AA 174A/CS 137A/EE 160A:
Principles of Robot Autonomy I
Exam 1
November 9, 2023

Name:

SUNet ID:

Instructions:

- Time allowed: 80 minutes.
- Total Points: 48.
- The exam consists of **four** equally weighted problems, with equally weighted subproblems.
- Please read all questions carefully before answering. Correct answers will receive full credit.
- **To get partial credit for an incorrect answer, you should explain your reasoning.** Please write all your answers in the provided printout.

Good luck!

1. **State-space models (12 points)**

   (i) Consider the differential equation: $\dot{x}(t) = \cos(x(t))$, with initial condition $x(0) = 0$. If we apply Euler's method for numerical integration with a discretization step of $\Delta t = 0.1$, then $x(0.1) = x(0 + \Delta t)$ can be approximated as:

   A. 0
   B. 0.1
   C. 1
   D. $\pi/2$

   Explain:

   (ii) A linear time-invariant (LTI) model is a model that takes the form $\dot{x}(t) = Ax(t) + Bu(t)$.

   True **False**

   Explain:

   (iii) Consider the one-dimensional state-space model, $\dot{x}(t) = \sin(x(t)) + u(t)$, with the equilibrium point $(x_{\text{eq}}, u_{\text{eq}}) = (0, 0)$. Define $\delta x(t) := x(t) - x_{\text{eq}}$ and $\delta u(t) := u(t) - u_{\text{eq}}$. The linearization of the model around the aforementioned equilibrium is given by:

   A. $\frac{d}{dt}\delta x(t) = \delta x(t)$
   B. $\frac{d}{dt}\delta x(t) = \delta u(t)$
   C. $\frac{d}{dt}\delta x(t) = \delta x(t) + \delta u(t)$
   D. 0

   Explain:

2. **Trajectory Optimization / Tracking (12 points)**

(i) Consider the following discrete-time linear dynamical system

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$
$$\mathbf{x}(t+1) = A\mathbf{x}(t) + Bu(t),$$

where $\mathbf{x} \in \mathbb{R}^2$ and $u \in \mathbb{R}$. Find the input $u(0)$ that solves the following one time-step optimal control problem

$$\begin{aligned} \min_{u(0)} \quad & \mathbf{x}(1)'\mathbf{x}(1) + u(0)^2, \\ \text{s.t.} \quad & \mathbf{x}(1) = A\mathbf{x}(0) + Bu(0), \\ & \mathbf{x}(0) = \begin{bmatrix} a \\ b \end{bmatrix}. \end{aligned}$$

Note: $\mathbf{x}'$ *denotes the transpose of a vector* $\mathbf{x}$.

Answer choices:

A. $u(0) = 0$

B. $u(0) = b - a$

C. $u(0) = \frac{b-a}{2}$

D. $u(0) = \frac{b-a}{3}$

Explain:

(ii) In an open-loop control system, the control action has no dependence on the current value of the state.

True **False**

Explain:

(iii) The tracking linear quadratic regulator (tracking LQR) method can be applied to both linear and nonlinear systems (in the latter case, provided that the system is linearized).

True **False**

Explain:

3. **Motion Planning (12 points)**

(i) To find the shortest path to a goal in a 2D grid environment, we ran two different graph-based motion planning algorithms: Dijkstra's shortest path algorithm and A*. The colored nodes depict the explored nodes for each algorithm. Determine which figure corresponds to which algorithm:

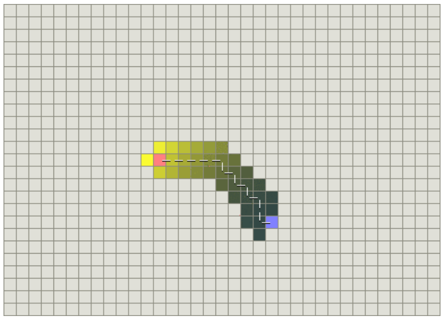


Figure 1

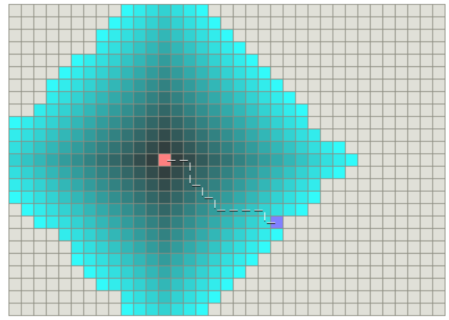


Figure 2

Dijkstra's algorithm = Figure _ (select 1 or 2)

A* = Figure _ (select 1 or 2)

Explain:

(ii) In robot path planning, graph search algorithms use a data structure, Q, to manage nodes during the search process. Match each algorithm with its corresponding data structure:

