Principles of Robot Autonomy I
Homework 4 (Part 2)
Due Tuesday, December 3rd (11:59pm)

Starter code for this homework will all be through Google Colab. Relevant links are listed for each problem.

You will submit your homework to Gradescope. Your submission will consist of a single pdf with your answers for written questions and relevant plots from code.

Your submission must be typeset in LaTeX.

Problem 1: Kalman Filter for 2D Landmark Localization

Objective: Implement a Kalman Filter to estimate the 2D global positions of four landmarks in an environment with a known robot pose.

Setup: For this problem, we will use a Google Colab notebook linked here:
https://drive.google.com/file/d/1wZw9E_kBAHQ2MThyqKNeDTV9RGn_VZjr/view?usp=sharing.
Please make a copy of this notebook to your drive to make changes.

Introduction: In this problem, we'll consider a robot navigating through an environment with the discrete-time TurtleBot Kinematics Model:

$$\begin{aligned} x^r_{t+1} &= x^r_t + V_t \cos(\theta^r_t)\Delta t, \\ y^r_{t+1} &= y^r_t + V_t \sin(\theta^r_t)\Delta t, \\ \theta^r_{t+1} &= \theta^r_t + \omega_t \Delta t. \end{aligned} \tag{1}$$

where the robot pose is known, and is defined by the state vector $\mathbf{x}^r_t = \begin{bmatrix} x^r_t & y^r_t & \theta^r_t \end{bmatrix}$. As the robot navigates through its environment, it receives noisy measurements of the global 2D positions of four landmarks in the environment. These landmarks are stationary objects in the environment, and their state vector is defined as follows:

$$\mathbf{x}^m_t = \begin{bmatrix} x^{m1}_t & y^{m1}_t & x^{m2}_t & y^{m2}_t & x^{m3}_t & y^{m3}_t & x^{m4}_t & y^{m4}_t \end{bmatrix}^T \tag{2}$$

The measurements have an associated measurement uncertainty of $R = 0.25I$.

(i) In this problem, the state vector we aim to estimate is $\mathbf{x}^m_t$. Assuming our linear Kalman Filter dynamics function takes the form $\mathbf{x}^m_{t+1} = A\mathbf{x}^m_t$, what is matrix A?

(ii) In this problem, the state vector we aim to estimate is $\mathbf{x}^m_t$. Assuming our linear Kalman Filter measurement function takes the form $\mathbf{z}_t = C\mathbf{x}^m_t$, where $\mathbf{z}_t$ is the measurement vector of 2D global landmark positions, what is matrix C?

(iii) Using the matrices determined in Parts (i) and (ii), implement the linear Kalman Filter on the 2D global landmark positions in the associated notebook. Include all resulting plots in your writeup.

Problem 2: Extended Kalman Filter for Robot Localization

Objective: Implement an Extended Kalman Filter to estimate the pose of the robot given 2D *relative* position measurements of four landmarks in the environment.

Setup: For this problem, we will use a Google Colab notebook linked here:
https://drive.google.com/file/d/1c9LI0GgOObBvpnssYCCUPf_eDlulmwHw/view?usp=sharing.
Please make a copy of this notebook to your drive to make changes.

Introduction: In this problem, we'll consider a robot navigating through an environment with uncertain discrete-time TurtleBot Kinematics Model:

$$\begin{aligned} x^r_{t+1} &= x^r_t + V_t \cos(\theta^r_t)\Delta t + w^x_t, \\ y^r_{t+1} &= y^r_t + V_t \sin(\theta^r_t)\Delta t + w^y_t, \\ \theta^r_{t+1} &= \theta^r_t + \omega_t \Delta t + w^\theta_t. \end{aligned} \tag{3}$$

where the robot pose is unknown, and is defined by the state vector $\mathbf{x}^r_t = \begin{bmatrix} x^r_t & y^r_t & \theta^r_t \end{bmatrix}$. The noise vector $\mathbf{w}_t = \begin{bmatrix} w^x_t & w^y_t & w^\theta_t \end{bmatrix}^T$ is drawn from a zero mean Gaussian distribution $\mathbf{w}_t \sim \mathcal{N}(0, Q)$, where $Q = 0.1\Delta t^2 I$. The global ground truth positions of these landmarks are known. These landmarks are stationary objects in the environment, and their state vector is defined as in Problem 1, Eq. (2). As the robot navigates through its environment, it receives noisy measurements of the 2D positions of four landmarks in the environment relative to the robot's current pose. The measurement equation for landmark 1 is as follows:

$$\mathbf{z}^{m1}_t = \begin{bmatrix} \hat{x}^{m1}_t \\ \hat{y}^{m1}_t \end{bmatrix} = \begin{bmatrix} \cos(\theta^r_t) & \sin(\theta^r_t) \\ -\sin(\theta^r_t) & \cos(\theta^r_t) \end{bmatrix} \left(\begin{bmatrix} x^{m1}_t \\ y^{m1}_t \end{bmatrix} - \begin{bmatrix} x^r_t \\ y^r_t \end{bmatrix} \right) + \mathbf{v}^{m1}_t, \tag{4}$$

where the measurements have an associated measurement uncertainty of $\mathbf{v}^m_t \sim \mathcal{N}(0, R)$, where $R = 0.25I$. The full measurement vector of all landmarks is represented as:

$$\mathbf{z}^m_t = \begin{bmatrix} \mathbf{z}^{m1}_t \\ \mathbf{z}^{m2}_t \\ \mathbf{z}^{m3}_t \\ \mathbf{z}^{m4}_t \end{bmatrix} \tag{5}$$

(i) In this problem, the state vector we aim to estimate is $\mathbf{x}^r_t$. What is the state Jacobian of the dynamics function, $G(\mathbf{x}^r_t)$, for our EKF?

(ii) In this problem, the state vector we aim to estimate is $\mathbf{x}^r_t$. What is the Jacobian of the measurement function, $H(\mathbf{x}^r_t)$, for our EKF?

(iii) Using the matrices determined in Parts (i) and (ii), implement the Extended Kalman Filter on the 2D relative landmark positions in the associated notebook. Include all resulting plots in your writeup.