

Principles of Robot Autonomy I

Simultaneous Localization and Mapping (SLAM)

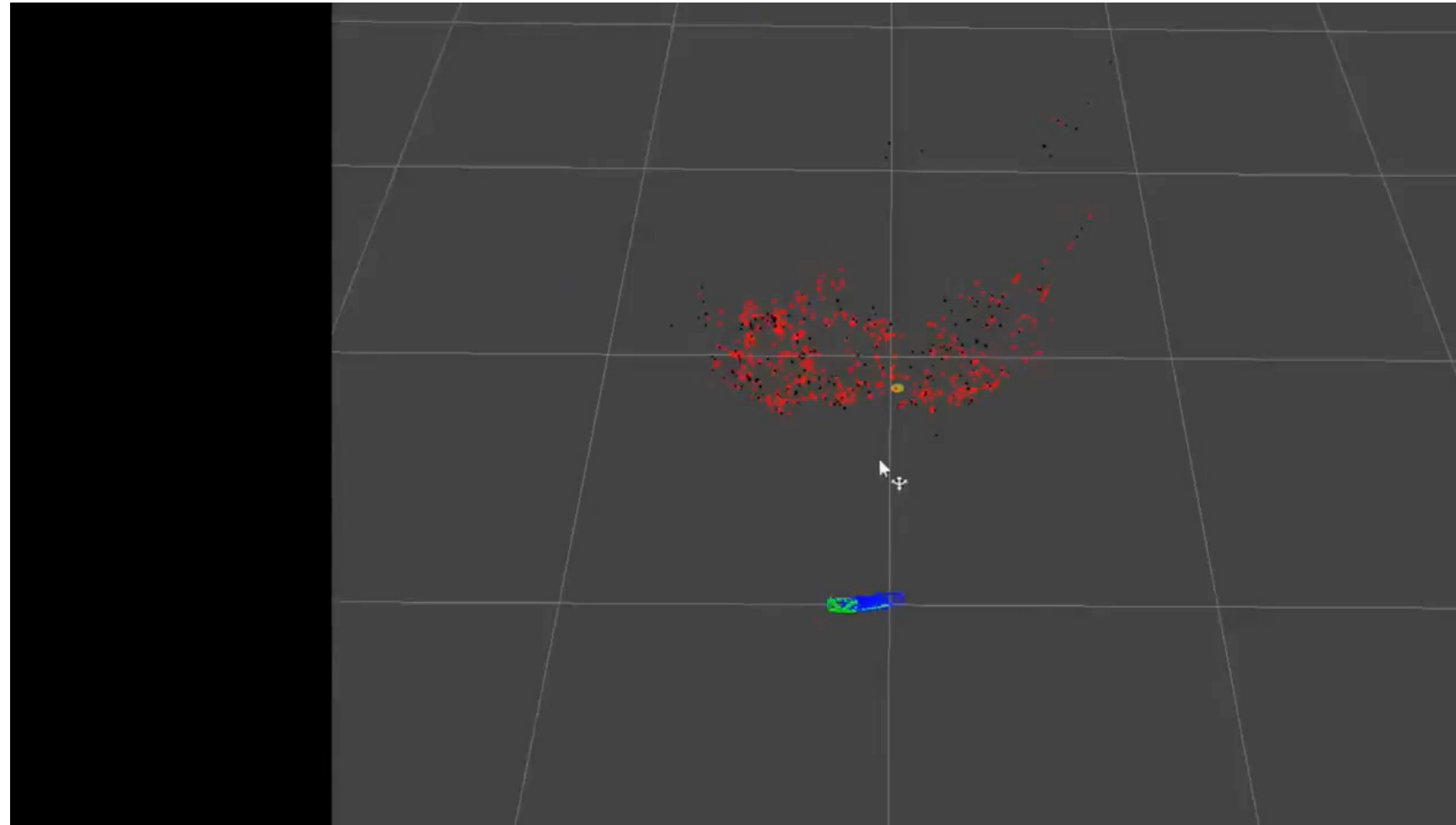


Agenda

- Aim
 - General SLAM problem
 - EKF SLAM
- Readings
 - Chapter 17 in PoRA lecture notes