1. Depth-First Search (DFS)
2. Breadth-First Search (BFS)
3. Best-First Search (BF) / Dijkstra's

a. 'Q' is a stack – Last in/first out

b. Select next 'q' from 'Q' by minimizing a cost (i.e., $q = \text{argmin}_{q \in Q} C(q)$)

c. 'Q' is a queue – First in/first out

Answer choices:

A. 1b, 2a 3c

B. 1a, 2c, 3b

C. 1c, 2a, 3b

D. 1b, 2c, 3a

Explain:

(iii) To find a feasible path in a 3D environment with rectangular obstacles, we ran two different sampling-based motion planning algorithms: the Rapidly-exploring Random Tree (RRT) algorithm and the Probabilistic Roadmap (PRM) algorithm. Determine which figure corresponds to which algorithm:

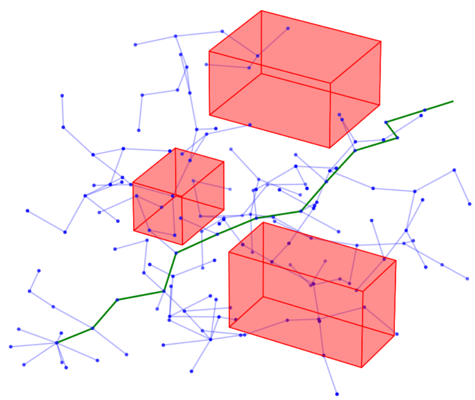


Figure 1

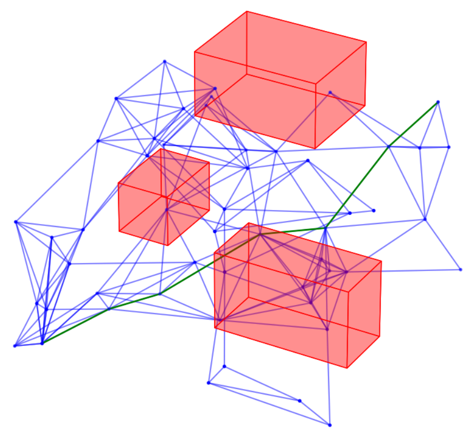


Figure 2

RRT = Figure _ (select 1 or 2)

PRM = Figure _ (select 1 or 2)

Explain:

4. **Robotic Perception (12 points)**

   (i) Determine the sensor classification for a Light Detection and Ranging (LiDAR) sensor.
   Answer choices:

   A. Passive

   B. Active

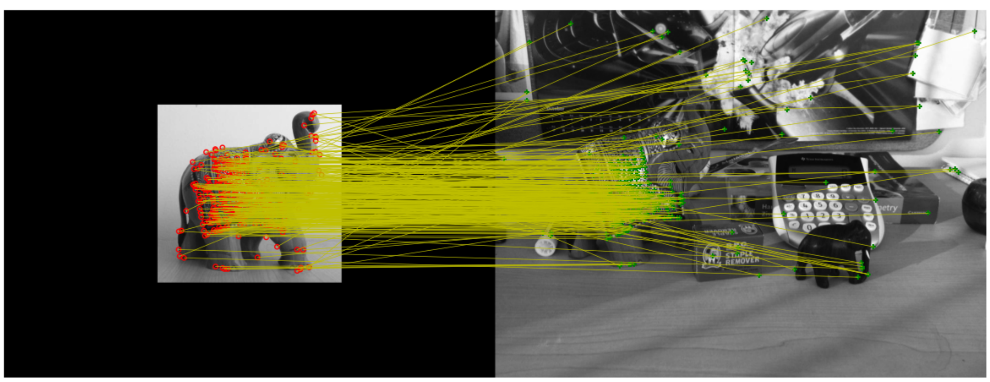
   Explain:

   (ii) Given the intrinsic parameters ($\mathbf{K}$) and extrinsic parameters ($\mathbf{R}$, $\mathbf{t}$), the *pinhole camera model* can be used to a project a 2D pixel coordinate into its corresponding 3D world point.

   True **False**

   Explain:

   (iii) In feature-based object detection, you are given an object in a source image and would like to localize the same object in a target image (see the figure below). After detecting keypoints and matching their features across both images, you observe that many of the keypoint matches are noisy outliers. Select the appropriate method to remove the outlier keypoint matches:



Matched keypoints in source image (left) and target image (right).

   Answer choices:

   A. Perform template matching

   B. Use the Random Sample Consensus (RANSAC) algorithm

   C. Correlate a Gaussian filter and rematch keypoints

   D. Perform Canny edge detection and rematch keypoints

   Explain: