Principles of Robot Autonomy I

Camera models and camera calibration

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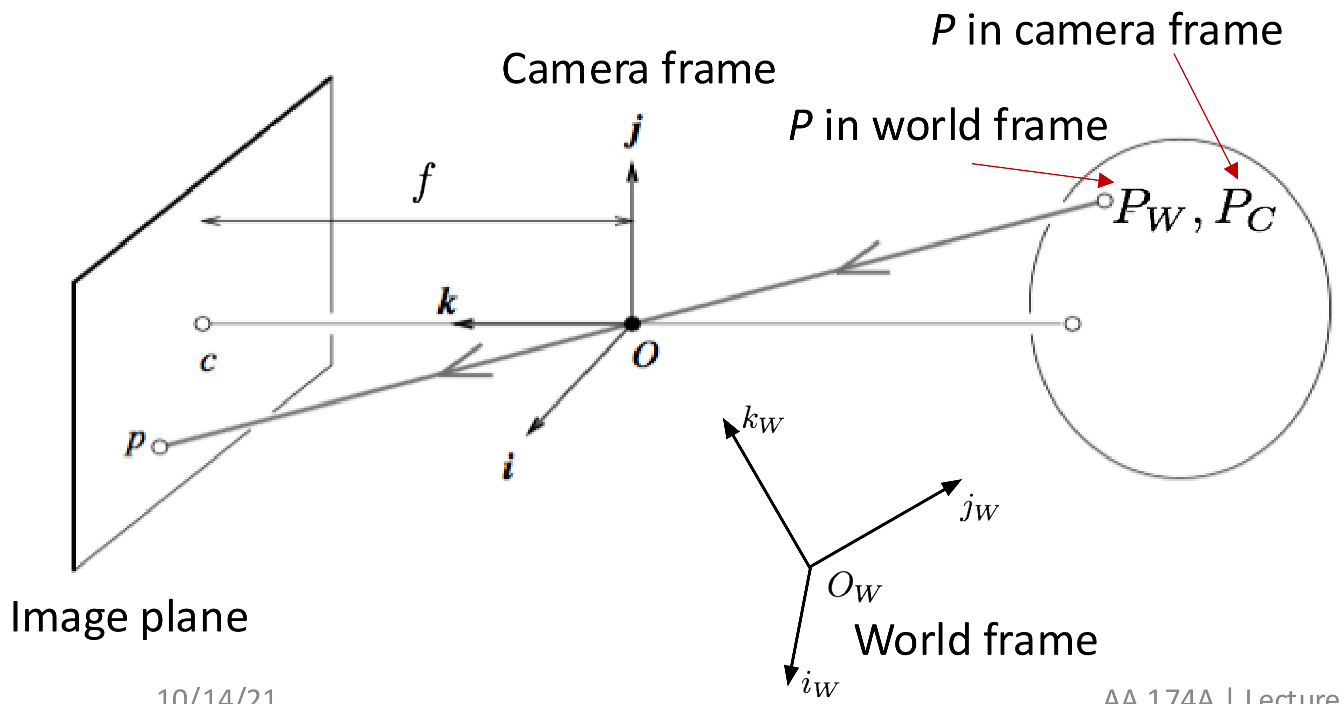
Agenda

- Agenda
  - Perspective projections
  - Camera calibration
  - Basic concepts in 3D reconstruction
- Readings:
  - Chapter 8 in PoRA lecture notes

Perspective projection

- Goal: find how world points map in the camera image
- Assumption: pinhole camera model (*all results also hold under thin lens model, assuming camera is focused at ∞*)



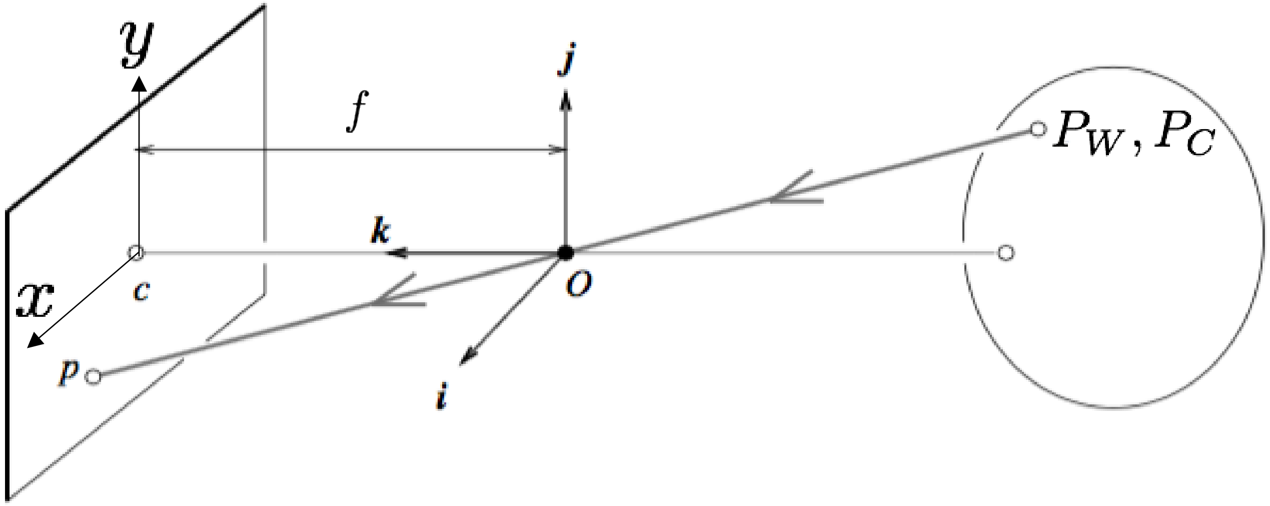
Roadmap:

1. Map P_c into *p* (image plane)
2. Map *p* into (u,v) (pixel coordinates)
3. Transform P_w into P_c

Step 1

- Task: Map $P_c = (X_C, Y_C, Z_C)$ into $p = (x, y)$ (image plane)
- From before

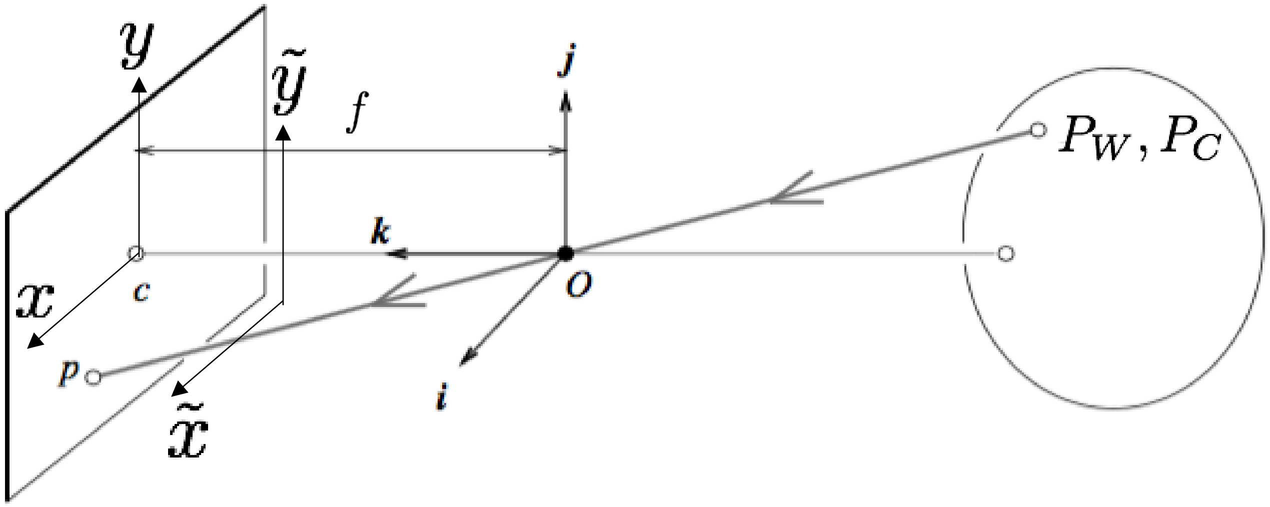
$$\begin{cases} x = f\,\frac{X_C}{Z_C} \\ y = f\,\frac{Y_C}{Z_C} \end{cases}$$

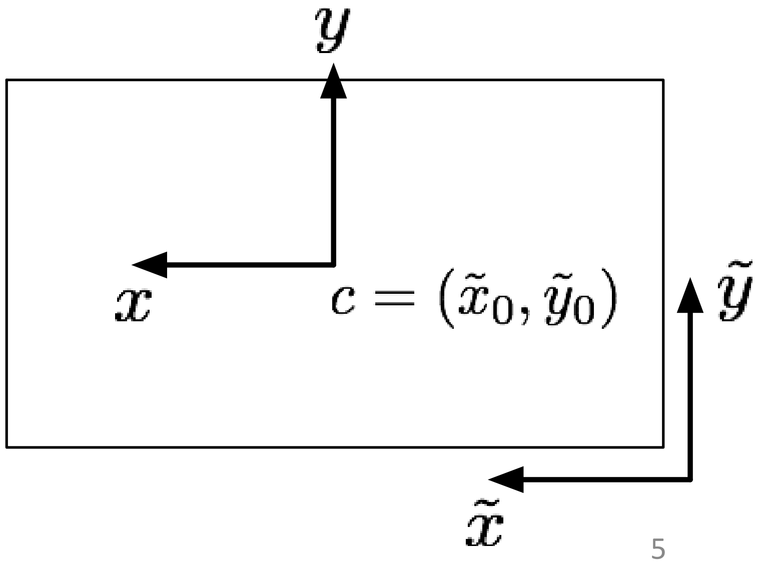


Step 2.a

- Actual origin of the camera coordinate system is usually at a corner (e.g., top left, bottom left)

$$\tilde{x} = f\,\frac{X_C}{Z_C} + \tilde{x}_0, \qquad \tilde{y} = f\,\frac{Y_C}{Z_C} + \tilde{y}_0,$$





Step 2.b

- Task: convert from image coordinates $(\tilde{x}, \tilde{y})$ to pixel coordinates (u, v)
- Let k_x and k_y be the number of pixels per unit distance in image coordinates in the *x* and *y* directions, respectively

$$u = k_x \tilde{x} = \overbrace{k_x f}^{\alpha} \frac{X_C}{Z_C} + \overbrace{k_x \tilde{x}_0}^{u_0}$$

$$v = k_y \tilde{y} = \underbrace{k_y f}_{\beta} \frac{Y_C}{Z_C} + \underbrace{k_y \tilde{y}_0}_{v_0}$$

$$\Rightarrow \quad \boxed{\begin{aligned} u &= \alpha \frac{X_C}{Z_C} + u_0 \\ v &= \beta \frac{Y_C}{Z_C} + v_0 \end{aligned}}$$

Nonlinear transformation

Homogeneous coordinates

- Goal: represent the transformation as a linear mapping
- Key idea: introduce homogeneous coordinates

Inhomogenous -> homogeneous

$$\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \lambda \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \lambda \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Homogenous -> inhomogeneous

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix} \Rightarrow \begin{pmatrix} x/w \\ y/w \end{pmatrix} \qquad \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Rightarrow \begin{pmatrix} x/w \\ y/w \\ z/w \end{pmatrix}$$

Perspective projection in homogeneous coordinates

- Projection can be equivalently written in homogeneous coordinates

$$\overbrace{\begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}}^{K}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\Bigg] \begin{pmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha X_c + u_0 Z_c \\ \beta Y_c + v_0 Z_c \\ Z_c \end{pmatrix}$$

Camera matrix/
Matrix of intrinsic parameters

P_c in homogeneous coordinates

Homogeneous pixel coordinates

- In homogeneous coordinates, the mapping is linear:

$$p^h = [K \quad 0_{3\times 1}] P_C^h$$

Point p in homogeneous pixel coordinates

Point P_c in homogeneous camera coordinates

Skewness

- In some (rare) cases

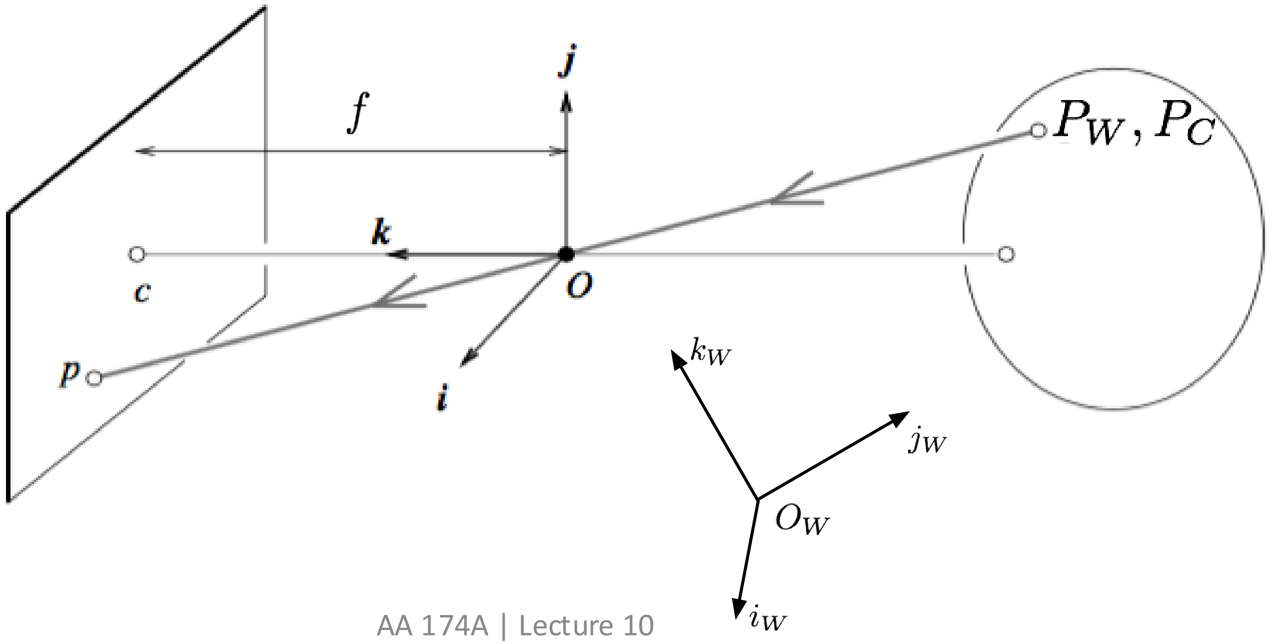
Skew parameter

$$K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

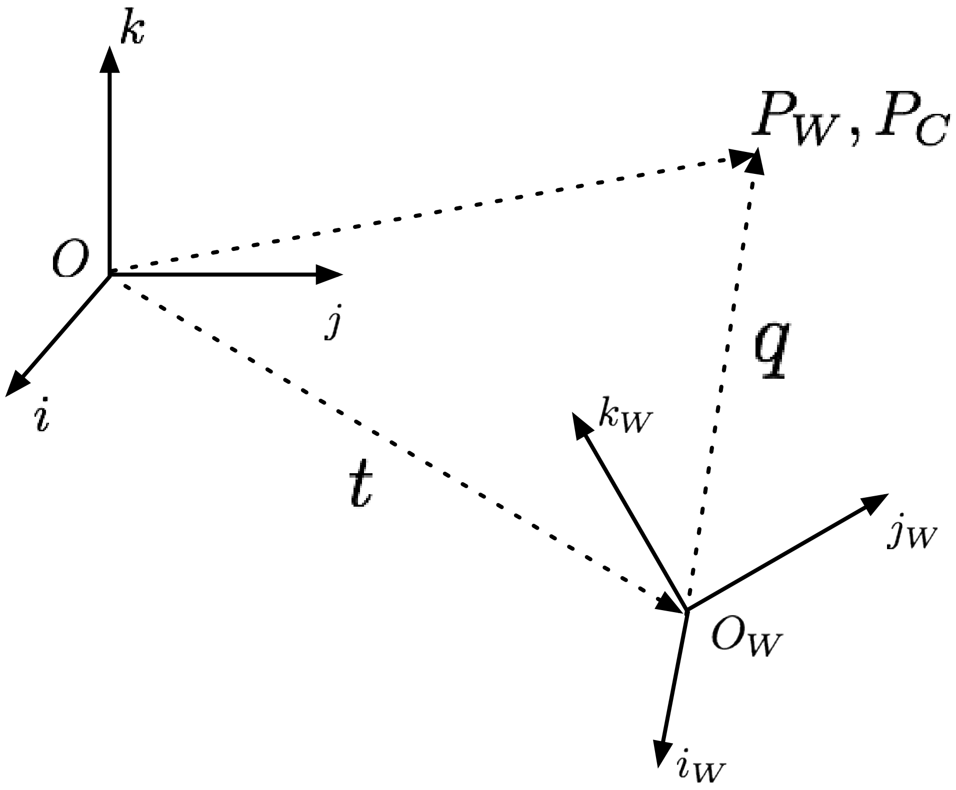
- When is $\gamma \neq 0$?
  - x- and y-axis of the camera are not perpendicular (unlikely)
  - For example, as a result of taking an image of an image
- Five parameters in total!

Step 3

- So far, we have derived a mapping between a point *P* in the 3D camera reference frame to a point *p* in the 2D image plane
- Last step is to include in our mapping an additional transformation to account for the difference between the world frame and the 3D camera reference frame



Rigid transformations



$$P_C = t + q$$

$$q = R\,P_W$$

where *R* is the rotation matrix relating camera and world frames

$$R = \begin{bmatrix} i_W \cdot i & j_W \cdot i & k_W \cdot i \\ i_W \cdot j & j_W \cdot j & k_W \cdot j \\ i_W \cdot k & j_W \cdot k & k_W \cdot k \end{bmatrix}$$

$$\boxed{\Rightarrow P_C = t + R\,P_W}$$

Rigid transformations in homogeneous coordinates

$$\begin{pmatrix} P_C \\ 1 \end{pmatrix} = \begin{bmatrix} R & t \\ 0_{1\times 3} & 1 \end{bmatrix} \begin{pmatrix} P_W \\ 1 \end{pmatrix}$$

Point P_c in homogeneous coordinates

Point P_w in homogeneous coordinates

Perspective projection equation

- Collecting all results

$$p^h = [K \quad 0_{3\times 1}]P_C^h = K[I_{3\times 3} \quad 0_{3\times 1}] \begin{bmatrix} R & t \\ 0_{1\times 3} & 1 \end{bmatrix} P_W^h$$

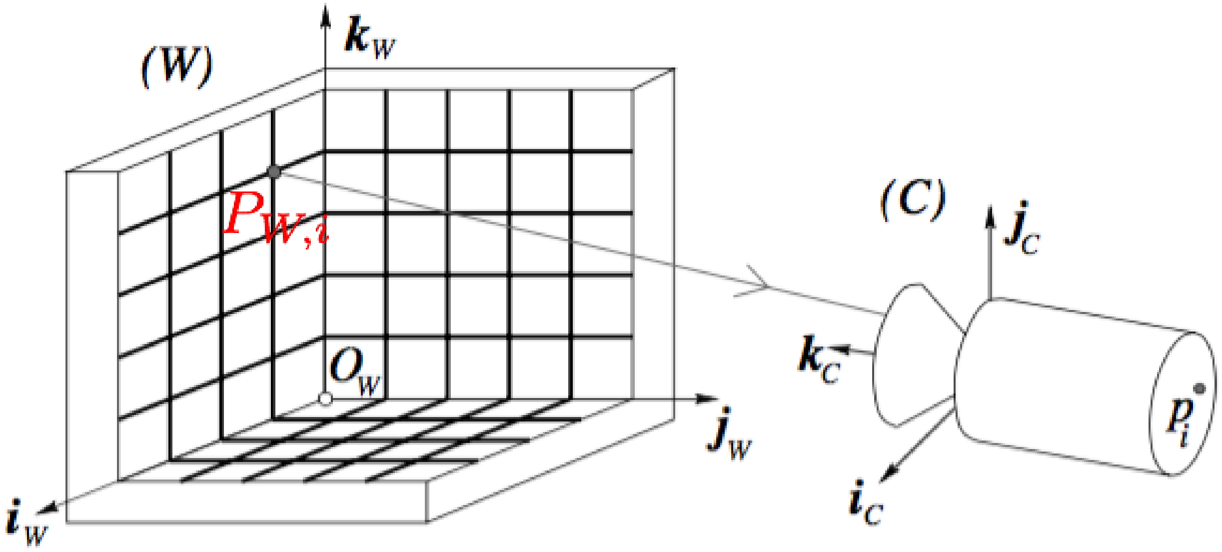
- Hence

$$\boxed{p^h = \underbrace{K}_{\text{Intrinsic parameters}} \overbrace{[\underbrace{R \quad t}_{\text{Extrinsic parameters}}]}^{\text{Projection matrix } M} P_W^h}$$

- Degrees of freedom: 4 for *K* (or 5 if we also include skewness), 3 for *R*, and 3 for *t*. Total is 10 (or 11 if we include skewness)

Camera calibration: direct linear transformation method

- Goal: find the intrinsic and extrinsic parameters of the camera



Strategy: given known correspondences $p_i \leftrightarrow P_{W,i}$, compute unknown parameters K, R, t by applying perspective projection

$P_{W,1}, P_{W,2}, \ldots, P_{W,n}$ with known positions in world frame

$p_1, p_2, \ldots, p_n$ with known positions in image frame

Step 1

- First consider combined parameters

1 × 4 vector

$$p_i^h = M\,P_{W,i}^h,\;\; i = 1,\ldots,n, \qquad \text{where} \quad M = K[R \quad t] = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

- This gives rise to $2n$ component-wise equations, for $i = 1, \ldots, n$

$$u_i = \frac{m_1 \cdot P_{W,i}^h}{m_3 \cdot P_{W,i}^h} \qquad\qquad u_i\,(m_3 \cdot P_{W,i}^h) - m_1 \cdot P_{W,i}^h = 0$$

or

$$v_i = \frac{m_2 \cdot P_{W,i}^h}{m_3 \cdot P_{W,i}^h} \qquad\qquad v_i\,(m_3 \cdot P_{W,i}^h) - m_2 \cdot P_{W,i}^h = 0$$

Calibration problem

- Stacking all equations together

$$\tilde{P}m = 0, \qquad \text{where } m = \begin{bmatrix} m_1^T \\ m_2^T \\ m_3^T \end{bmatrix}_{12 \times 1}$$

2*n* x 12 matrix of known coefficients

12 x 1 vector of unknown coefficients

- $\tilde{P}$ contains in block form the known coefficients stemming from the given correspondences
- To estimate 11 coefficients, we need at least 6 correspondences

Solution

- To find non-zero solution

$$\min_{m \in R^{12}} \quad \|\tilde{P}m\|^2$$
$$\text{subject to} \quad \|m\|^2 = 1$$

- Solution: select eigenvector of $\tilde{P}^T \tilde{P}$ with the smallest eigenvalue
- Readily computed via SVD (singular value decomposition)

Step 2

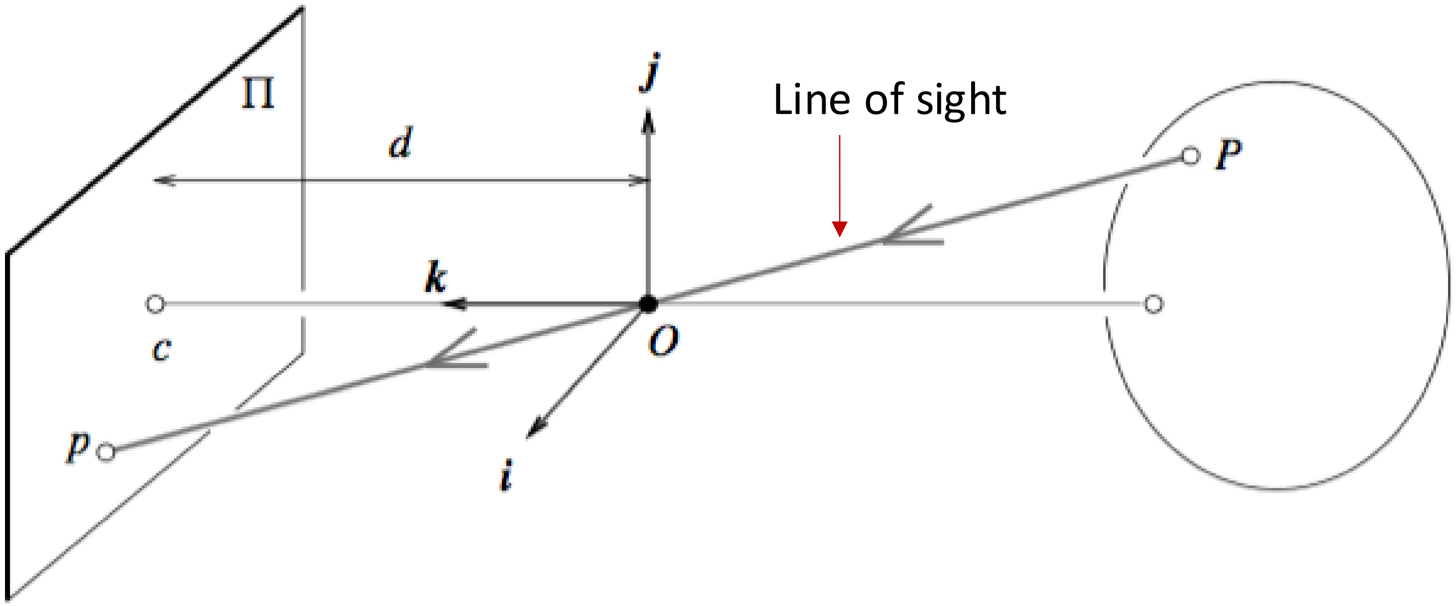
- Next, we need to extract the camera parameters, i.e., we want to factorize *M* as

$$M = [KR \quad Kt]$$

- This can be done efficiently (indeed, explicitly) by using RQ factorization, whereby the submatrix $\mathrm{M}_{1:3,1:3}$ is decomposed into the product of an upper triangular matrix *K* and a rotation matrix *R*

- These concepts will be investigated further in Problem 1 in HW3

Measuring depth



$$p^h = K[R \quad t]P_W^h$$

Homogeneous coordinates

Once the camera is calibrated, can we measure the location of a point *P* in 3D given its known observation *p*?

- No: one can only say that *P* is located *somewhere* along the line joining *p* and *O*!

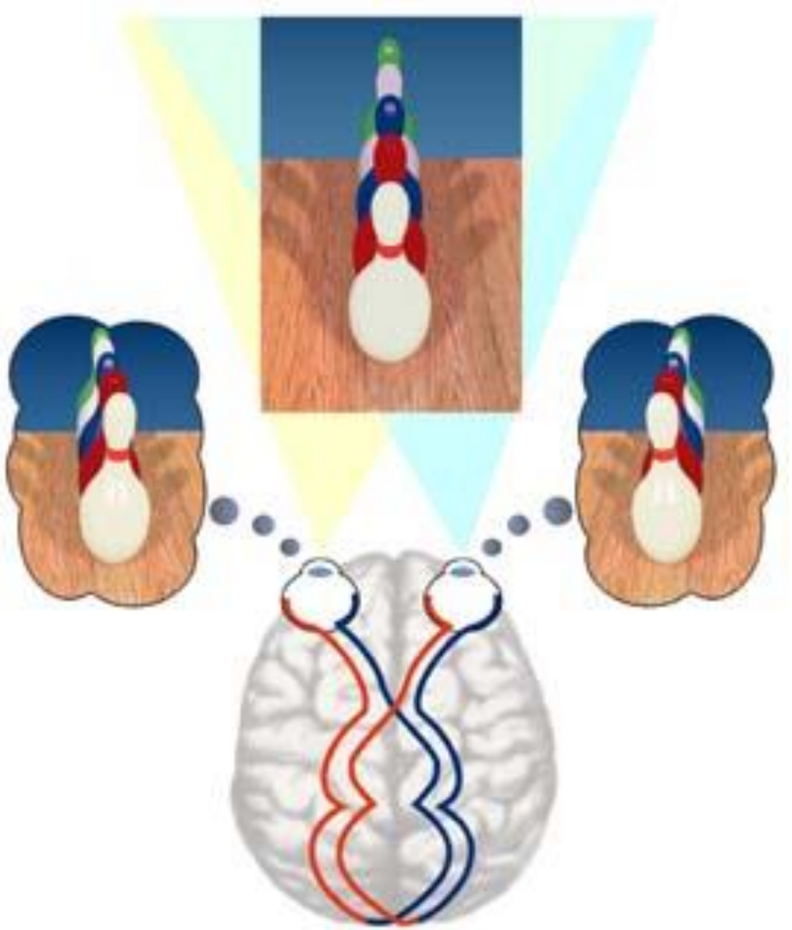
Issues with recovering structure

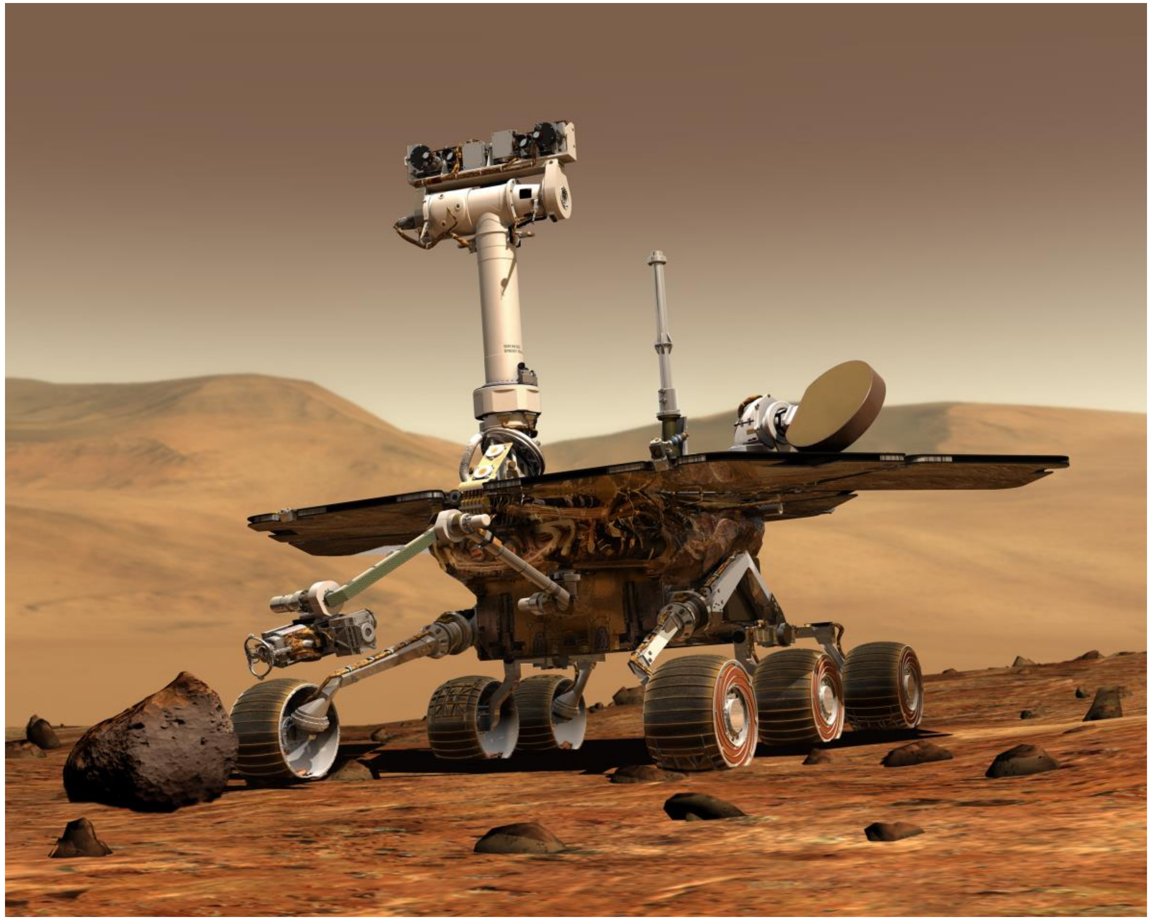
Recovering structure

- Structure: 3D scene to be reconstructed by having access to 2D images

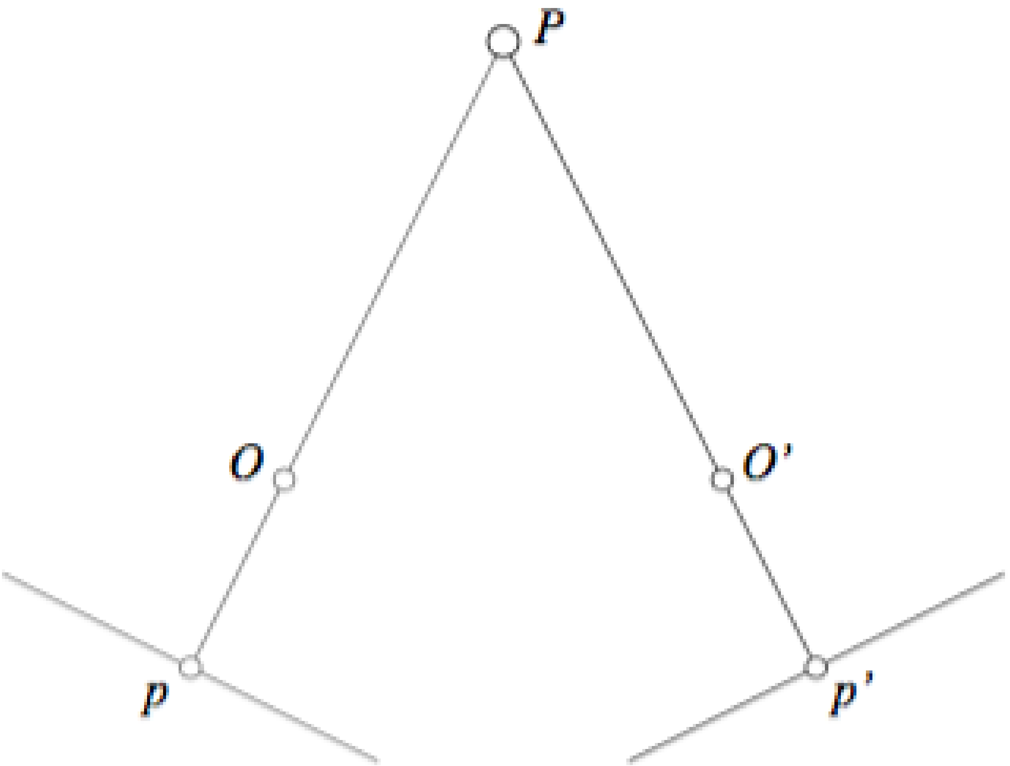
- Common methods
  1. Through recognition of landmarks (e.g., orthogonal walls)
  2. Depth from focus: determines distance to one point by taking multiple images with better and better focus
  3. Stereo vision: processes two distinct images taken at the *same time* and assumes that the relative pose between the two cameras is *known*
  4. Structure from motion: processes two images taken with the same or different cameras at *different times* and from different *unknown* positions

Stereopsis, or why we have two eyes



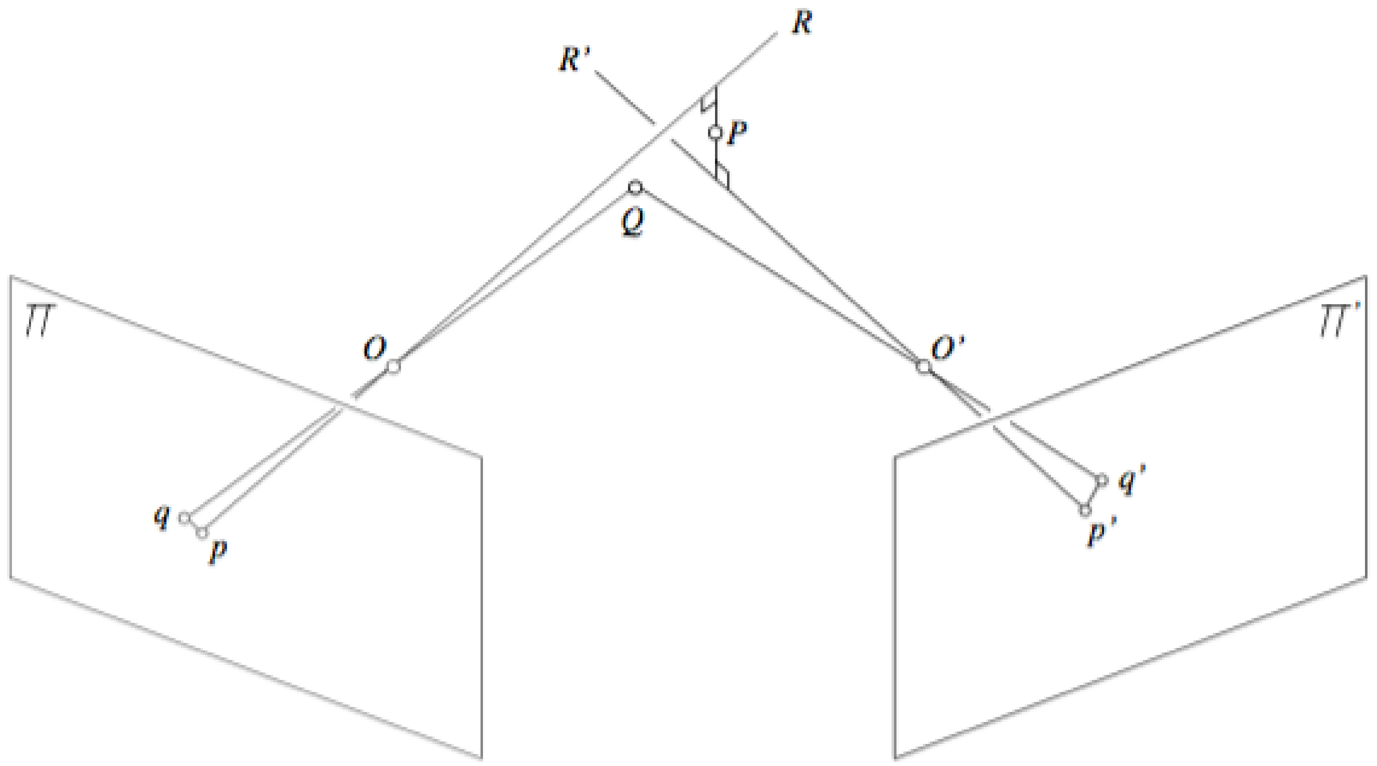


Binocular reconstruction



- Given: *calibrated* stereo rig and two image matching points p and p'
- Find corresponding scene point by intersecting the two rays $\overline{Op}$ and $\overline{O'p'}$ (process known as triangulation)

Approximate triangulation



- Due to noise, triangulation problem is often solved as finding the point Q with images q and q' that minimizes

$$\underbrace{d^2(p,q) + d^2(p',q')}_{\text{Re-projection error}}$$

Next time: image processing, feature detection & description