Simultaneous Localization and Mapping

The SLAM problem:
given measurements $z_{1:t}$ and controls $u_{1:t}$,
find the path (or pose) of the robot and
acquire a map of the environment



Forms of SLAM

- **Online SLAM problem:** estimate the posterior over the momentary pose along with the map

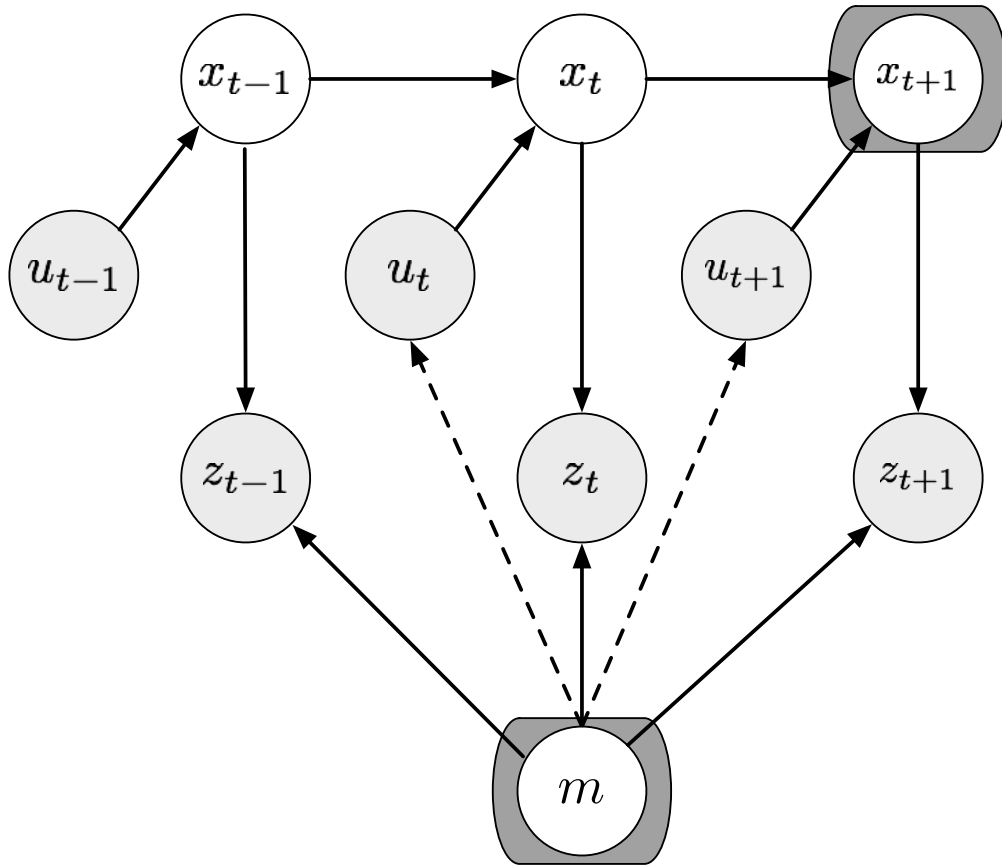
$$p(x_t, m \mid z_{1:t}, u_{1:t}) \quad \text{or} \quad p(x_t, m, c_t \mid z_{1:t}, u_{1:t})$$

- **Full SLAM problem:** estimate posterior over the entire path along with the map

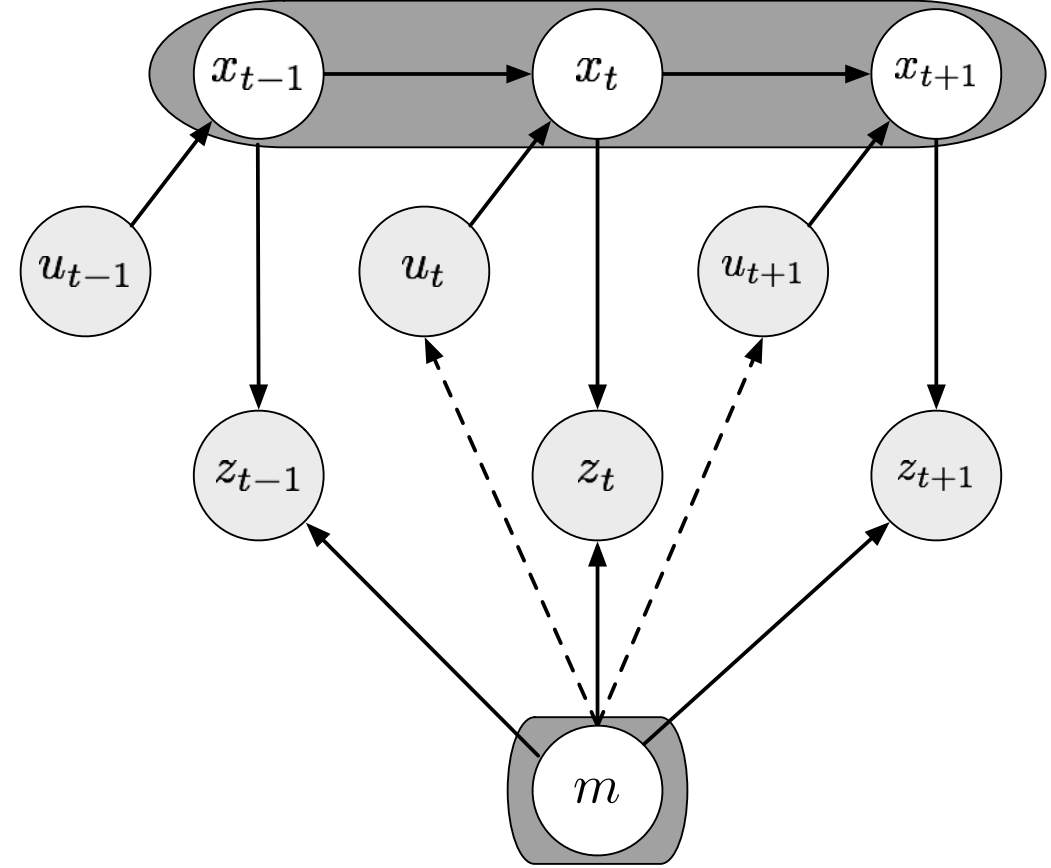
$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t}) \quad \text{or} \quad p(x_{1:t}, m, c_t \mid z_{1:t}, u_{1:t})$$

Graphical models of SLAM

Online SLAM



Full SLAM



EKF SLAM

- Historically the earliest SLAM algorithm
- **Key idea:** apply EKF to online SLAM using maximum likelihood data association
- Assumptions:
 1. Gaussian assumption for motion and perception noise, and Gaussian approximation for belief (essential)
 2. Feature-based maps (essential)
- Two versions of the problem
 1. Correspondence variables are known
 2. Correspondence variables are not known (usual case)

EKF SLAM with known correspondences

- Similar to EKF localization algorithm with known correspondences
- **Key difference**: in addition to estimate the robot pose x_t , the EKF SLAM algorithm also estimates the coordinates of **all** landmarks
- Define combined state vector

$$y_t := \begin{pmatrix} x_t \\ m \end{pmatrix} = (x, y, \theta, m_{1,x}, m_{1,y}, m_{2,x}, m_{2,y} \dots m_{N,x}, m_{N,y})^T$$

3 + 2N vector

- **Goal**: calculate the **online posterior**

$$p(y_t \mid z_{1:t}, u_{1:t})$$

Motion and sensing model

- (Following discussion is for illustration purposes; setup can be generalized to other motion and sensing models)
- Assume motion model with state $x_t = (x, y, \theta)$

$$y_t = g(u_t, y_{t-1}) + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, R_t), \quad G_t := J_g(u_t, \mu_{t-1})$$

where we assume that the landmarks are *static*, that is

1. $g(u_t, y_{t-1})$ is a $3+2N$ vector, whose last $2N$ components are the same as those in y_{t-1}
2. R_t has zero entries, except for the top left 3×3 block

Motion and sensing model

- Assume range and bearing measurement model

$$z_t^i = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \end{pmatrix}}_{:=h(y_t, j)} + \delta_t, \quad \delta_t \sim \mathcal{N}(0, Q_t), \quad Q_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{pmatrix}$$

- Usual linear approximation for sensing model (with $j = c_t^i$)

$$h(y_t, j) \approx h(\bar{\mu}_t, j) + H_t^i (y_t - \bar{\mu}_t), \quad \text{where } H_t^i := \frac{\partial h(\bar{\mu}_t, j)}{\partial y_t}$$

- Since h depends only on x_t and m_j , H_t^i can be factored as

$$H_t^i = h_t^i F_{x,j}$$

Motion and sensing model

- First term, a 2 x 5 matrix, is the Jacobian of $h(y_t, j)$ at $\bar{\mu}_t$ w.r.t. x_t and m_j :

$$h_t^i = \frac{\partial h(\bar{\mu}_t, j)}{\partial(x_t, m_j)} = \begin{pmatrix} \frac{\bar{\mu}_{t,x} - \bar{\mu}_{j,x}}{\sqrt{q_{t,j}}} & \frac{\bar{\mu}_{t,y} - \bar{\mu}_{j,y}}{\sqrt{q_{t,j}}} & 0 & \frac{\bar{\mu}_{j,x} - \bar{\mu}_{t,x}}{\sqrt{q_{t,j}}} & \frac{\bar{\mu}_{j,y} - \bar{\mu}_{t,y}}{\sqrt{q_{t,j}}} \\ \frac{\bar{\mu}_{j,y} - \bar{\mu}_{t,y}}{q_{t,j}} & \frac{\bar{\mu}_{t,x} - \bar{\mu}_{j,x}}{q_{t,j}} & -1 & \frac{\bar{\mu}_{t,y} - \bar{\mu}_{j,y}}{q_{t,j}} & \frac{\bar{\mu}_{j,x} - \bar{\mu}_{t,x}}{q_{t,j}} \end{pmatrix}$$

where $q_{t,j} := (\bar{\mu}_{j,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y} - \bar{\mu}_{t,y})^2$

- Second term, a 5 x (3+2N) matrix, maps h_t^i into H_t^i :

$$F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{2j-2} & 0 & 1 & \underbrace{0 \cdots 0}_{2N-2j} \end{pmatrix}$$

Initialization

- Initial belief expressed as

$$\mu_0 = (0, 0, 0 \dots 0)^T$$

Initialization
for pose
variables

$\Sigma_0 =$

$(3+2N) \times (3+2N)$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \infty & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \infty \end{pmatrix}$$

Initialization

- When a landmark is observed for the first time, the landmark estimate $(\bar{\mu}_{j,x}, \bar{\mu}_{j,y})^T$ is initialized with the expected position, that is

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{pmatrix}$$

EKF SLAM algorithm

- Similar to EKF localization; main differences:
 - Augmented state vector
 - Augmented dynamics (with trivial dynamics for the landmarks)
 - Initialization of unseen landmarks
 - Augmented measurement Jacobian

Data: $(\mu_{t-1}, \Sigma_{t-1}), u_t, z_t, c_t$

Result: (μ_t, Σ_t)

$\bar{\mu}_t = g(u_t, \mu_{t-1});$

$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t;$

foreach $z_t^i = (r_t^i, \phi_t^i)^T$ **do**

$j = c_t^i;$

if landmark j never seen before **then**

$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{pmatrix};$

end

$\hat{z}_t^i = \begin{pmatrix} \sqrt{(\bar{\mu}_{j,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y} - \bar{\mu}_{t,y})^2} \\ \text{atan2}(\bar{\mu}_{j,y} - \bar{\mu}_{t,y}, \bar{\mu}_{j,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix};$

$H_t^i = h_t^i F_{x,j};$

$S_t^i = H_t^i \bar{\Sigma}_t [H_t^i]^T + Q_t;$

$K_t^i = \bar{\Sigma}_t [H_t^i]^T [S_t^i]^{-1};$

$\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i);$

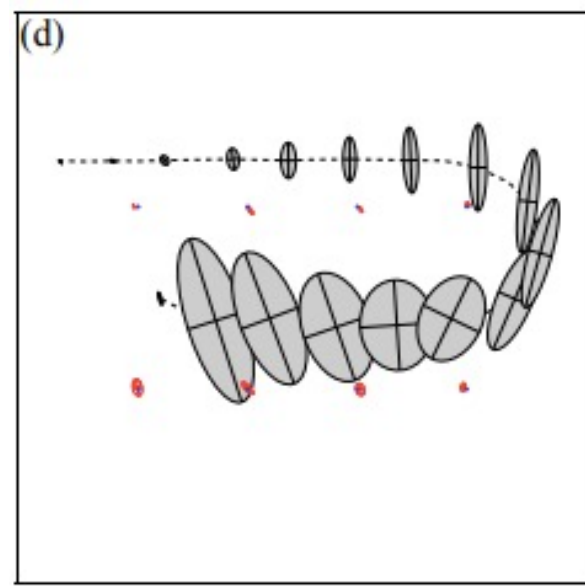
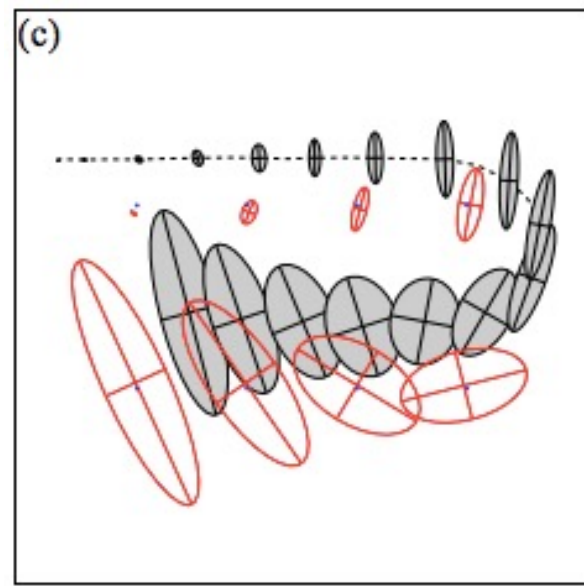
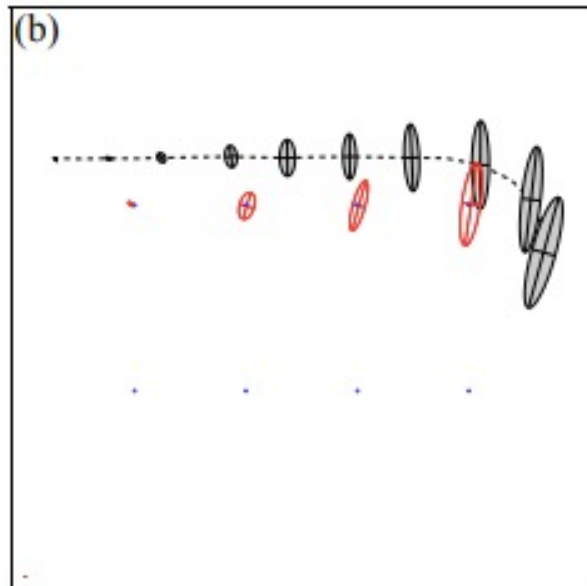
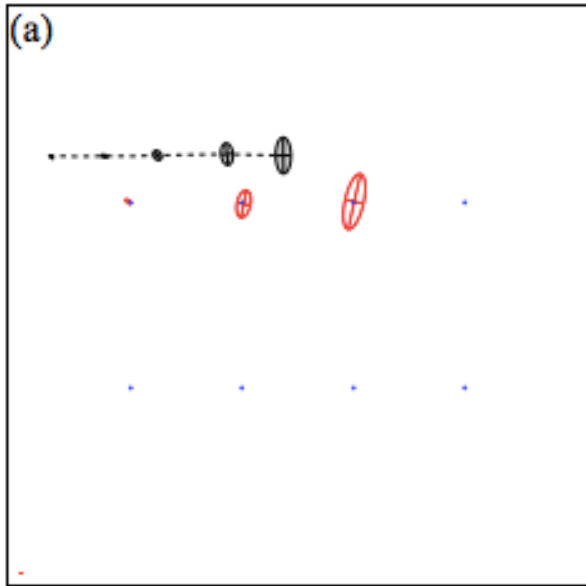
$\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t;$

end

$\mu_t = \bar{\mu}_t$ and $\Sigma_t = \bar{\Sigma}_t;$

Return (μ_t, Σ_t)

Example



EKF SLAM with unknown correspondences

- **Key idea:** use an incremental maximum likelihood estimator to determine correspondences
- Similar to EKF localization with unknown correspondences, but now we also need to create hypotheses for new landmarks
- **Caveat:** maximum likelihood data association often makes the algorithm brittle, as it is not possible to revise past data associations

EKF SLAM with unknown correspondences

- In the measurement update loop, we first create the hypothesis of a new landmark
- A new landmark is created if the Mahalanobis distance to all existing landmarks exceeds the value α

Data: $(\mu_{t-1}, \Sigma_{t-1}), u_t, z_t, N_{t-1}$

Result: (μ_t, Σ_t)

$N_t = N_{t-1};$

$\bar{\mu}_t = g(u_t, \mu_{t-1});$

$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t;$

Hypothesis
for new
landmark

foreach $z_t^i = (r_t^i, \phi_t^i)^T$ **do**

$\begin{pmatrix} \bar{\mu}_{N_t+1,x} \\ \bar{\mu}_{N_t+1,y} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{pmatrix};$

for $k = 1$ **to** $N_t + 1$ **do**

$\hat{z}_t^k = \begin{pmatrix} \sqrt{(\bar{\mu}_{j,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y} - \bar{\mu}_{t,y})^2} \\ \text{atan2}(\bar{\mu}_{j,y} - \bar{\mu}_{t,y}, \bar{\mu}_{j,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix};$

$H_t^k = h_t^k F_{x,k};$

$S_t^k = H_t^k \bar{\Sigma}_t [H_t^k]^T + Q_t;$

$\pi_k = (z_t^i - \hat{z}_t^k)^T [S_t^k]^{-1} (z_t^i - \hat{z}_t^k);$

end

Mahalanobis
distance

$\pi_{N_t+1} = \alpha;$

$j(i) = \text{argmin}_k \pi_k;$

Hypothesis test

$N_t = \max\{N_t, j(i)\};$

$K_t^i = \bar{\Sigma}_t [H_t^{j(i)}]^T [S_t^{j(i)}]^{-1};$

$\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^{j(i)});$

$\bar{\Sigma}_t = (I - K_t^i H_t^{j(i)}) \bar{\Sigma}_t;$

end

$\mu_t = \bar{\mu}_t$ and $\Sigma_t = \bar{\Sigma}_t;$

Return (μ_t, Σ_t)

Making EKF SLAM robust

- A key issue is represented by the fact that **fake landmarks** might be created; furthermore, EKF can **diverge** if nonlinearities are large
- Several techniques exist to mitigate such issues
 1. Outlier rejection schemes, for example via provisional landmark lists
 2. Strategies to enhance the distinctiveness of landmarks
 - Spatial arrangement
 - Signatures
 - Enforcing geometric constraints
- **Dilemma of EKF SLAM**: accurate localization typically requires dense maps, but EKF requires sparse maps due to quadratic update complexity

Summary: Gaussian filtering

- **Key ideas:**
 - Represent a belief with a Gaussian distribution
 - Assume all uncertainty sources are Gaussian
- **Pros:**
 - Runs online
 - Well understood
 - Works well when uncertainty is low
- **Cons:**
 - Unimodal estimate
 - States must be well approximated by a Gaussian
 - Works poorly when uncertainty is high

Final considerations

- **A recent overview of SLAM (with strong focus on graph SLAM):** C. Cadena, L. Carlone, H. Carrillo, Y. Latif, D. Scaramuzza, J. Neira, I. Reid, and J. J. Leonard. "Past, present, and future of simultaneous localization and mapping: Toward the robust-perception age." IEEE Transactions on Robotics 32, no. 6 (2016): 1309-1332.
- **Popular software packages**
 - <https://www.openslam.org/>: comprehensive list of open-source SLAM software
 - <https://github.com/pamela-project/slambench>: popular benchmark framework
 - Commercial SDKs: ARCore/ARKit from Google/Apple, Oculus Insight

Thanks for a great quarter!