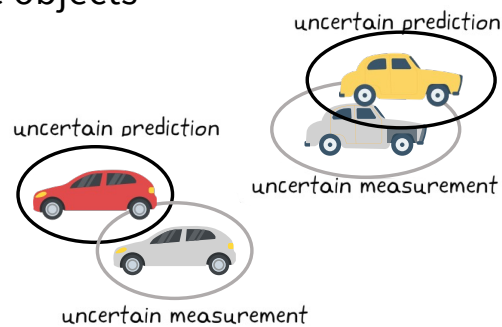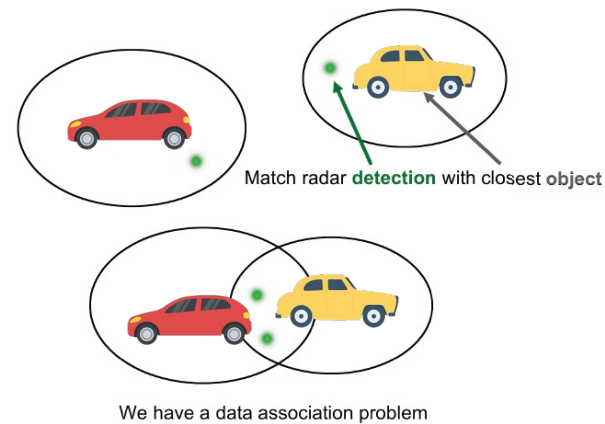We have a data association problem

3. Track maintenance

deletion

Track creation

The difficulty of multi-object tracking

1. Uncertainties in predictions and in measurements of the objects

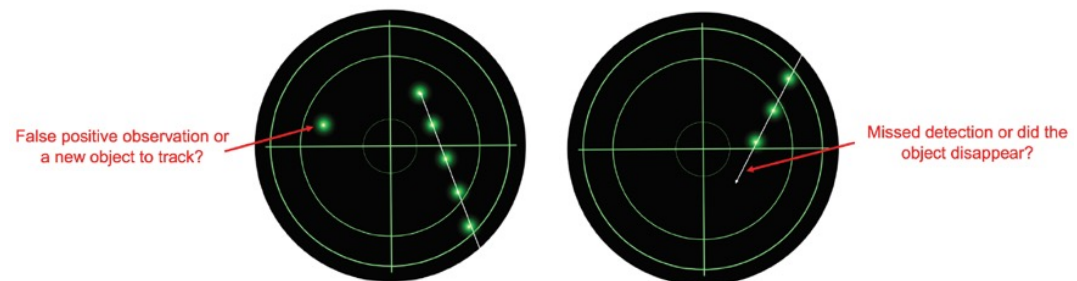uncertain prediction

uncertain prediction

uncertain measurement

uncertain measurement
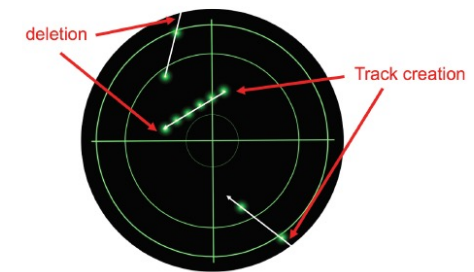
4. Track maintenance due to uncertainties

False positive observation or a new object to track?

Missed detection or did the object disappear?

Multi-object tracking

When tracking multiple objects:

- What are the ways to approach the data association problem?

- What are the ways to address the track maintenance problem?
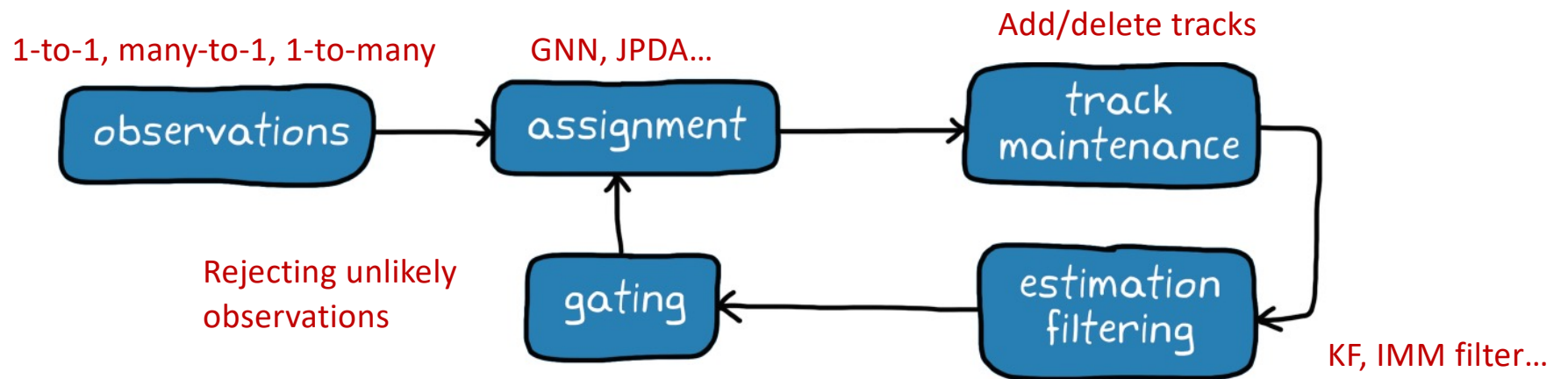
- **Multi-object tracking flow chart**

1-to-1, many-to-1, 1-to-many   GNN, JPDA…   Add/delete tracks

observations → assignment → track maintenance

Rejecting unlikely observations
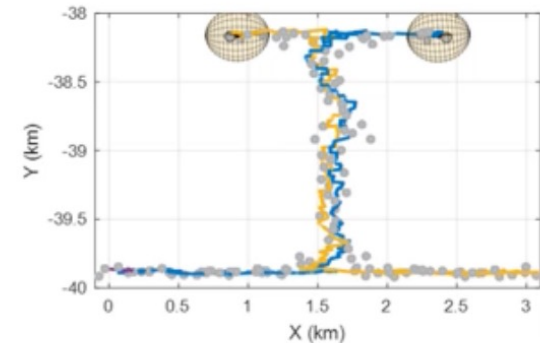
gating ← estimation filtering

KF, IMM filter…

Figure adapted from Design and Analysis of Modern Tracking Systems by Samuel Blackman and Robert Popoli (Artech House Radar Library).
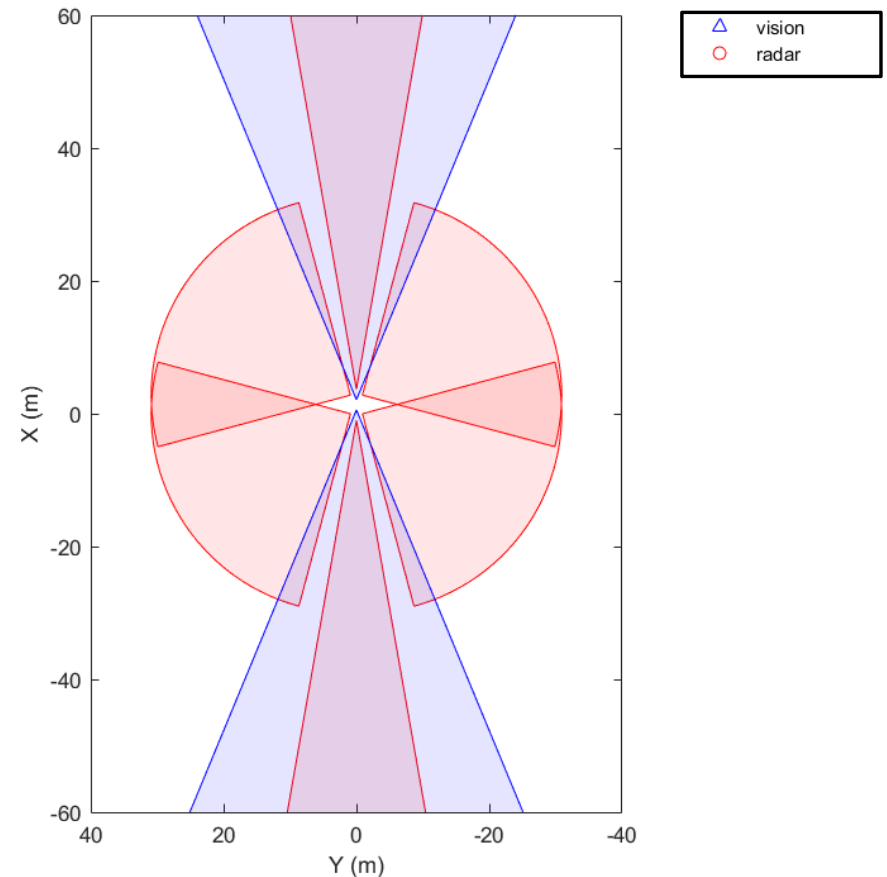
Sensor fusion using radar and vision data

Define Radar and Vision Sensors

Sensors defined / ego vehicle:

- 6 radar sensors:
  - 2 long-range radar sensors covering 20 degrees (in front and back),
  - 4 short-range radar sensors covering 90 degrees (two per side),
- 2 vision sensors:
  - Front-facing camera located at front windshield,
  - Rear-facing camera located at rear windshield,
- sensors have some overlap and some coverage gap.

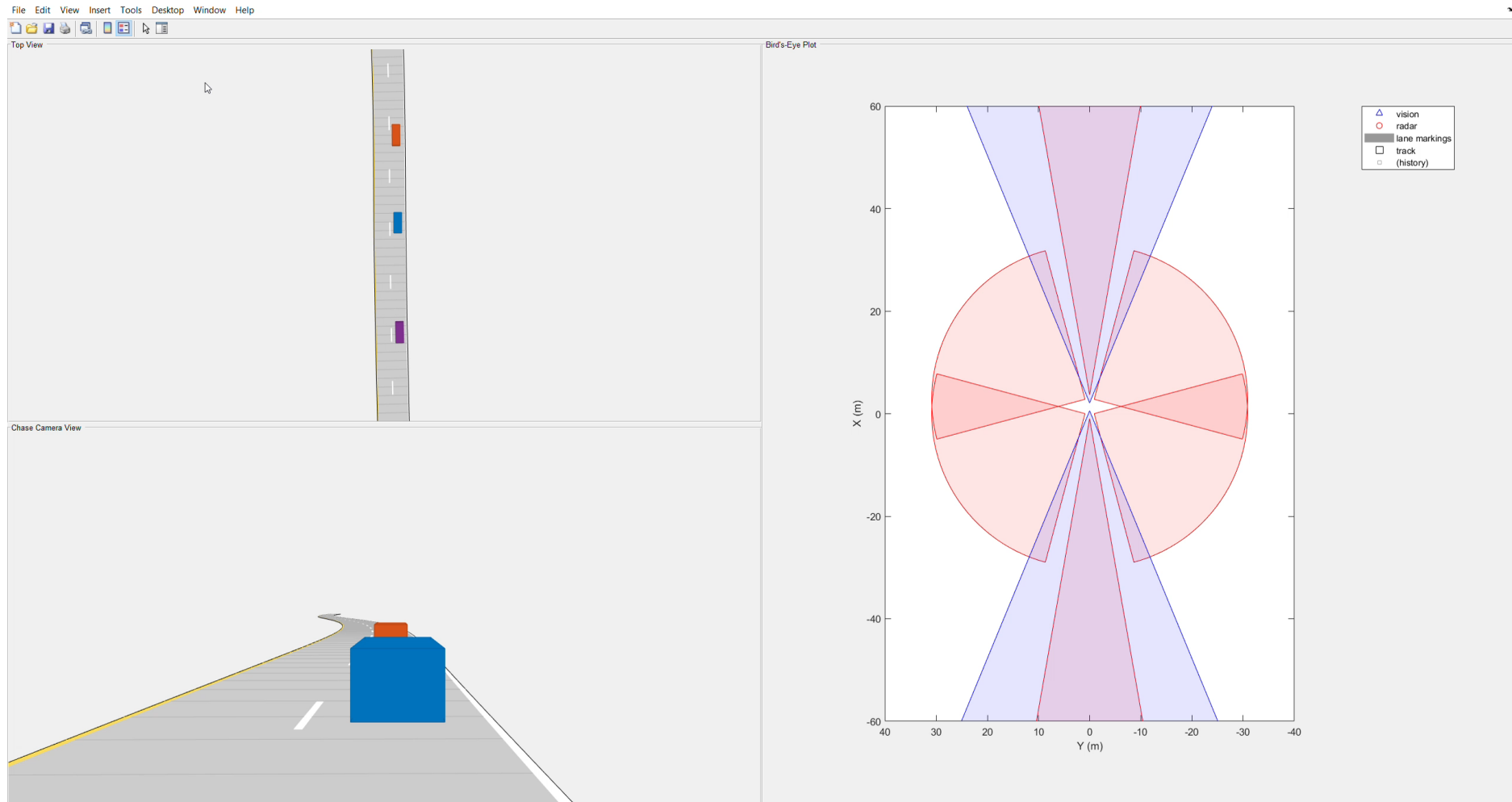# Sensor fusion using radar and vision data

Create a Tracker

- to track the vehicles that are close to the ego vehicle,
- Note:
  1. initialize the motion model
  2. initialize the EKF that works with position and velocity

What is the tracker responsible for?

A. Assigning detections to tracks.

B. Initializing new tracks based on unassigned detections. All tracks are initialized as 'Tentative', accounting for the possibility that they resulted from a false detection.

C. Confirming tracks if they have more than M assigned detections in N frames.

D. Updating existing tracks based on assigned detections.

E. Coasting (predicting) existing unassigned tracks.

F. Deleting tracks if they have remained unassigned (coasted) for too long

Sensor fusion using radar and vision data

Sensor fusion using radar and vision data
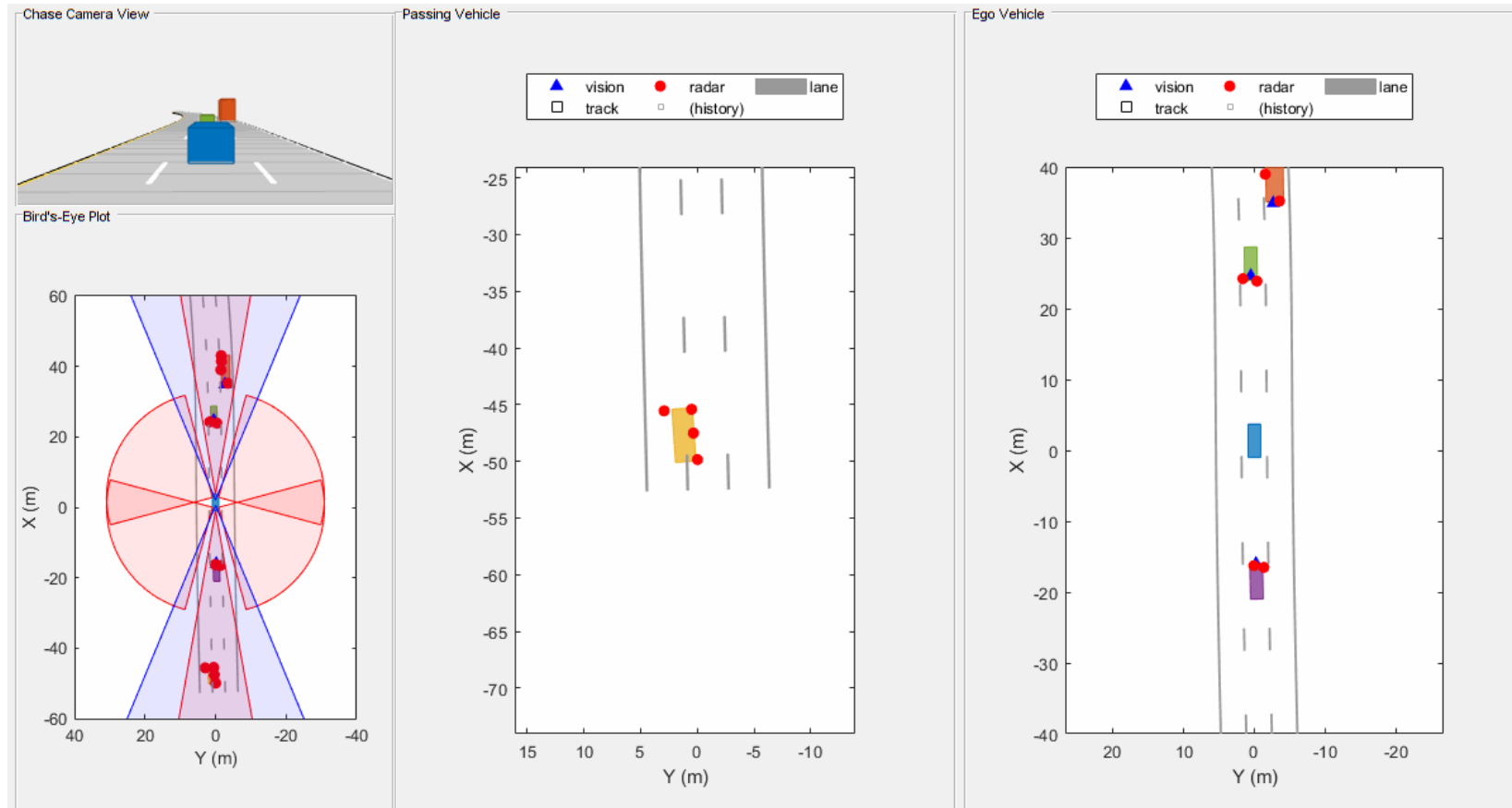
- Gaussian mixture phd tracker (here: MATLAB implementation)
- Can handle multiple detections per object per sensor (here: 6 radars, 2 cameras)
- It estimates the size and orientation of the object (along with pose and velocity)

Takeaways

- **Multiple target tracking (MTT), multi-object tracking (MOT)**
    - multiple detections from multiple targets,
    - use of one or more sensors,
    - one or more tracks are used to estimate the states of the targets.

- **Note: Extended object tracking**
    - high-resolution radar/lidar sensors,
    - can handle multiple detections per object.

Emerging mathematical frameworks - outlook

How to handle the unpredictable nature of real-world environments?

- **Dynamic Bayesian filtering: from static to dynamic uncertainty**
  - Standard: sensor uncertainty are static, pre-defined probabilities.
  - Dynamically adapt probability models based on sensor performance / environmental conditions.
  - E.g., robot moves into a shadowy corner → system adjusts the weighting of visual vs. Lidar data.
- **Context-aware filtering**
  - Standard KFs rely on fixed noise models.
  - Adaptive noise covariance matrices that evolve based on environmental conditions.
  - Effect: reduction of estimation error e.g., in scenarios involving rapid environmental changes.
- **Hybrid architectures**
  - They combine classical filtering with modern machine learning.
  - Use of neural networks to learn optimal filter parameters from experience, rather than manual tuning.
 - Effect: reduced calibration time while improving accuracy.

Sensor fusion attention mechanisms

- Combines data from multiple sensors by **dynamically weighting** their contributions depending on context, reliability, and relevance.

- Main benefits:
  - Prioritize high-quality signals
  - Suppress noisy or unreliable data
  - Adapt to changing environments
  - Improve robustness in edge cases

- Examples:
  - Rely more on LiDAR in low-light conditions
  - Switch focus to camera when fine texture is needed
  - Elevate radar signals in fog or rain

How it works (conceptually)

- Each sensor provides a feature representation
  - e.g., RGB image features, Lidar point cloud features, radar Doppler features
- The attention module computes similarity scores between sensor features (queries, keys, values).
- These scores determine which sensors or modalities contribute more to the fused output.
- The result is a context-dependent weighted fusion.

Possible architecture
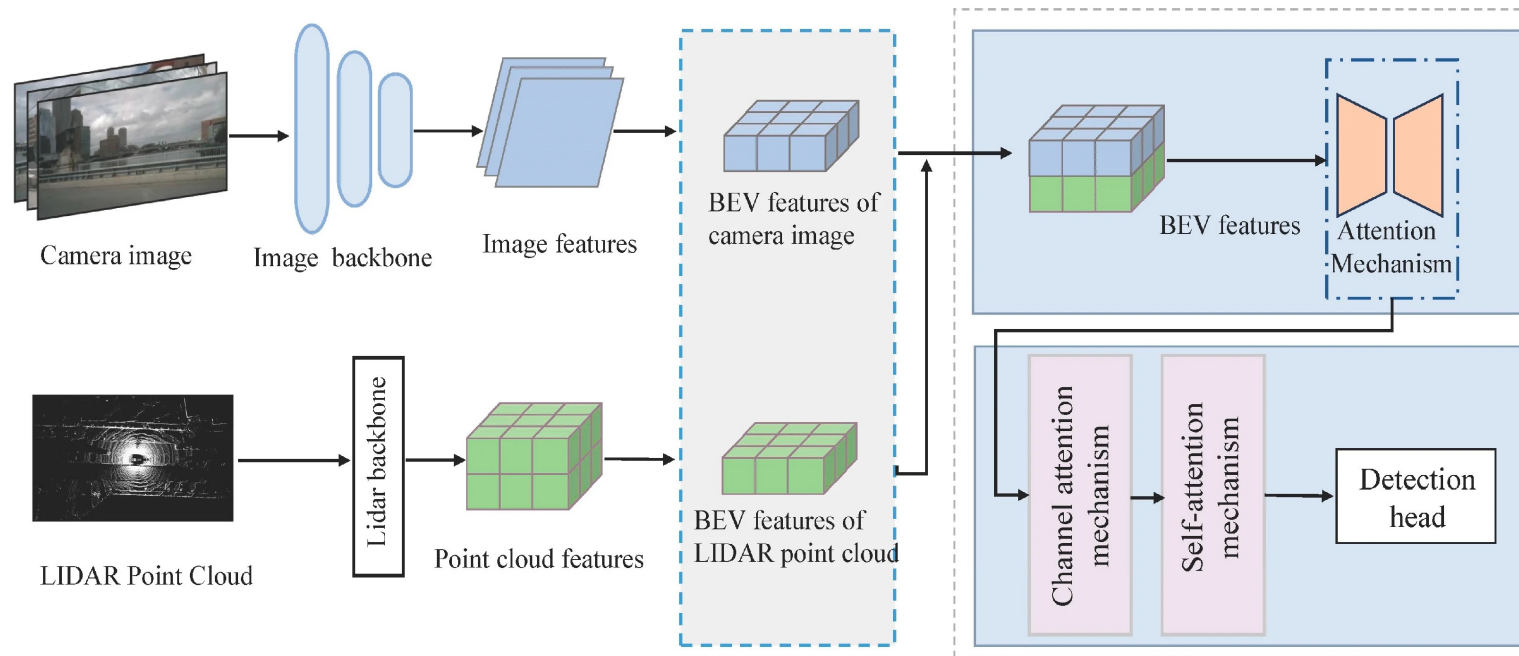
[Source: P. Shi et al., Att-BEVFusion: An Object Detection Algorithm for Camera and LiDAR Fusion Under BEV Features, *World Electr. Veh. J.* 2024, *15*(11), 539; https://doi.org/10.3390/wevj15110539]

Common problems in multi-sensor data fusion

- **Registration:** Coordinates (both time and space) of different sensors or fusion agents must be aligned.

- **Bias:** Even if the coordinate axis are aligned, due to the transformations, biases can result. These have to be compensated.

- **Correlation:** Even if the sensors are independently collecting data, processed information to be fused can be correlated.

- **Data association:** multi-target tracking problems introduce a major complexity to the fusion system.

- **Out-of-sequence measurements:** Due to delayed communications between local agents, measurements belonging to a target whose more recent measurement has already been processed, might arrive to a fusion center.

- …

Principles of Robot Autonomy I

Multi-sensor perception and sensor fusion

Daniel Watzenig